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# COUPLED NEUTRONICS AND THERMAL-HYDRAULICS TRANSIENT CALCULATIONS BASED ON A FISSION MATRIX APPROACH: APPLICATION TO THE MOLTEN SALT FAST REACTOR

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# SUMMARY

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## INTRODUCTION: OBJECTIVE OF THE CURRENT DEVELOPMENTS

### I. TRANSIENT FISSION MATRIX

- PRESENTATION
- TFM KINETICS EQUATIONS
- KINETICS PARAMETERS CALCULATION
- TFM SIMPLIFIED KINETICS EQUATIONS

### II. GENERAL COUPLING STRATEGY

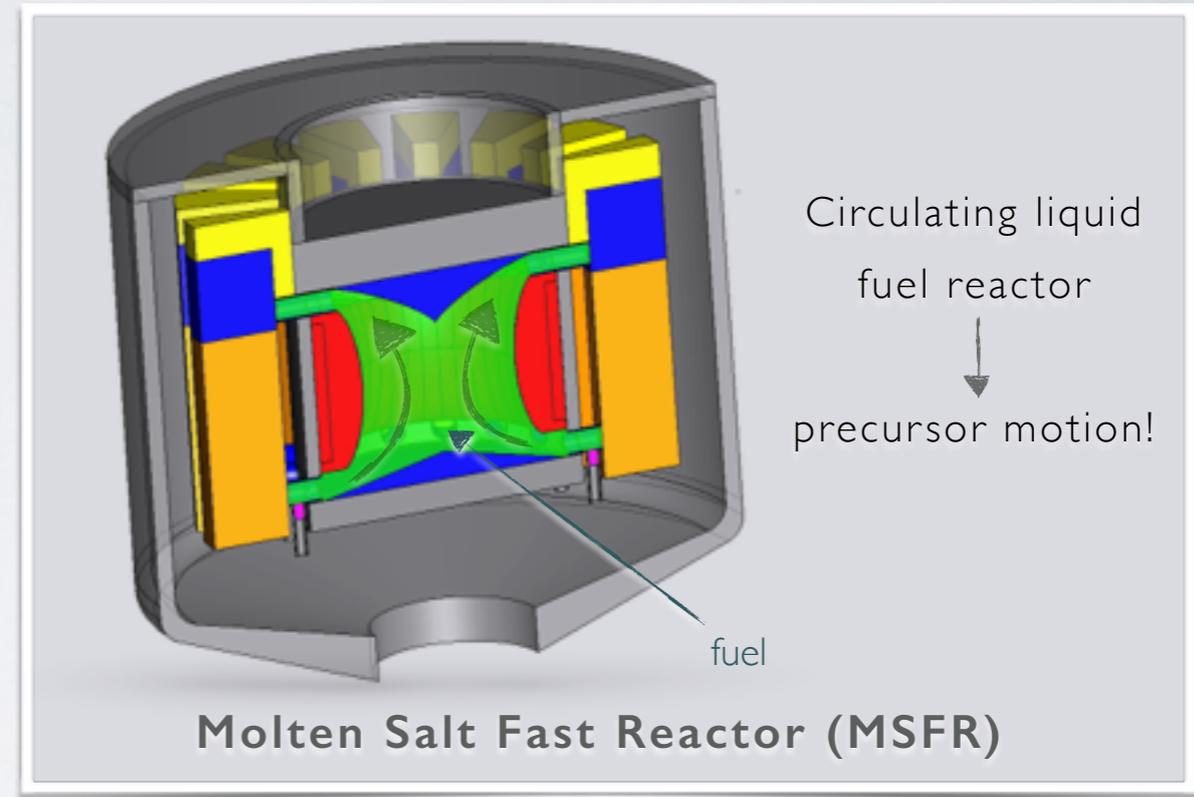
### III. APPLICATION CASES

- MSFR PRESENTATION
- OVER COOLING TRANSIENT CALCULATION
- REACTIVITY INSERTION

# INTRODUCTION: OBJECTIVE OF THE CURRENT DEVELOPMENTS

## Context:

- Need to perform transient calculations for the MSFR
  - neutronics / thermal-hydraulics coupling



## Objectives:

- with a high precision of the T&H modeling (flow distribution, precursor transport, ...)
  - CFD code (OpenFOAM)
- with a high precision of the neutronics modeling
  - Monte Carlo code (MCNP and SERPENT) ...
- ... with a low computational cost (need to perform many cases)
  - Diffusion? Improved point kinetics? ... something else?

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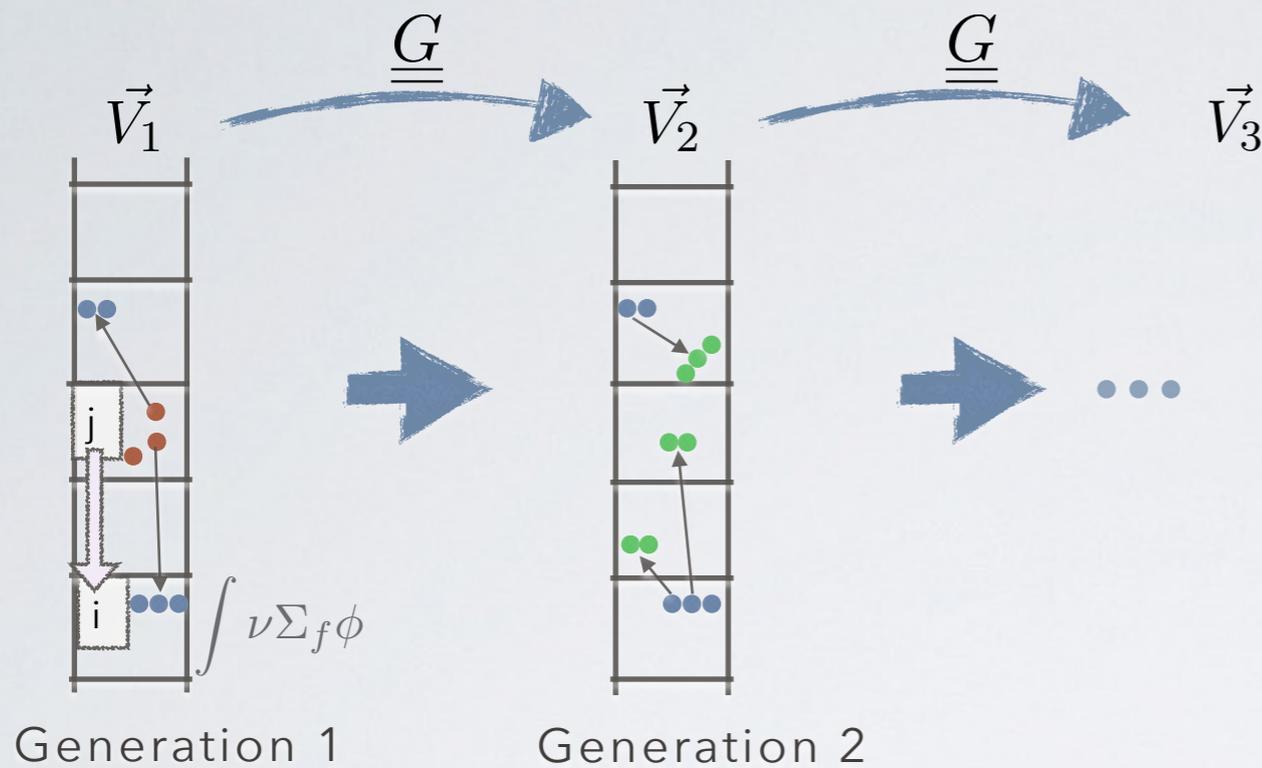
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# TRANSIENT FISSION MATRIX: PRESENTATION



OVERALL PRINCIPLE:  
CHARACTERIZE THE SYSTEM RESPONSE  
ON ONE GENERATION (GREEN FUNCTION)

step 1: Matrix element  $ij$ : volume  $i$  neutron production  
probability induced by an incoming source neutron injected in  $j$

↓ IFM

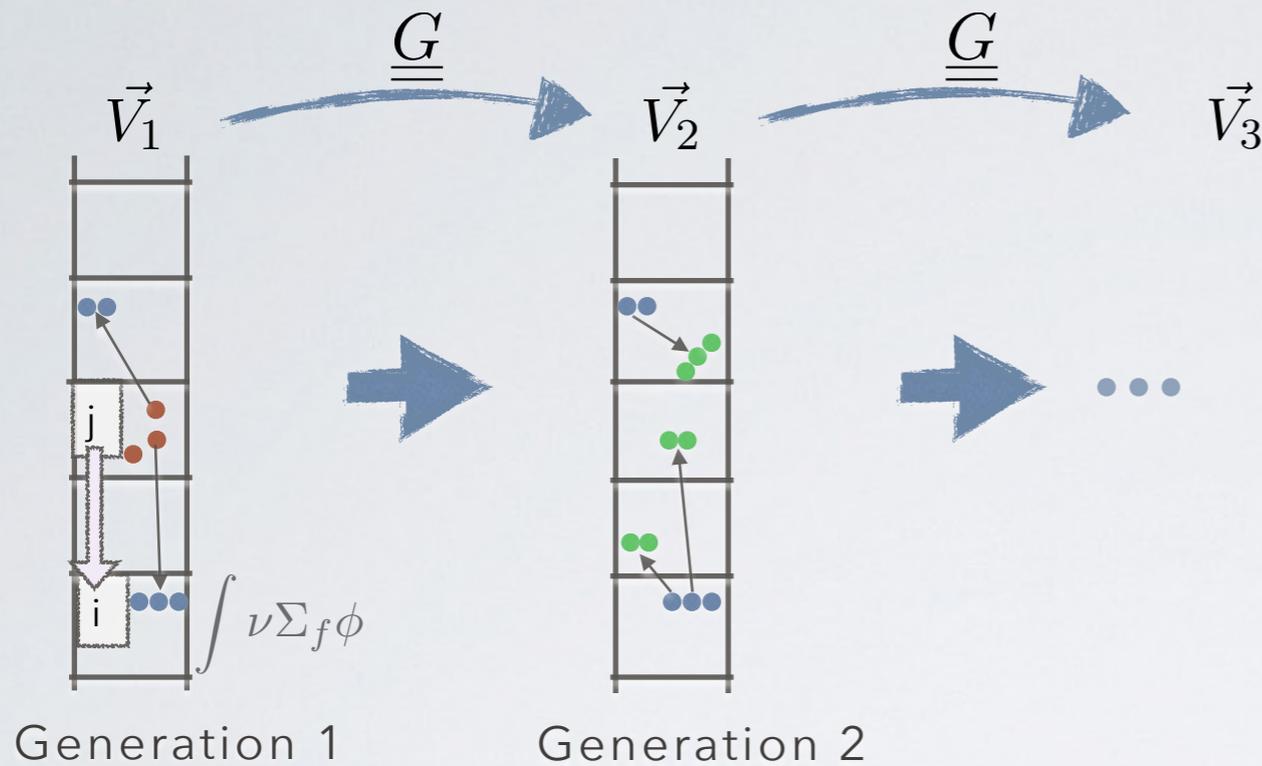
step 2: Discretization of this matrix through time to get  
the temporal response

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step 3: Interpolation (Doppler & density feedback effects)

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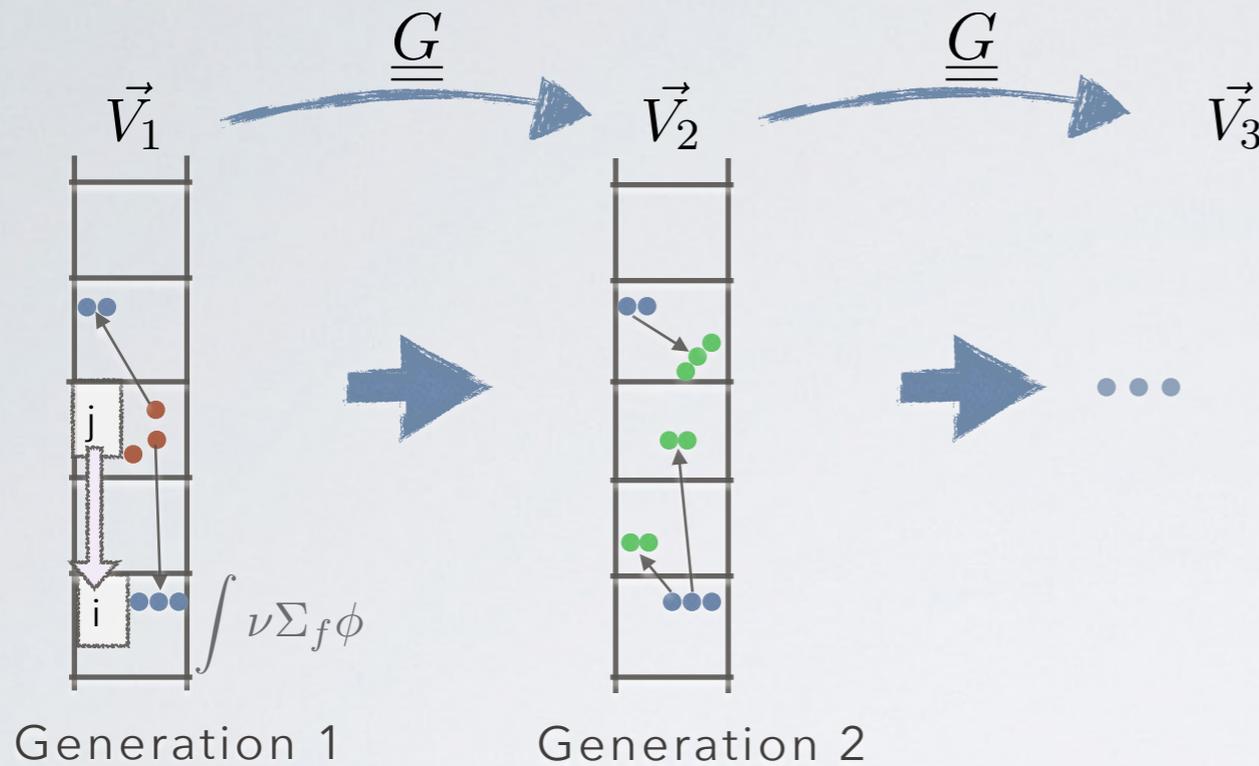
With  $S(\mathbf{r}, t)$  the prompt source neutron distribution rate at time  $t$  in  $\mathbf{r}$

With  $G_{\chi_p \nu_p}(t' - t, \mathbf{r}', \mathbf{r})$  the continuous operator associated to the transient fission matrix:  
 the probability that a neutron created in  $\mathbf{r}', t'$  induces a new neutron in  $\mathbf{r}, t$

prompt emission spectrum      prompt production

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prompt emission spectrum

prompt production

The kinetics of a prompt neutron population is given by:

$$S(\mathbf{r}, t) = \left| G_{\chi_p \nu_p}(t' - t, \mathbf{r}', \mathbf{r}) \left| S(\mathbf{r}', t') \right. \right\rangle = \iint_{t' < t, \mathbf{r}' \in \mathcal{R}} G_{\chi_p \nu_p}(t' - t, \mathbf{r}', \mathbf{r}) \cdot S(\mathbf{r}', t') d\mathbf{r}' dt'$$

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# TRANSIENT FISSION MATRIX: KINETICS EQUATIONS

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AND WITH THE DELAYED NEUTRON PRECURSORS:

- Prompt source neutron distribution rate

$$S(t, \mathbf{r}) = \overset{\text{prompt} \longleftarrow}{\left| G_{\chi_p \nu_p}(t - t', \mathbf{r}', \mathbf{r}) \right| S(t', \mathbf{r}') \rangle} + \overset{\text{prompt} \longleftarrow}{\left| G_{\chi_d \nu_p}(t - t', \mathbf{r}', \mathbf{r}) \right| \sum_f \lambda_f P_f(t', \mathbf{r}') \rangle}$$

- Precursor family  $f$

$$\frac{dP_f}{dt}(t, \mathbf{r}) = \overset{\text{delayed} \longleftarrow}{\frac{\beta_f}{\beta_0} \left[ \left| G_{\chi_p \nu_d}(t - t', \mathbf{r}', \mathbf{r}) \right| S(t', \mathbf{r}') \rangle + \left| G_{\chi_d \nu_d}(t - t', \mathbf{r}', \mathbf{r}) \right| \sum_f \lambda_f P_f(t', \mathbf{r}') \rangle \right]} - \overset{\text{decay constant}}{\lambda_f P_f}$$

↑ family ratio
↑

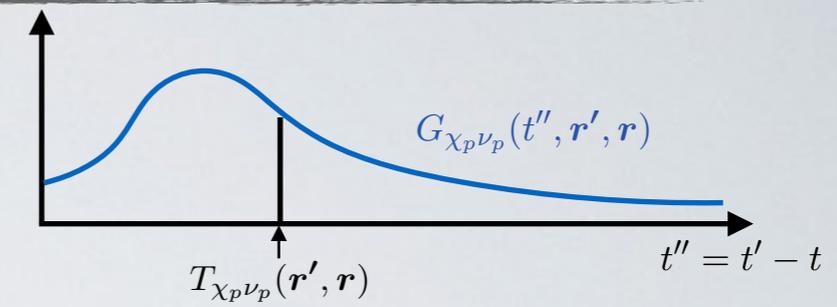
The time integration requires one matrix-vector product by time discretization of the Green operators...

→ Too long for our objective

# I. TRANSIENT FISSION MATRIX: KINETICS PARAMETERS CALCULATION

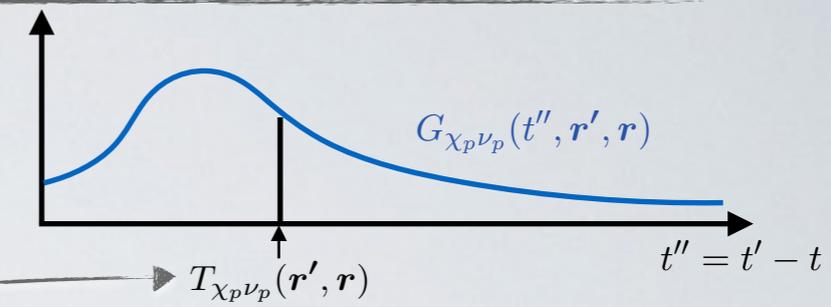
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EFFECTIVE LIFE TIME  $l_{eff}$  CALCULATION:



# I. TRANSIENT FISSION MATRIX: KINETICS PARAMETERS CALCULATION

EFFECTIVE LIFE TIME  $l_{eff}$  CALCULATION:



We need the average time response:  
directly computed in the SERPENT code

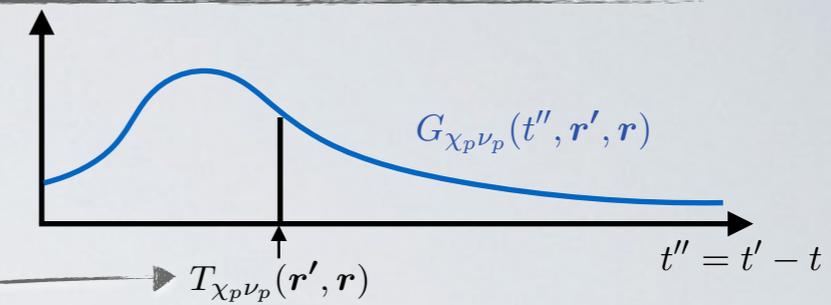
$$T_{\chi_p \nu_p}(\mathbf{r}', \mathbf{r}) = \frac{\int_{t'' > 0} G_{\chi_p \nu_p}(t'', \mathbf{r}', \mathbf{r}) \cdot t'' dt''}{\int_{t'' > 0} G_{\chi_p \nu_p}(t'', \mathbf{r}', \mathbf{r}) dt''}$$

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EFFECTIVE LIFE TIME  $l_{eff}$  CALCULATION:

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With the total response through time:  
the classic FM operator



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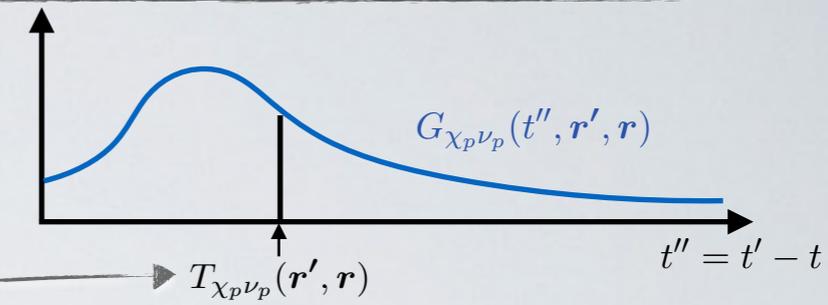
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The adjoint operator and its Eigenvector  
the neutron goes backward in generation = importance!



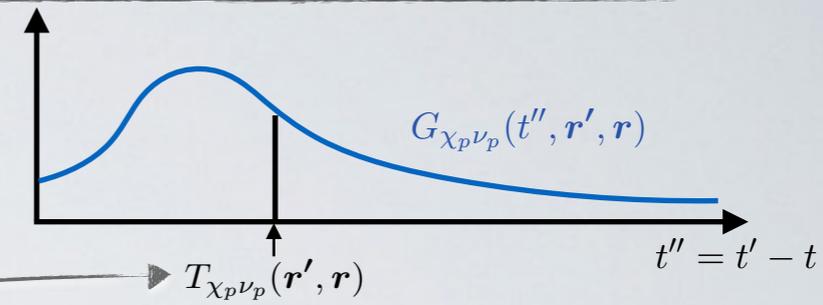
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Finally:

$$l_{eff} = \frac{\iint_{\mathbf{r}' \in \mathcal{R}, \mathbf{r} \in \mathcal{R}} N_p^*(\mathbf{r}) \left[ T_{\chi_p \nu_p}(\mathbf{r}', \mathbf{r}) \cdot \tilde{G}_{\chi_p \nu_p}(\mathbf{r}', \mathbf{r}) \right] N_p(\mathbf{r}') d\mathbf{r}' d\mathbf{r}}{\iint_{\mathbf{r}' \in \mathcal{R}, \mathbf{r} \in \mathcal{R}} N_p^*(\mathbf{r}) \tilde{G}_{\chi_p \nu_p}(\mathbf{r}', \mathbf{r}) N_p(\mathbf{r}') d\mathbf{r}' d\mathbf{r}}$$

And its discretized version:

$$l_{eff} = \frac{\sum_{\mathcal{R}} N_p^* \left( \underline{T_{\chi_p \nu_p}} \cdot \underline{\tilde{G}_{\chi_p \nu_p}} \right) N_p}{\sum_{\mathcal{R}} N_p^* \underline{\tilde{G}_{\chi_p \nu_p}} N_p}$$

aimed average time × neutron production per incoming neutron × neutron population

importance weighting

produced neutron

# I. TRANSIENT FISSION MATRIX: TFM SIMPLIFIED KINETIC EQUATIONS

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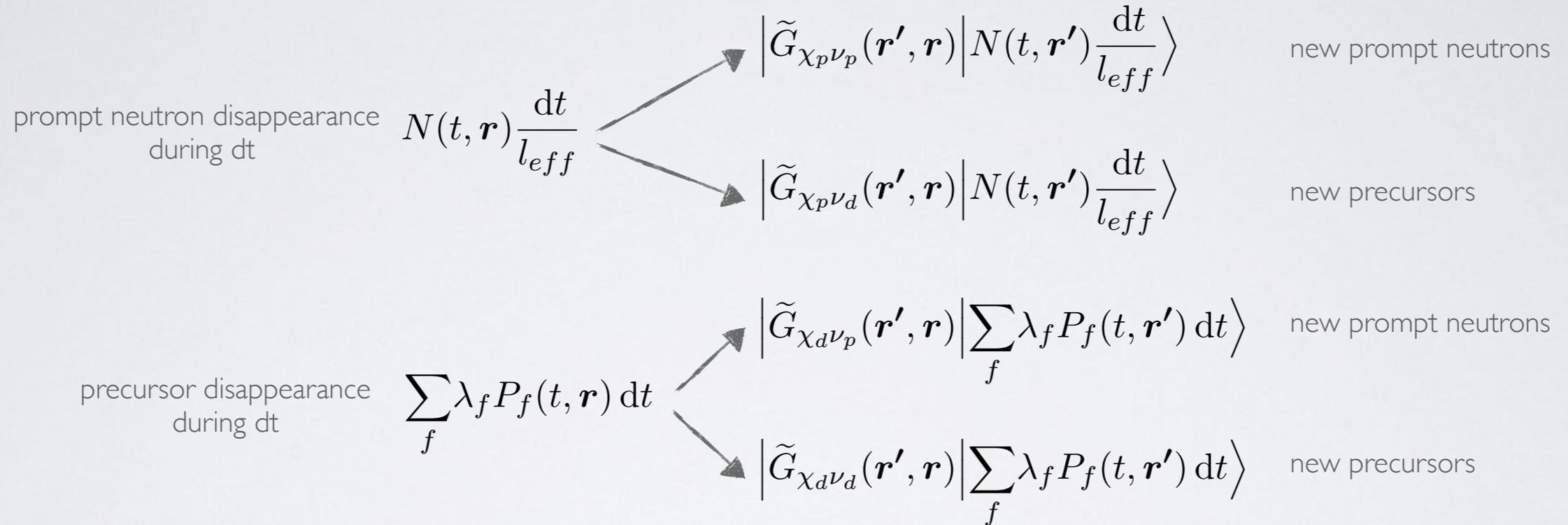
## OVERALL PRINCIPLE:

Replace the neutron production rate  $S(\mathbf{r}, t)$  by a neutron population  $N(t, \mathbf{r})$  associated to a time constant  $l_{eff}$ :  
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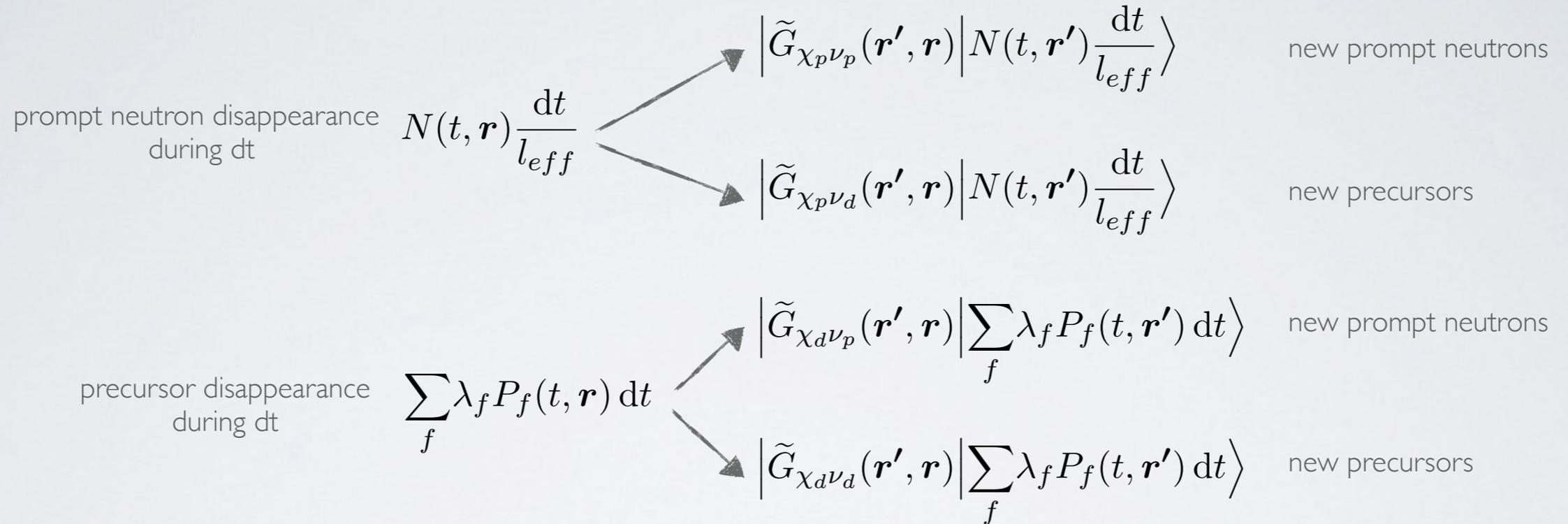
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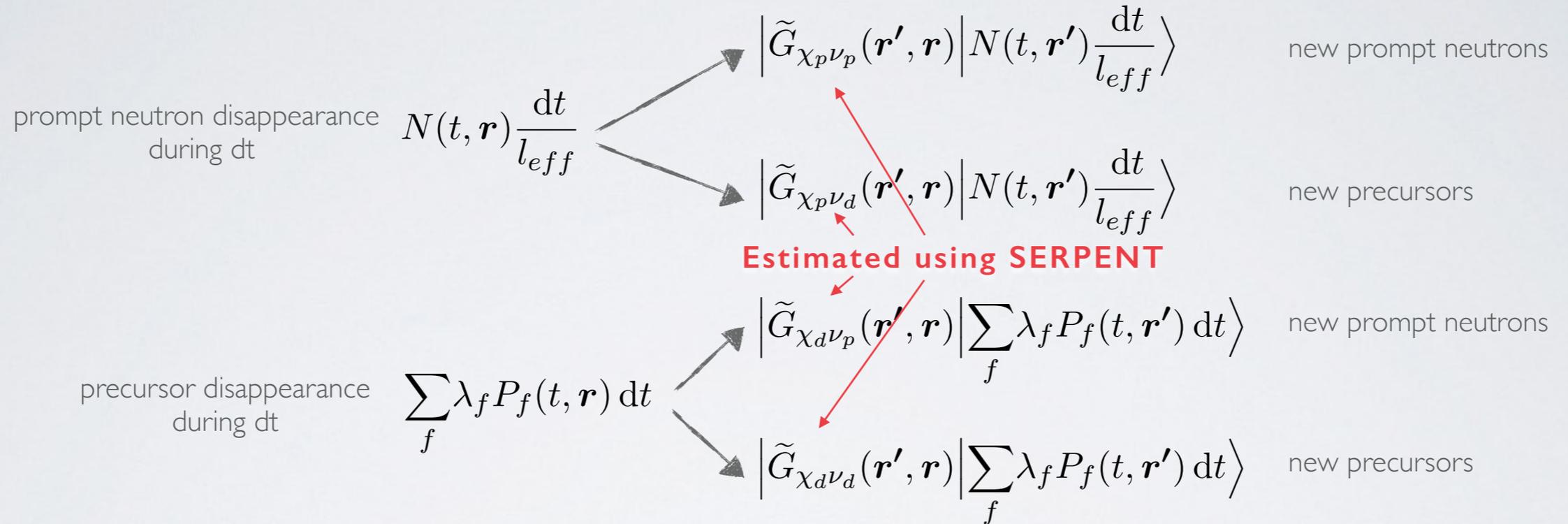
$$\frac{dN}{dt}(t, \mathbf{r}) = \frac{1}{l_{eff}} \left| \tilde{G}_{\chi_p \nu_p}(\mathbf{r}', \mathbf{r}) \left| N(t, \mathbf{r}') \right. \right\rangle + \left| \tilde{G}_{\chi_d \nu_p}(\mathbf{r}', \mathbf{r}) \left| \sum_f \lambda_f P_f(t, \mathbf{r}') \right. \right\rangle - \frac{1}{l_{eff}} N(t, \mathbf{r})$$

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# I. TRANSIENT FISSION MATRIX: FISSION MATRIX INTERPOLATION

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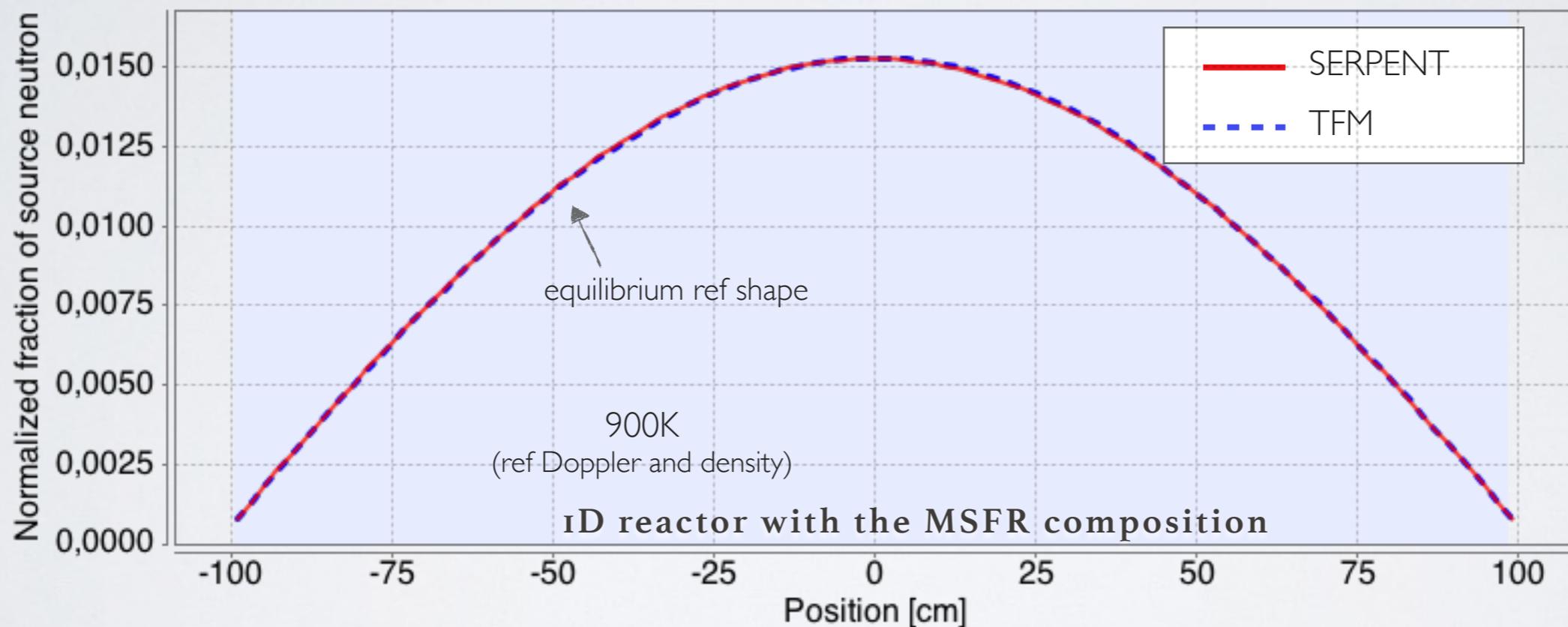
AND FOR TRANSIENT CALCULATIONS?

Matrix interpolation! 
$$\tilde{G}(\mathbf{r}', \mathbf{r}) = \tilde{G}_{ref}(\mathbf{r}', \mathbf{r}) + \underbrace{(T(\mathbf{r}') - T_{ref})}_{\text{linear dependency}} \cdot \Delta_{\rho} \tilde{G}(\mathbf{r}', \mathbf{r}) + \underbrace{\log \frac{T(\mathbf{r}')}{T_{ref}}}_{\text{logarithmic dependency}} \cdot \Delta_{Doppler} \tilde{G}(\mathbf{r}', \mathbf{r})$$

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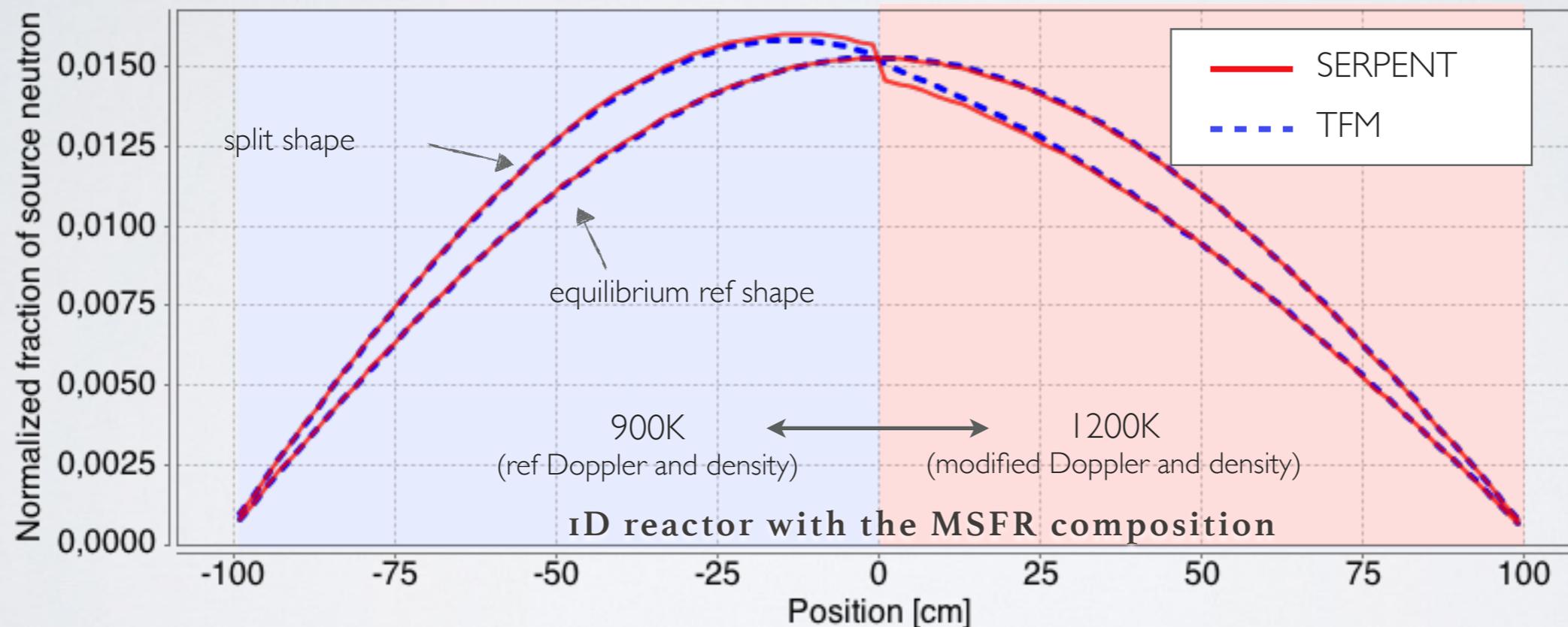
- good modeling of the neutron shift
- good prediction of the multiplication factor variation (~1-2% error on 1000pcm)

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## GENERAL COUPLING STRATEGY

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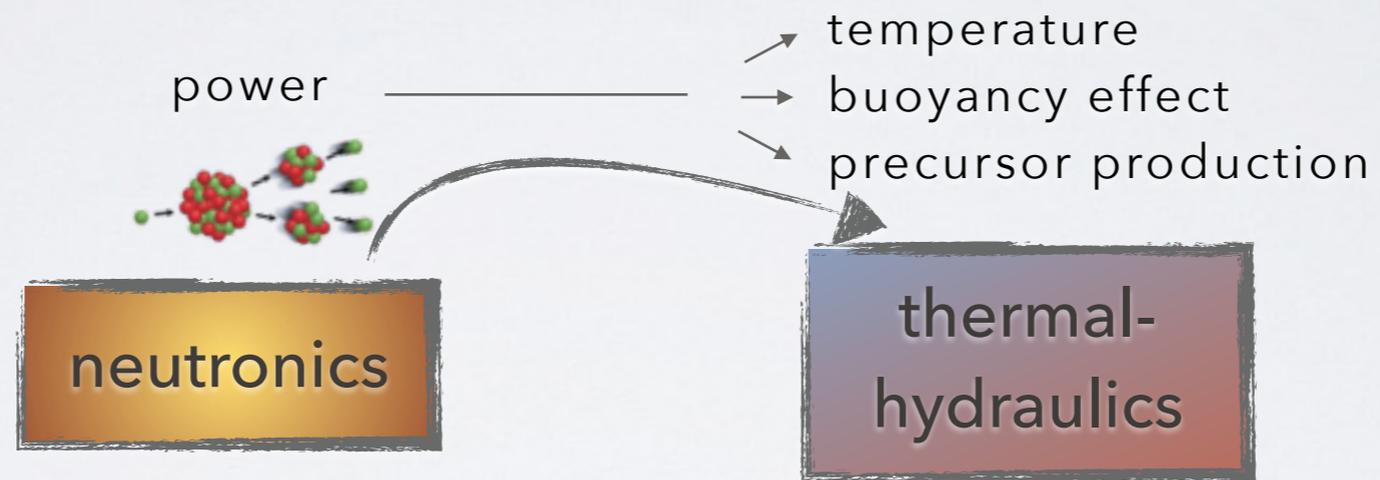
neutronics

thermal-  
hydraulics

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# GENERAL COUPLING STRATEGY

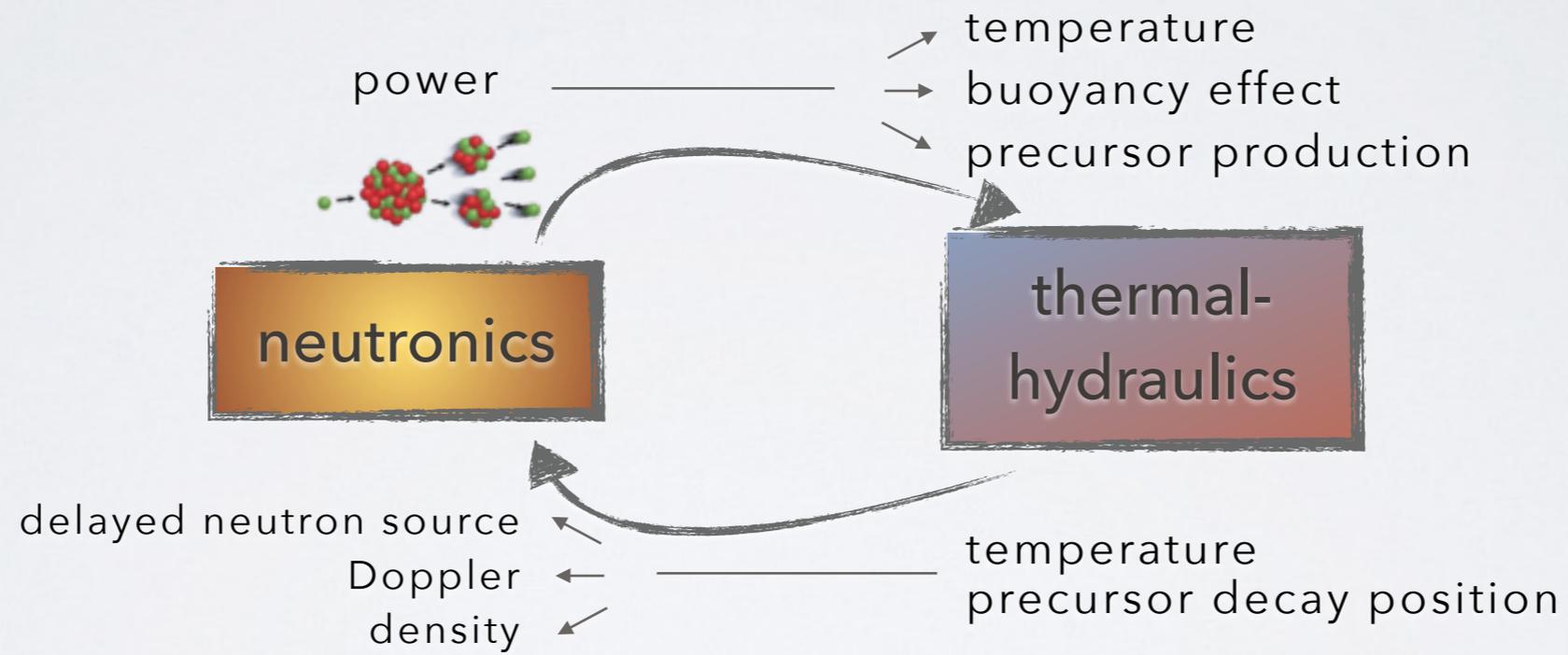
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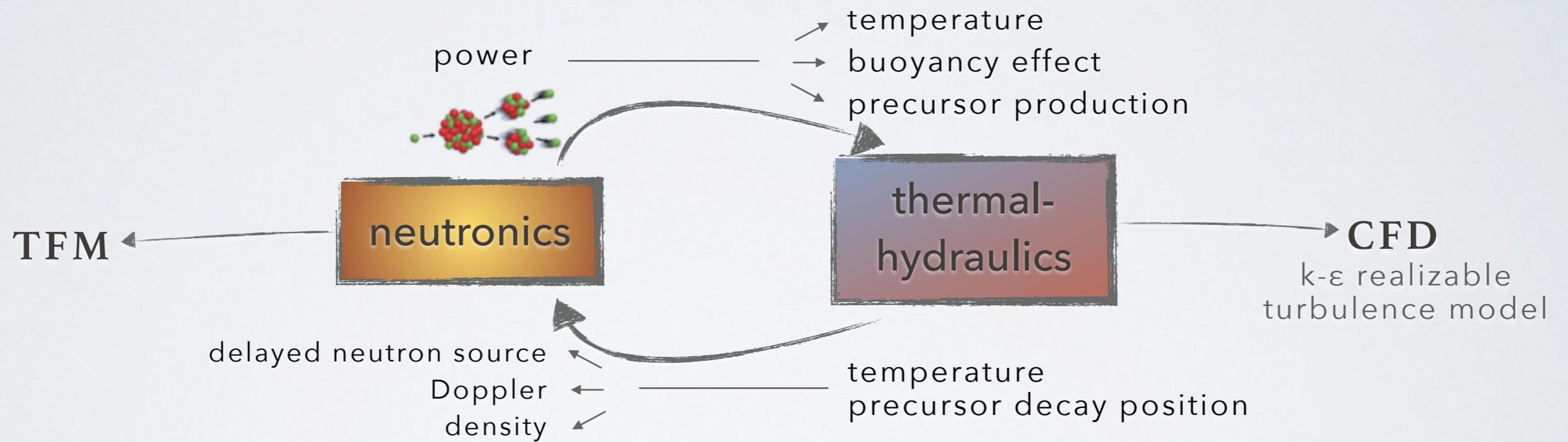
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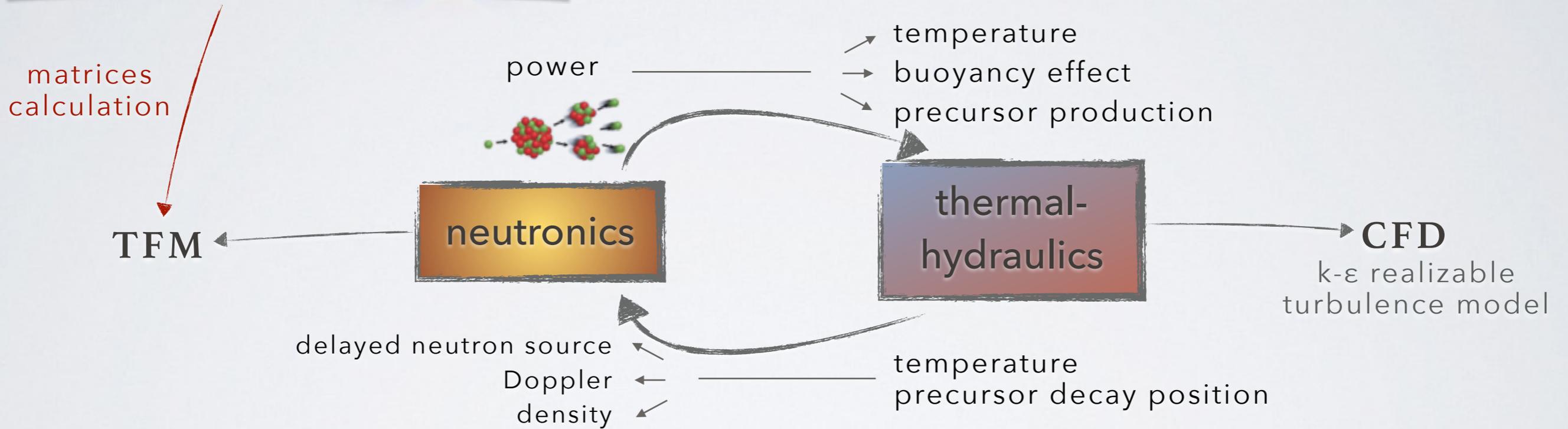
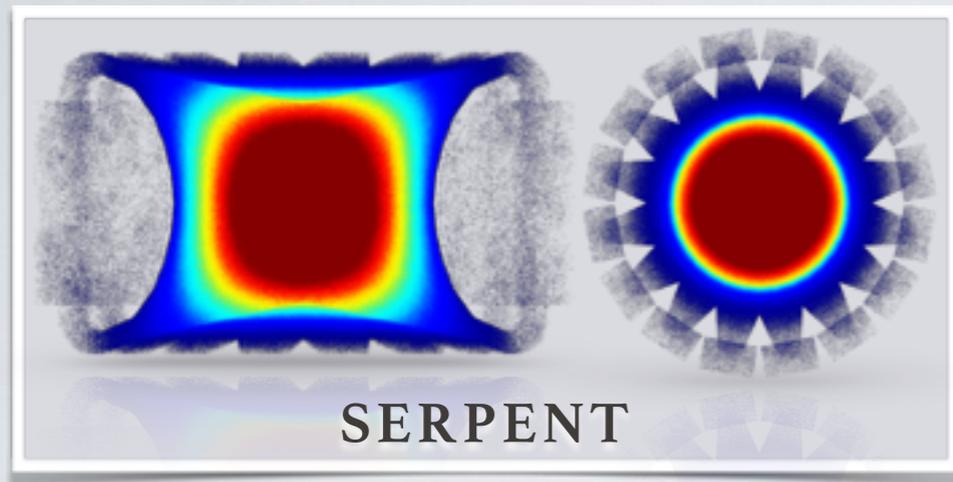
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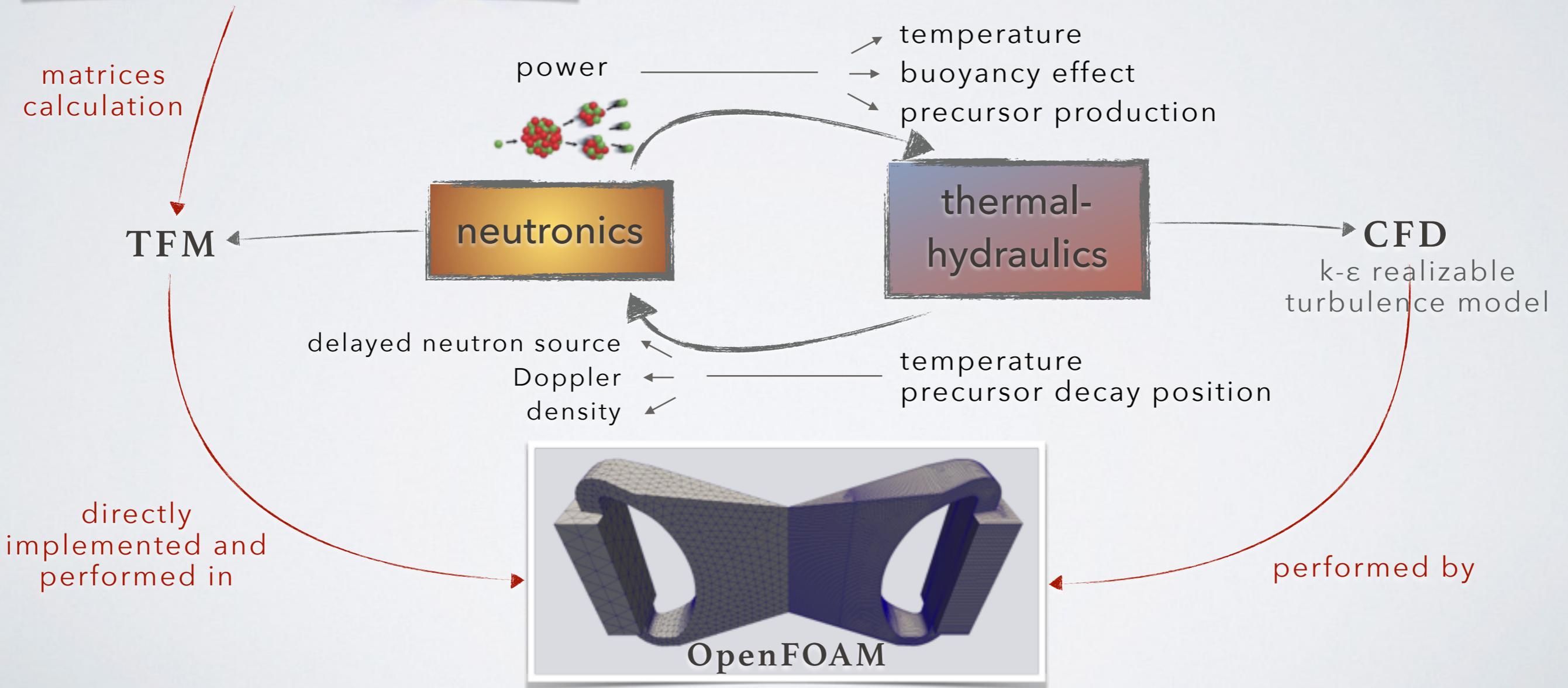
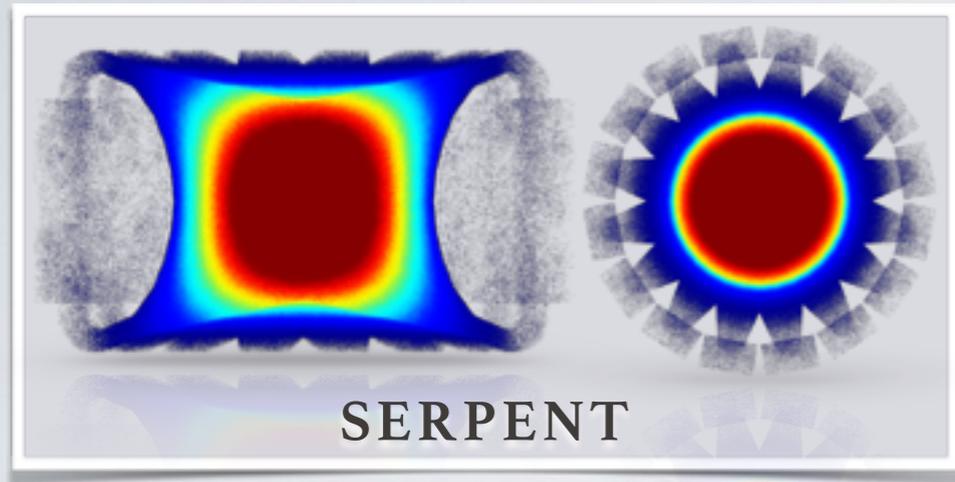
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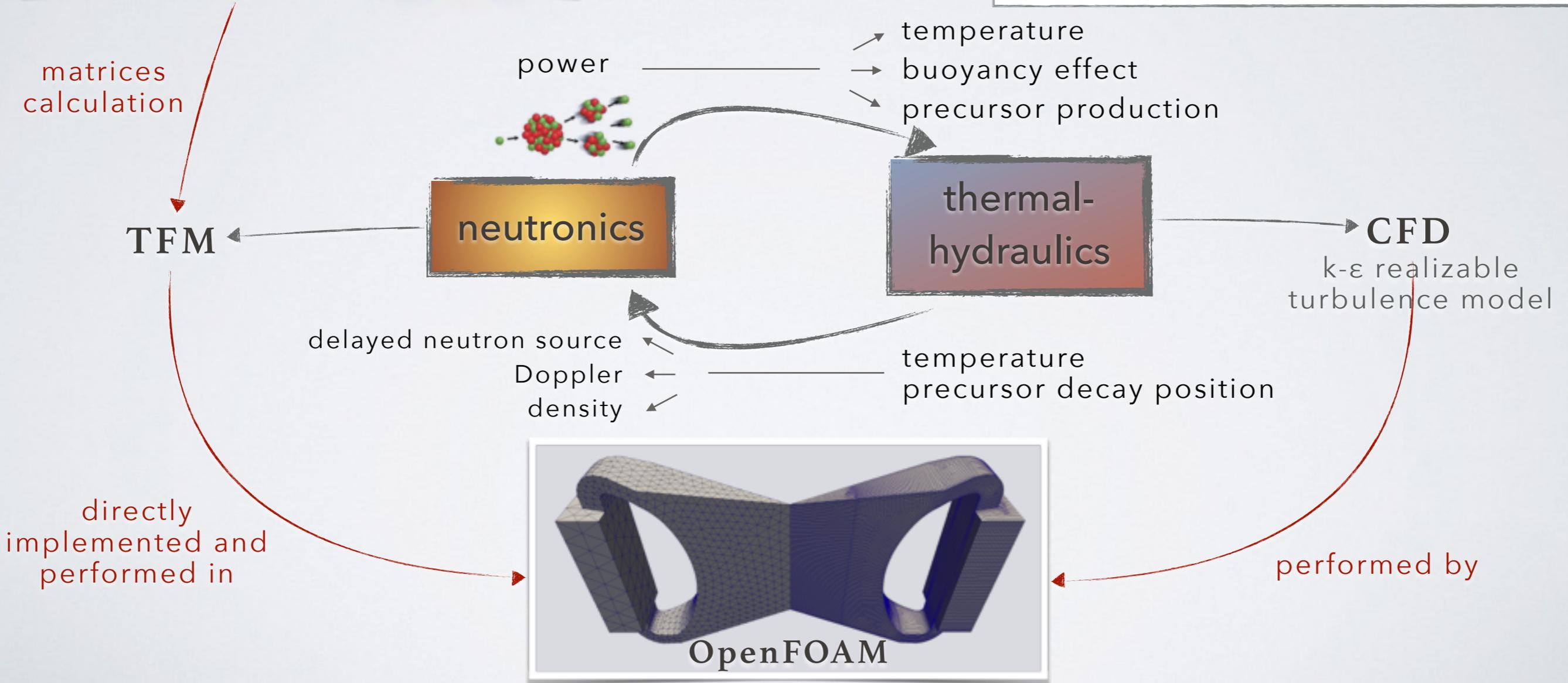
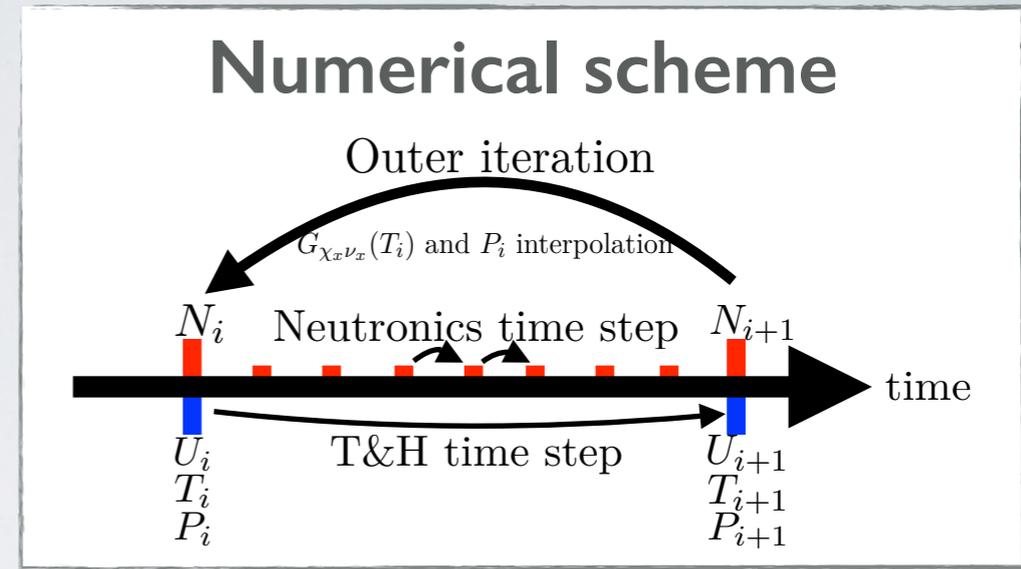
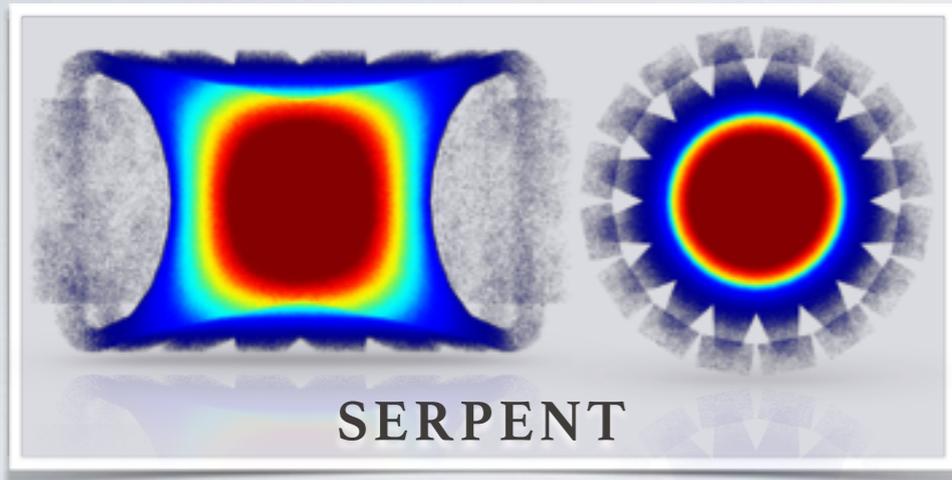
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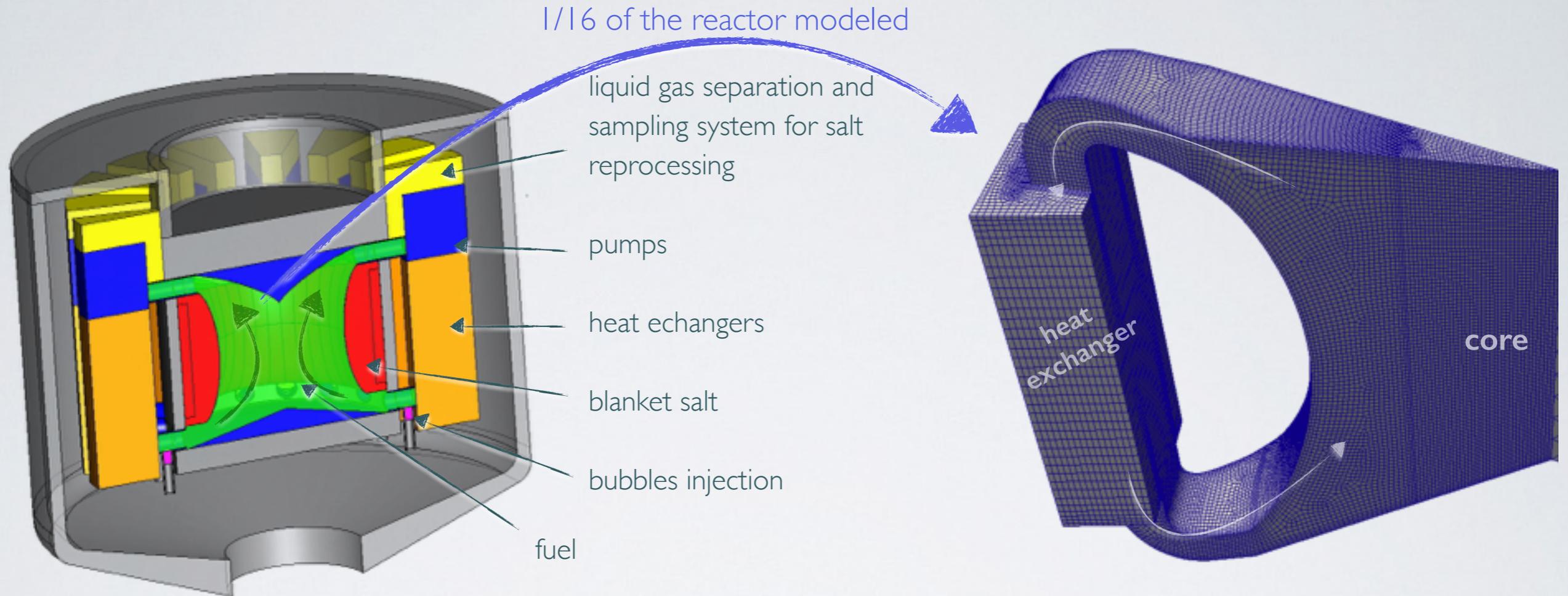
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## II. APPLICATION CASE: MOLTEN SALT FAST REACTOR (MSFR) PRESENTATION

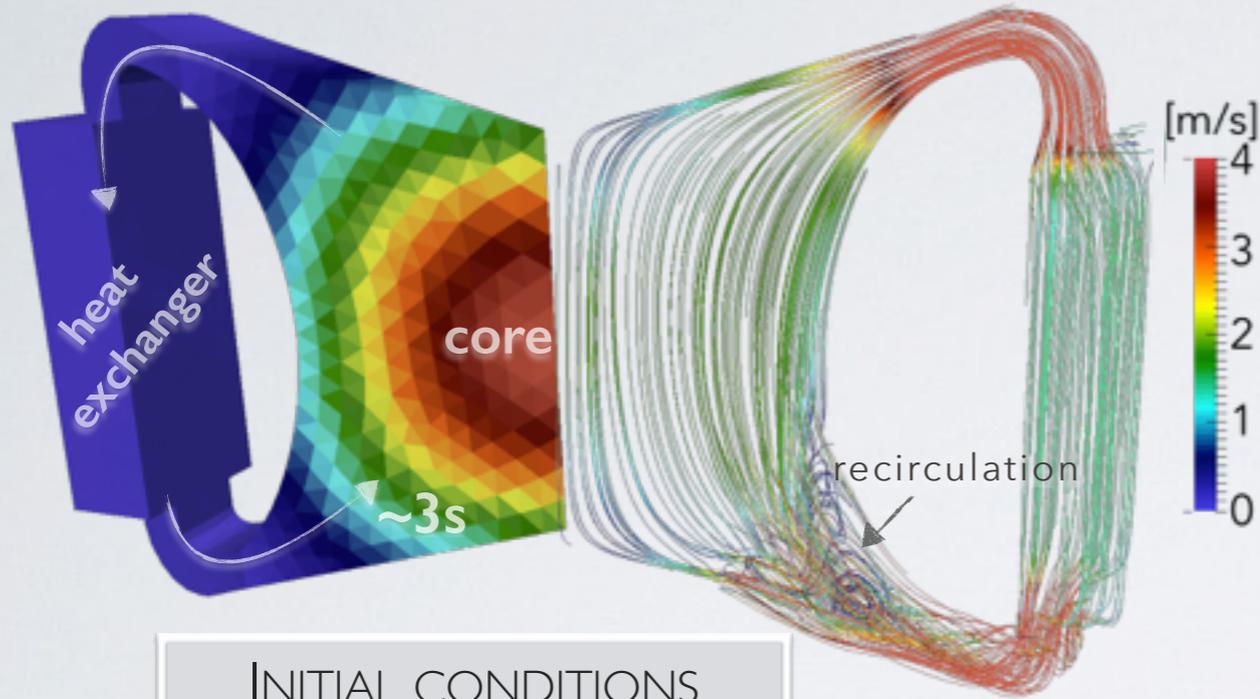


### Molten Salt Fast Reactor (MSFR)

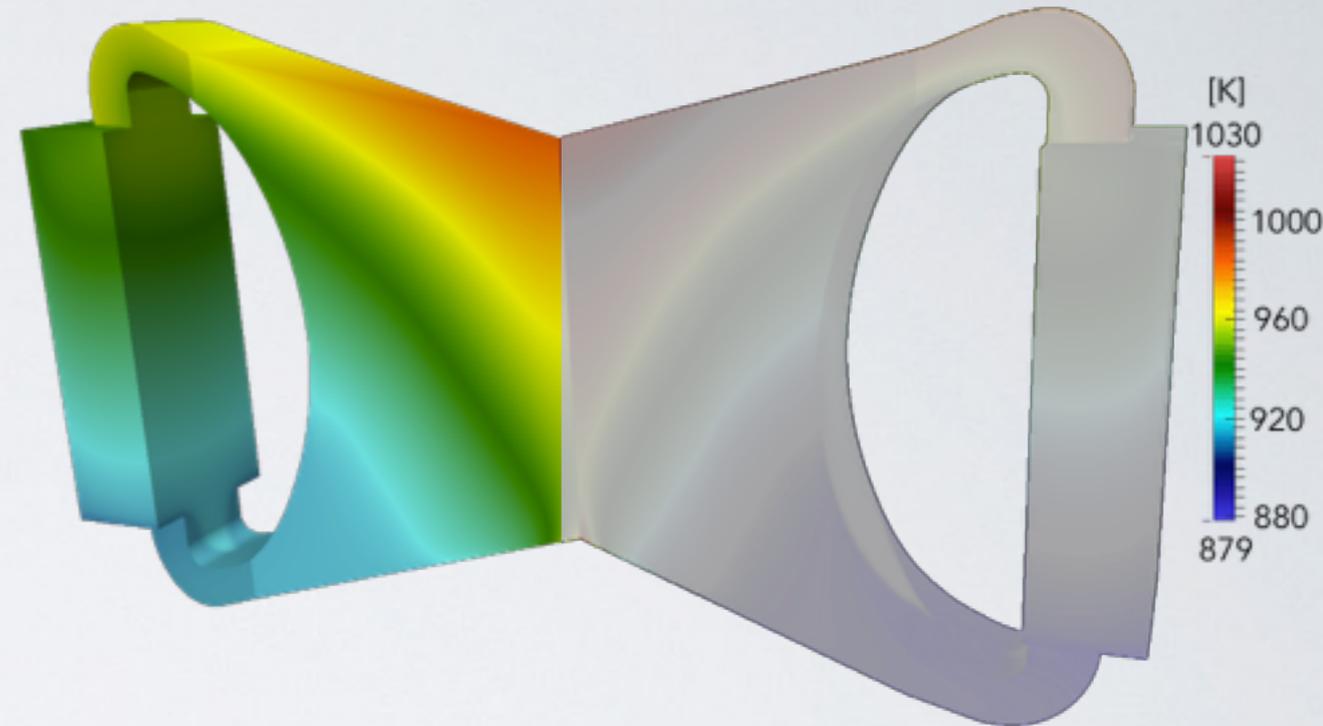
- Liquid fuel (precursor motion)
- Fuel = coolant
- Fast neutron spectrum
- Circulation time  $\sim 3$  s
- Reynolds in core:  $\sim 500000$
- Power: 3GWth
- Molten Salt :  $\text{LiF} - (\text{Th}/^{233}\text{U})\text{F}_4$ 
  - density: 4 x water
  - viscosity: 2 x water (oil  $\sim 1000$ x water)
  - low pressure
  - mean fuel temperature  $\sim 900$  K

## II. APPLICATION CASES: INSTANTANEOUS OVER COOLING TRANSIENT

Intermediate fluid temperature variation in the heat exchanger



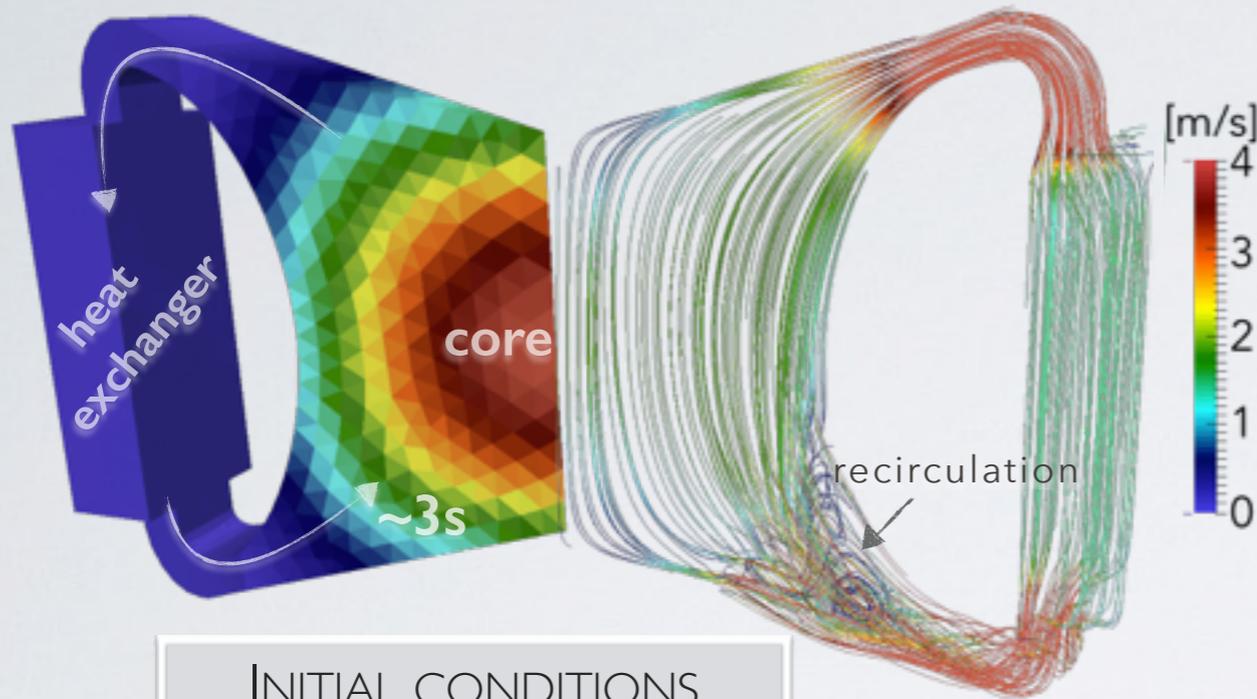
INITIAL CONDITIONS  
(STEADY STATE)



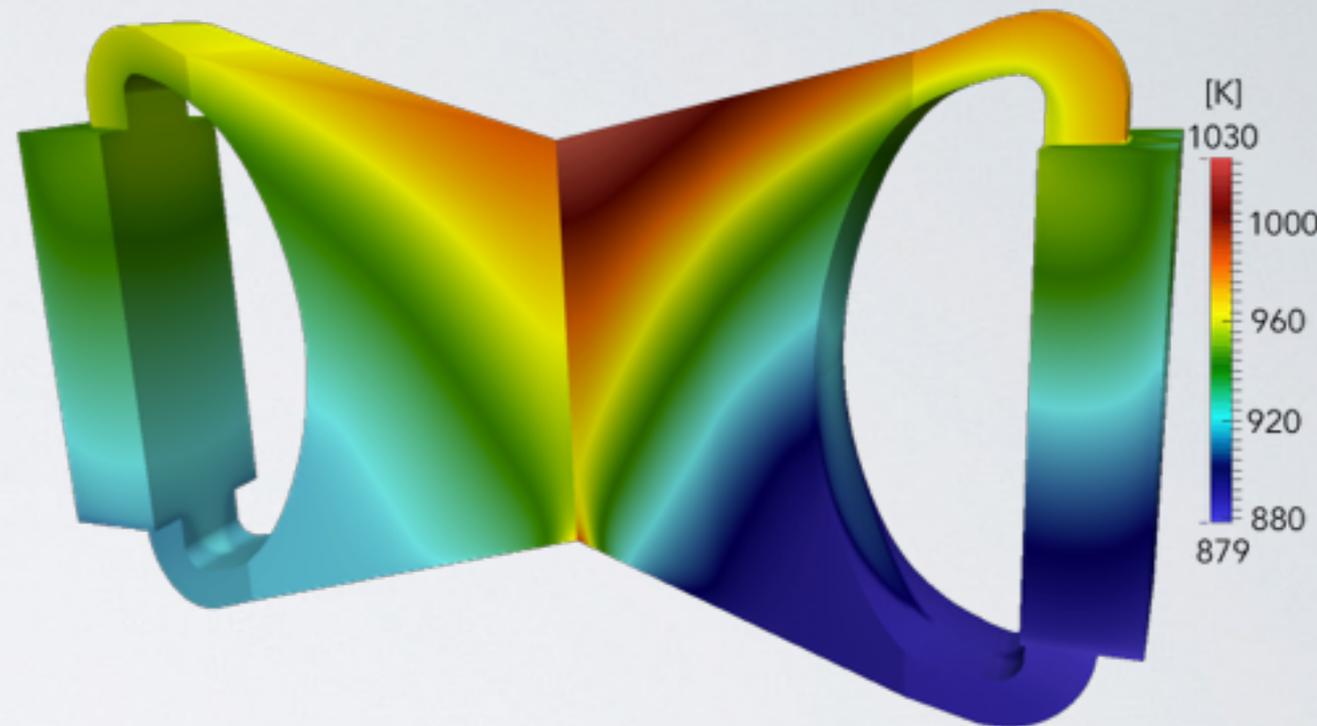
INITIAL / FINAL TEMPERATURE DISTRIBUTION

## II. APPLICATION CASES: INSTANTANEOUS OVER COOLING TRANSIENT

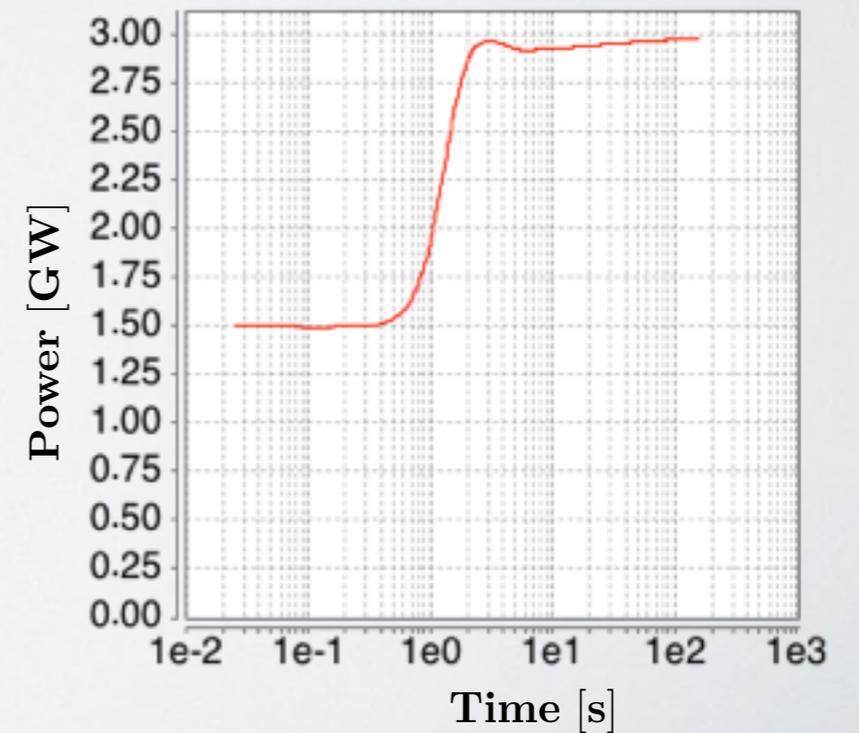
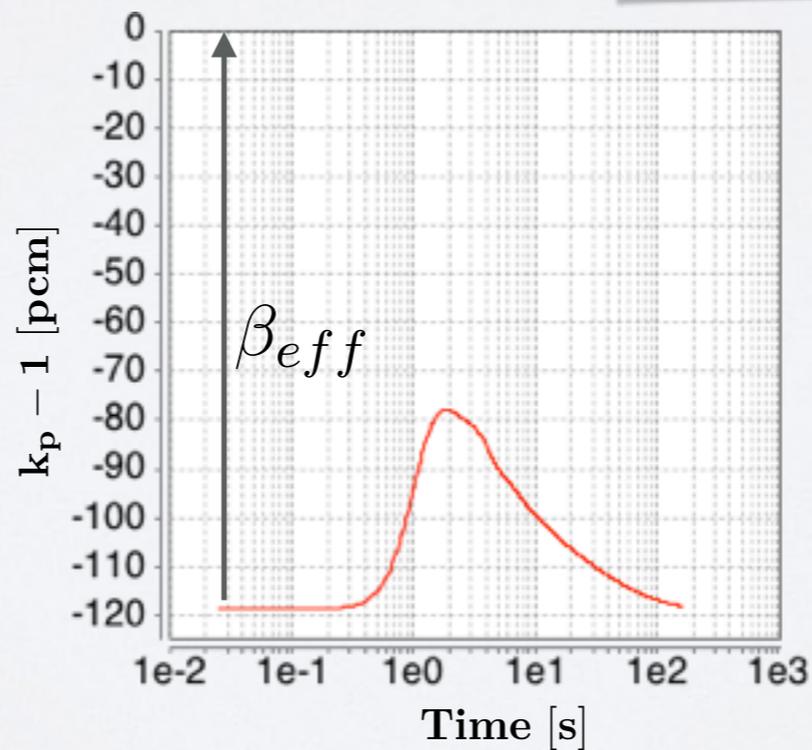
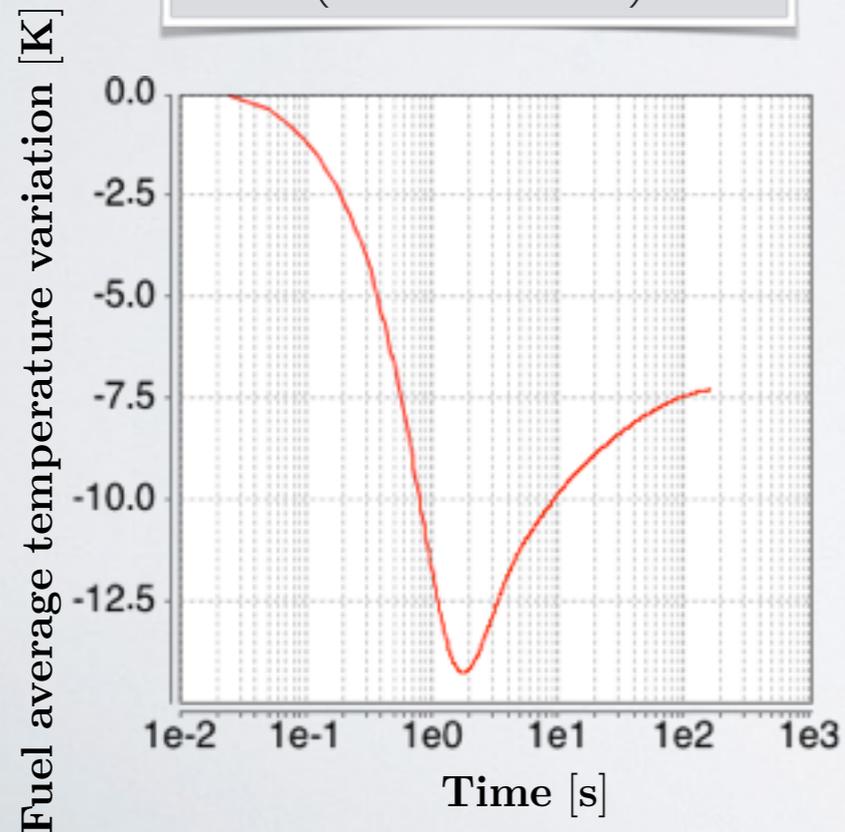
Intermediate fluid temperature variation in the heat exchanger



INITIAL CONDITIONS  
(STEADY STATE)

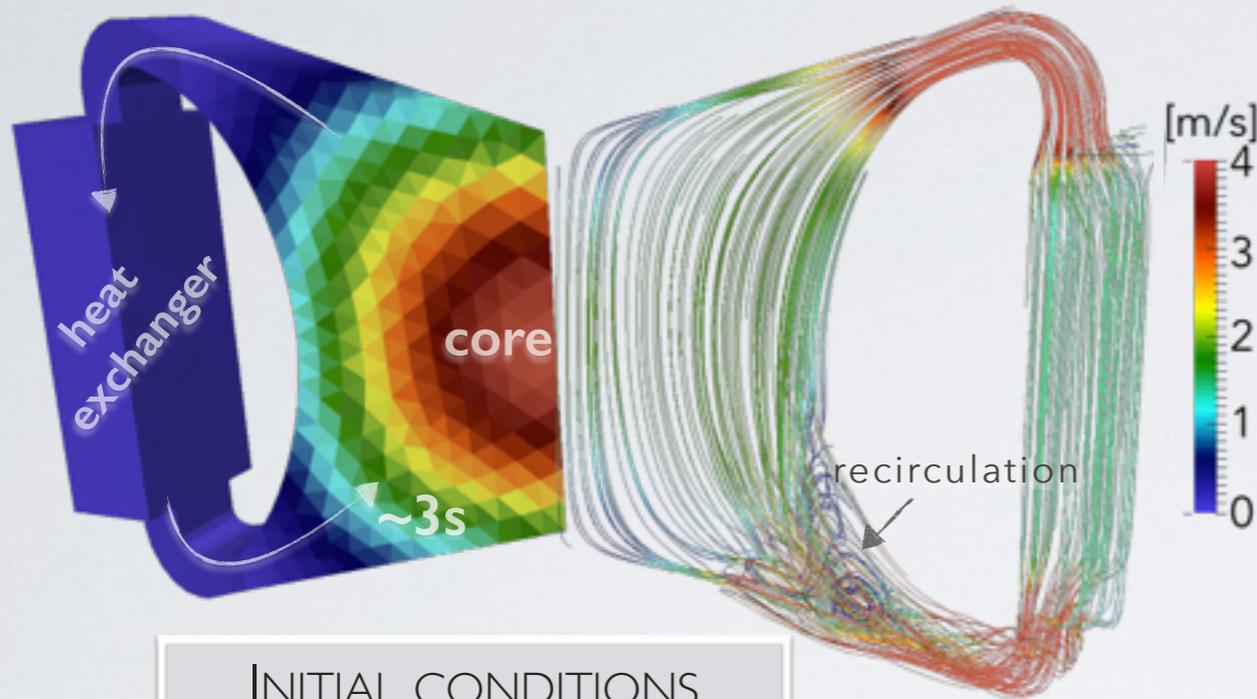


INITIAL / FINAL TEMPERATURE DISTRIBUTION

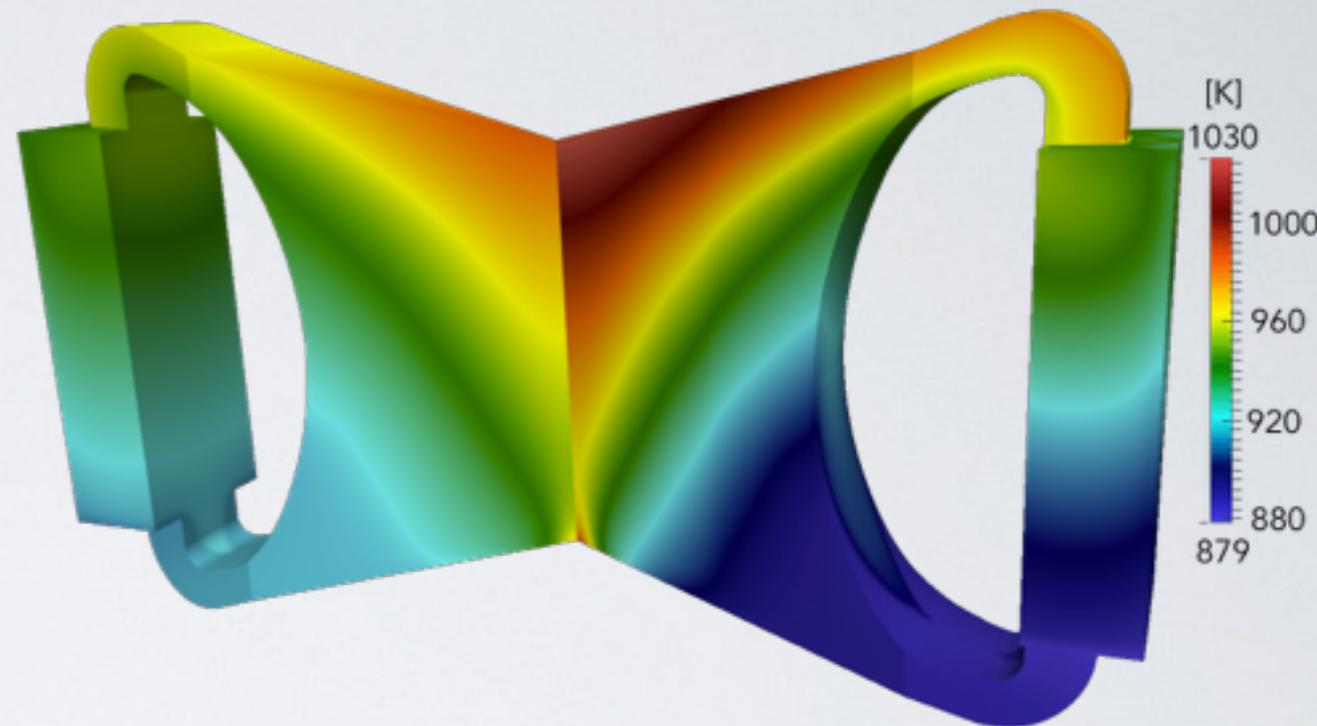


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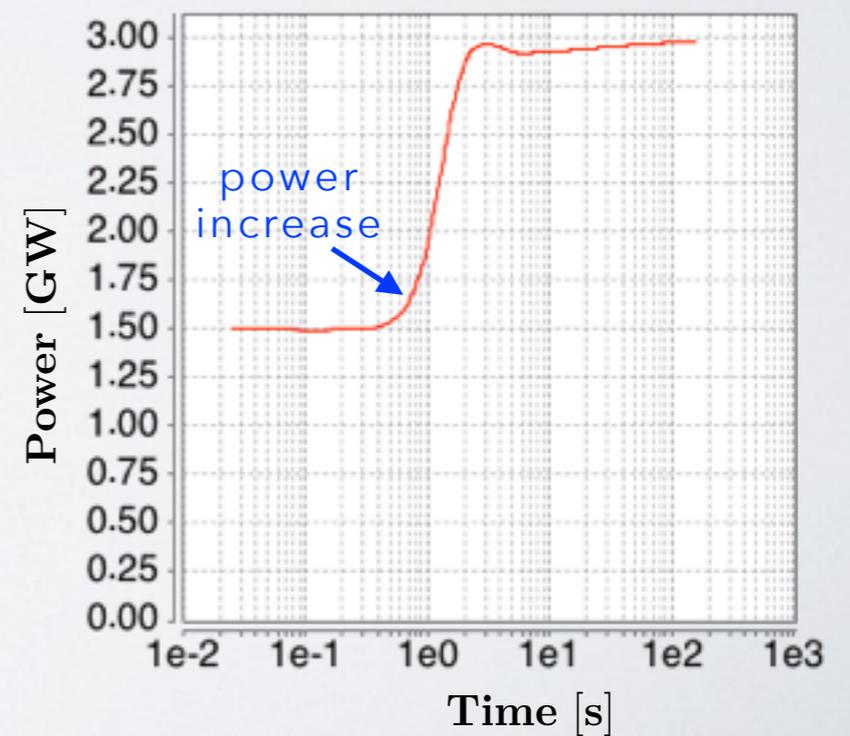
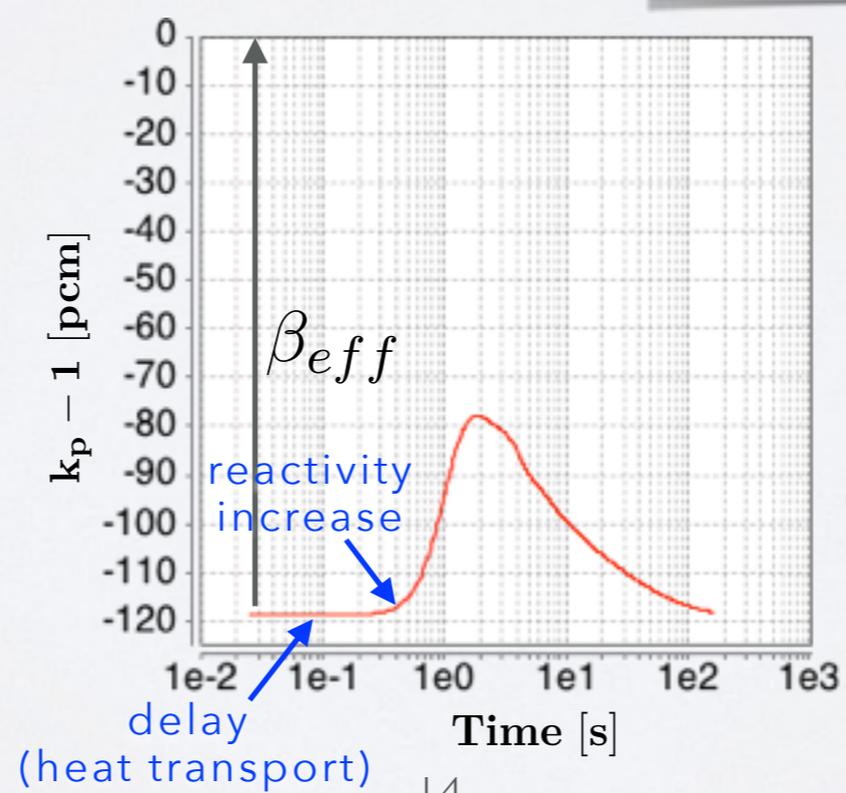
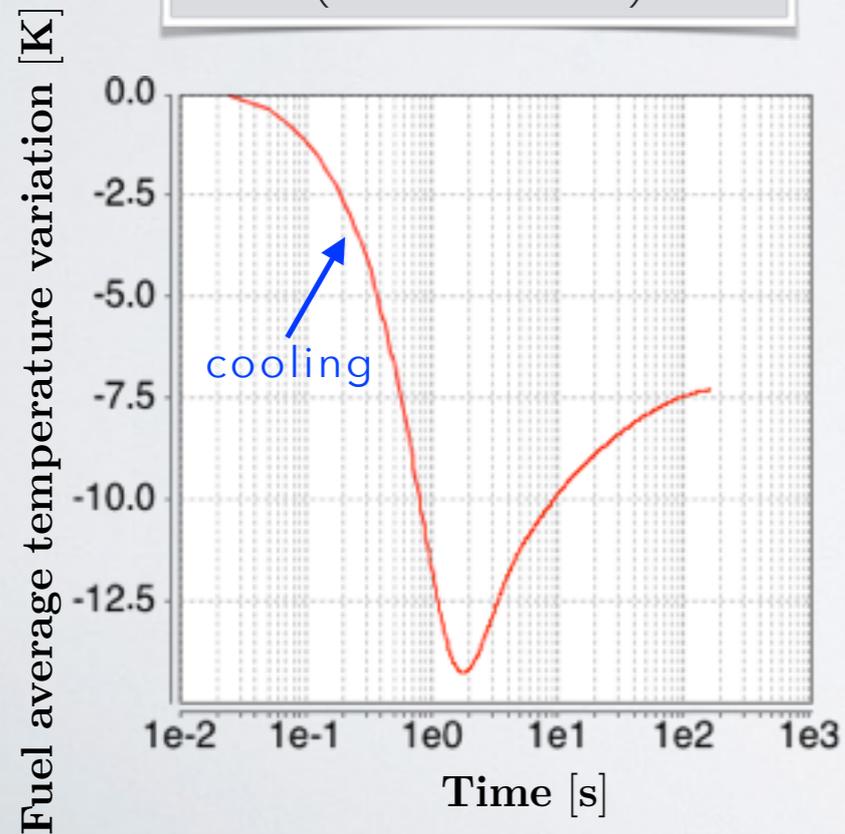
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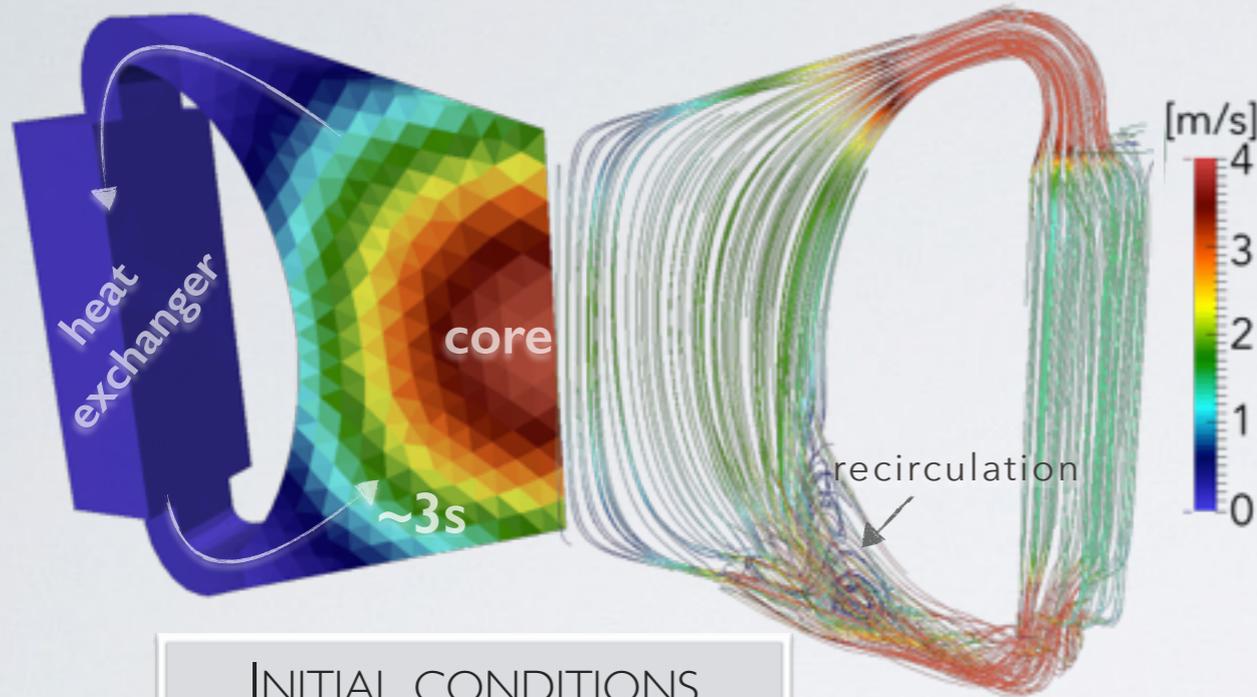


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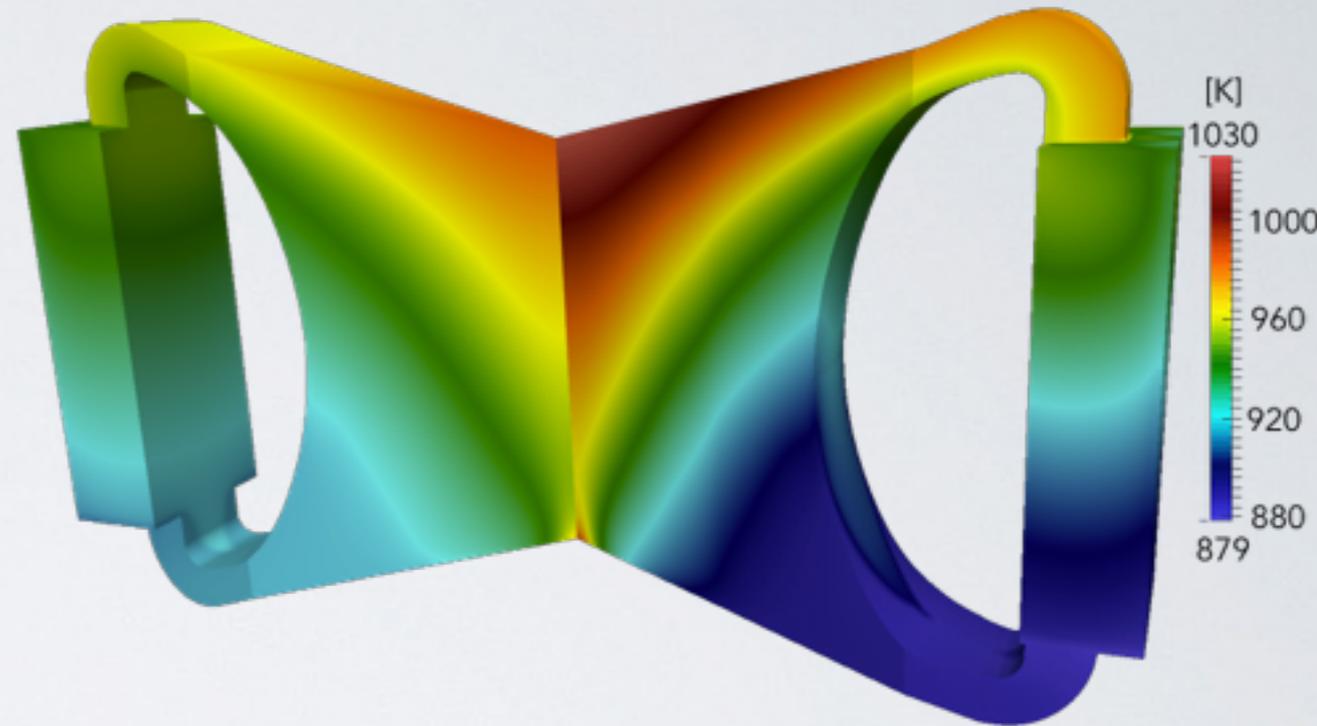


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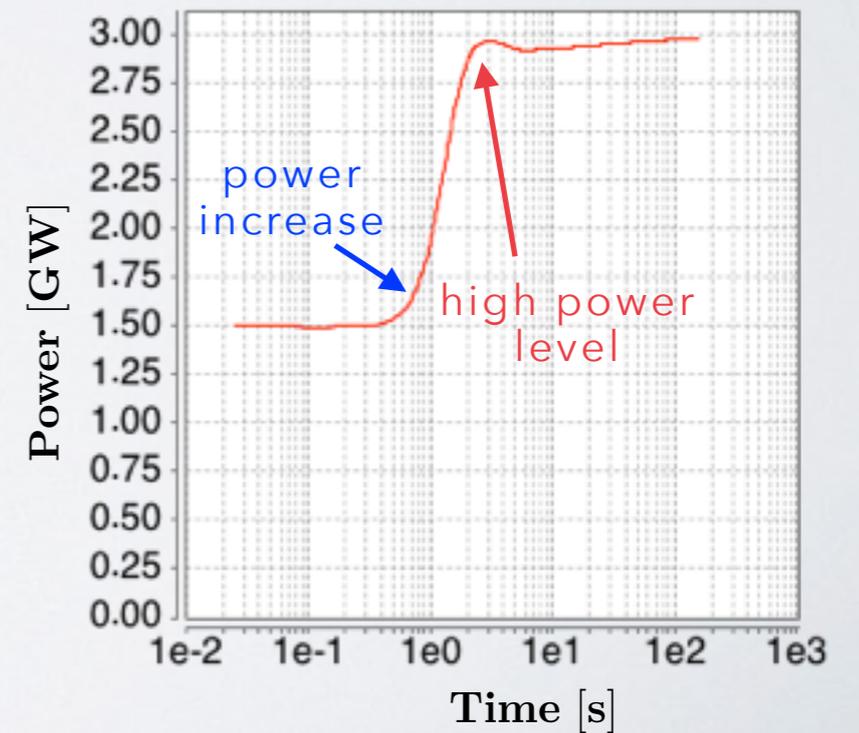
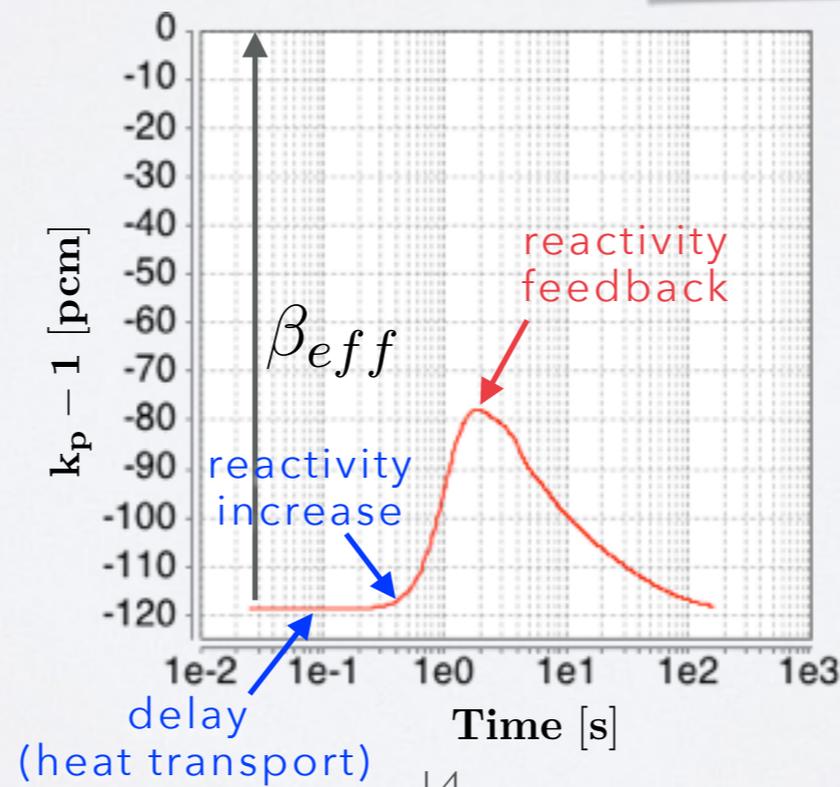
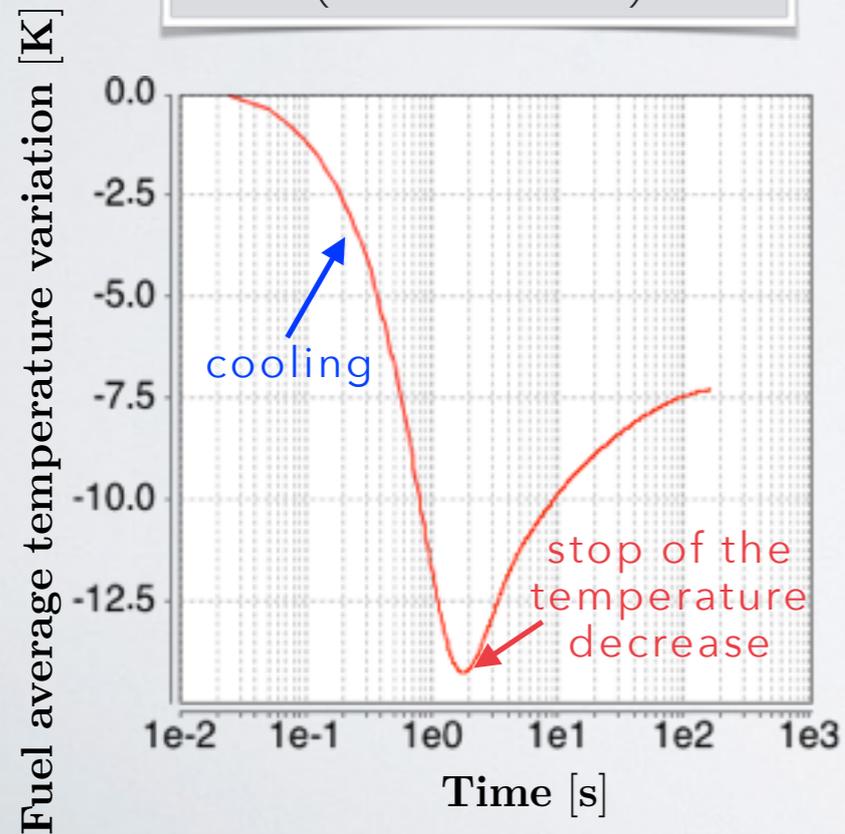
Intermediate fluid temperature variation in the heat exchanger



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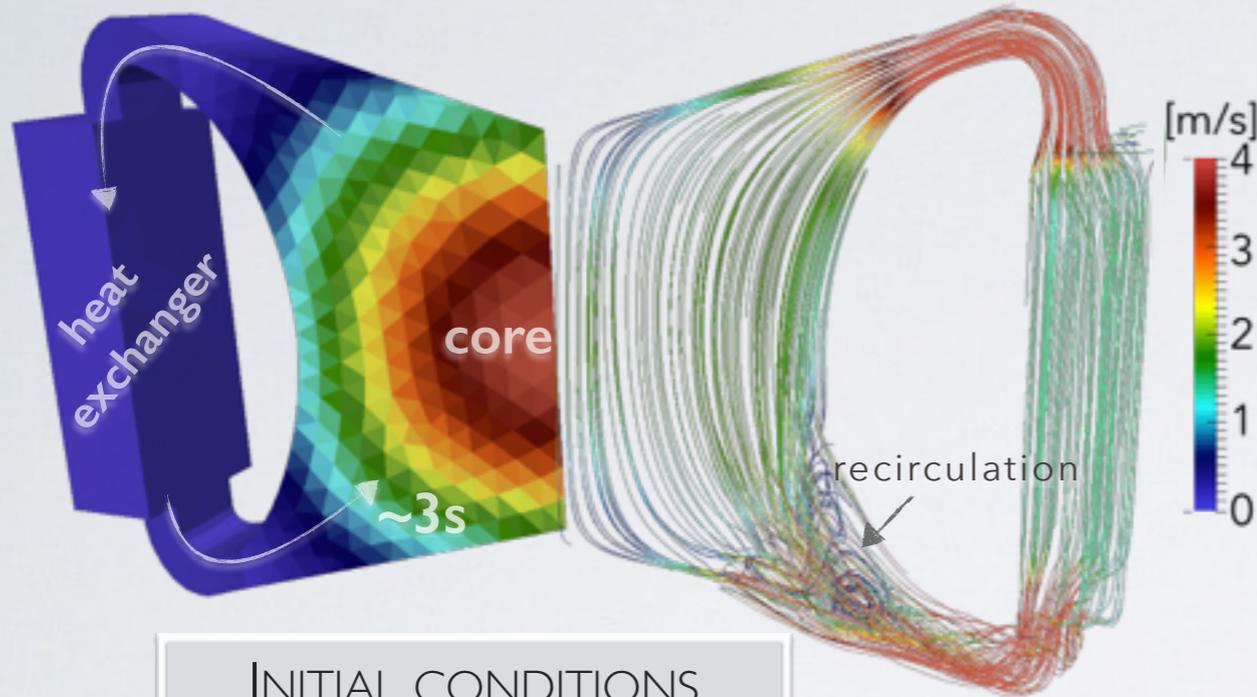


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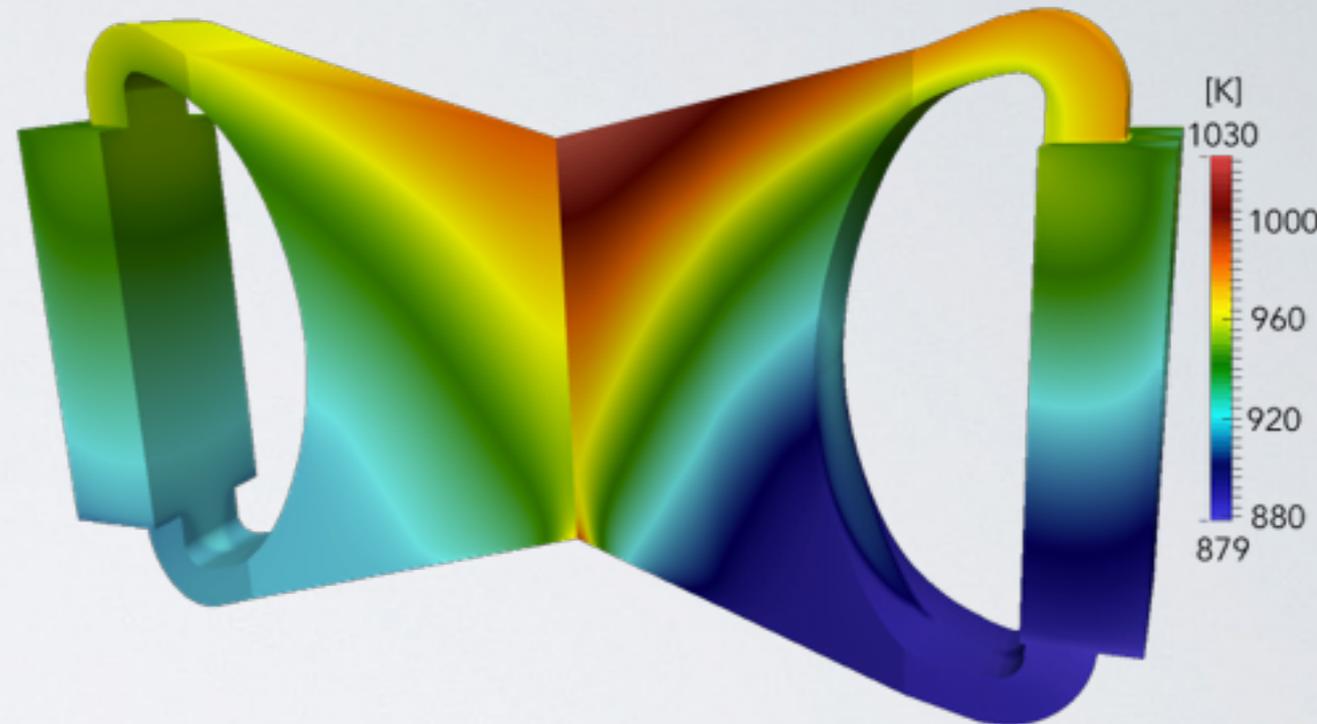


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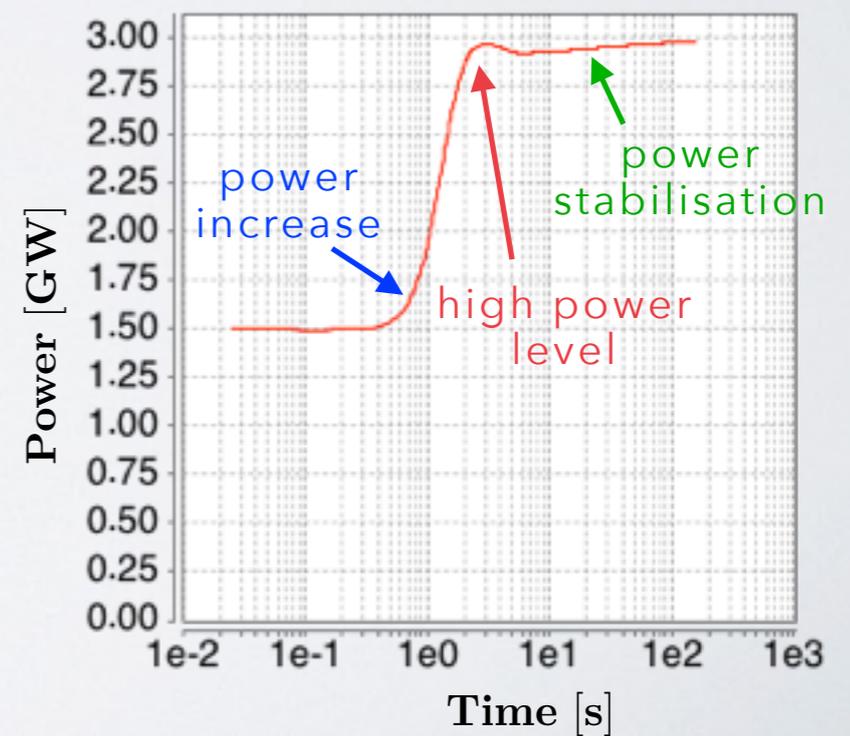
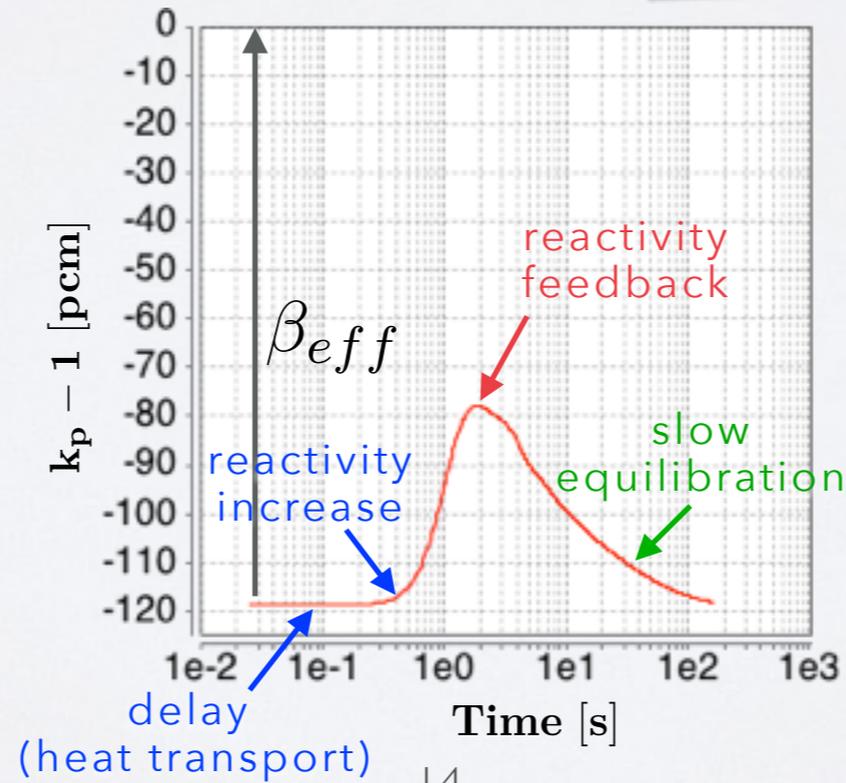
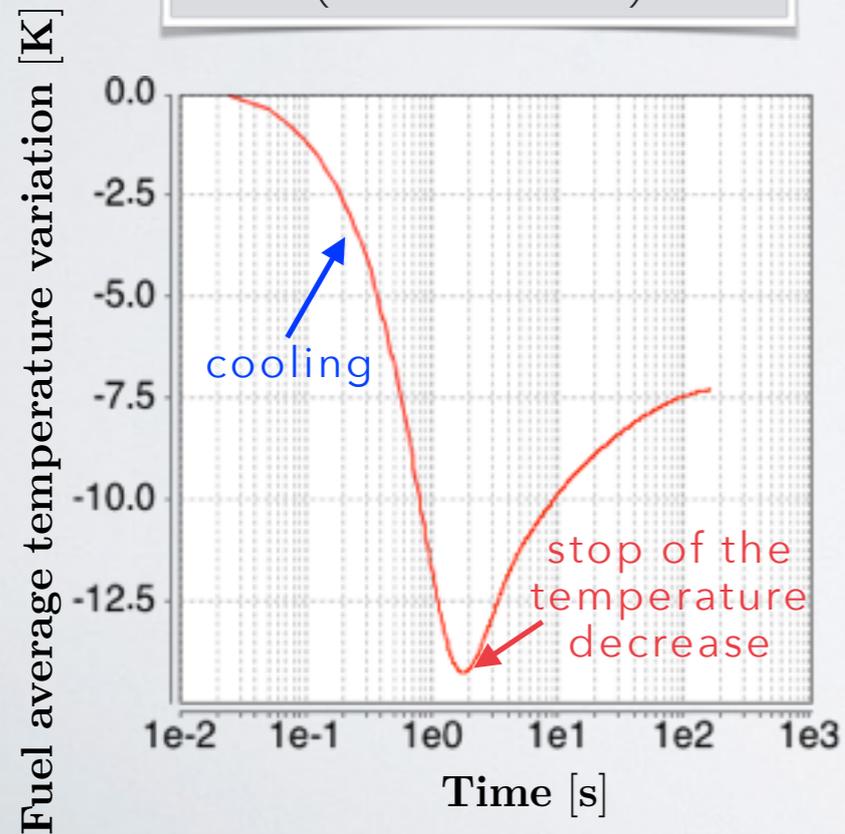
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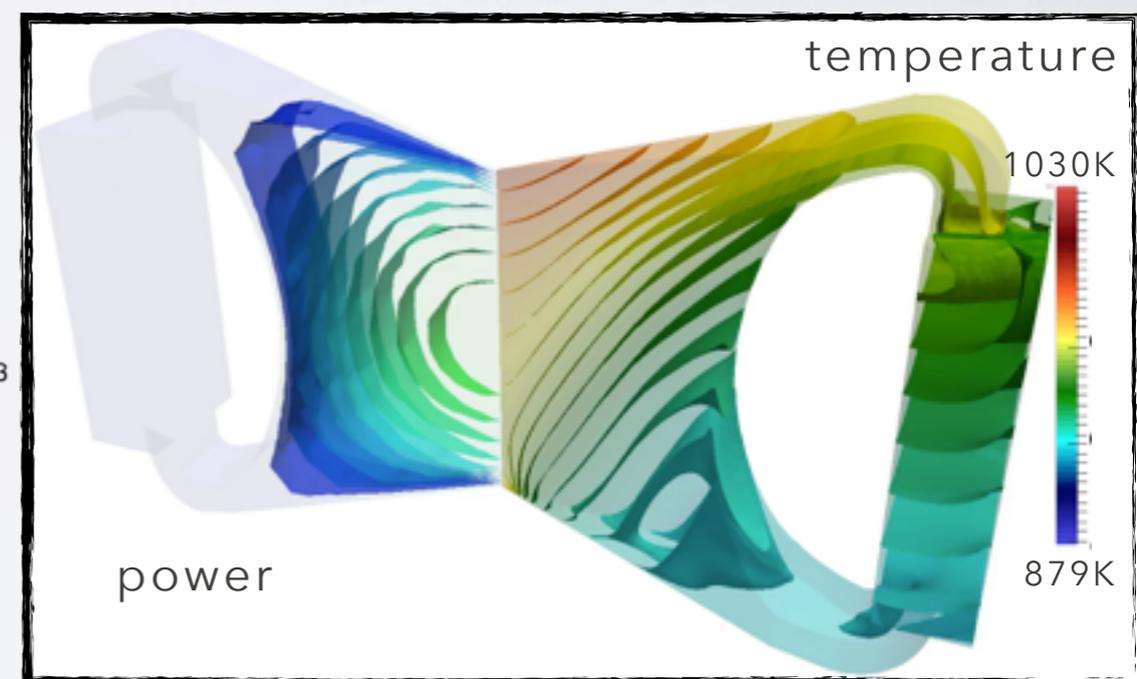
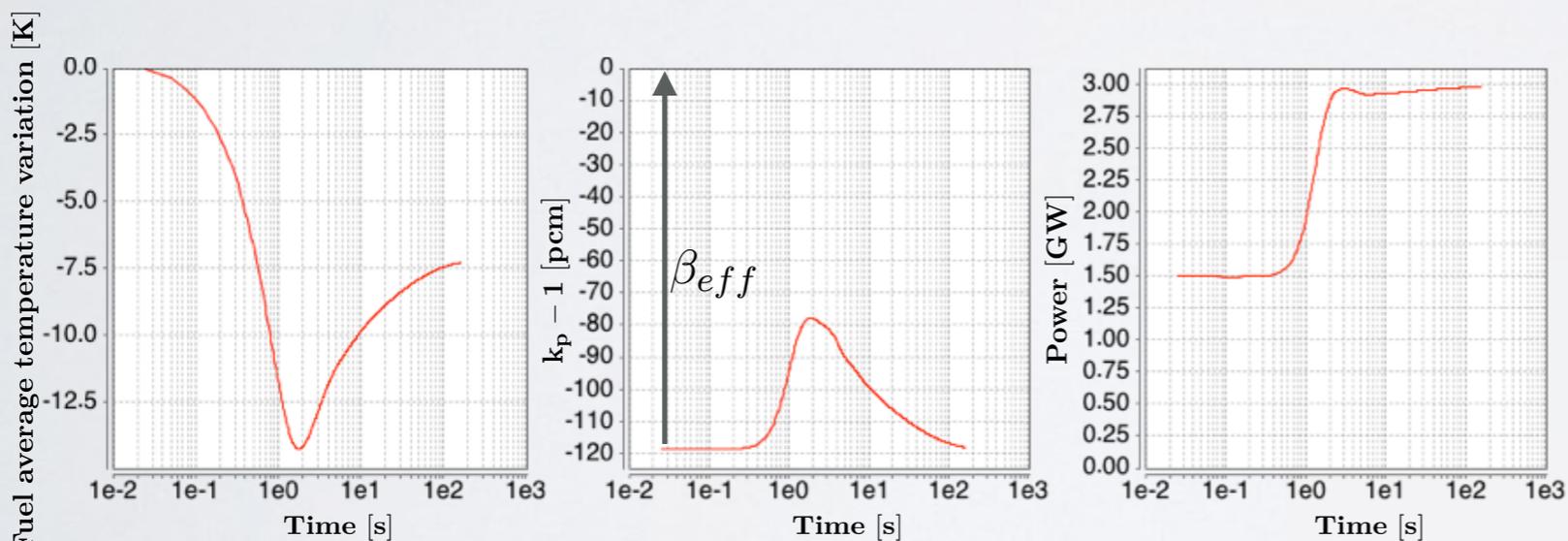
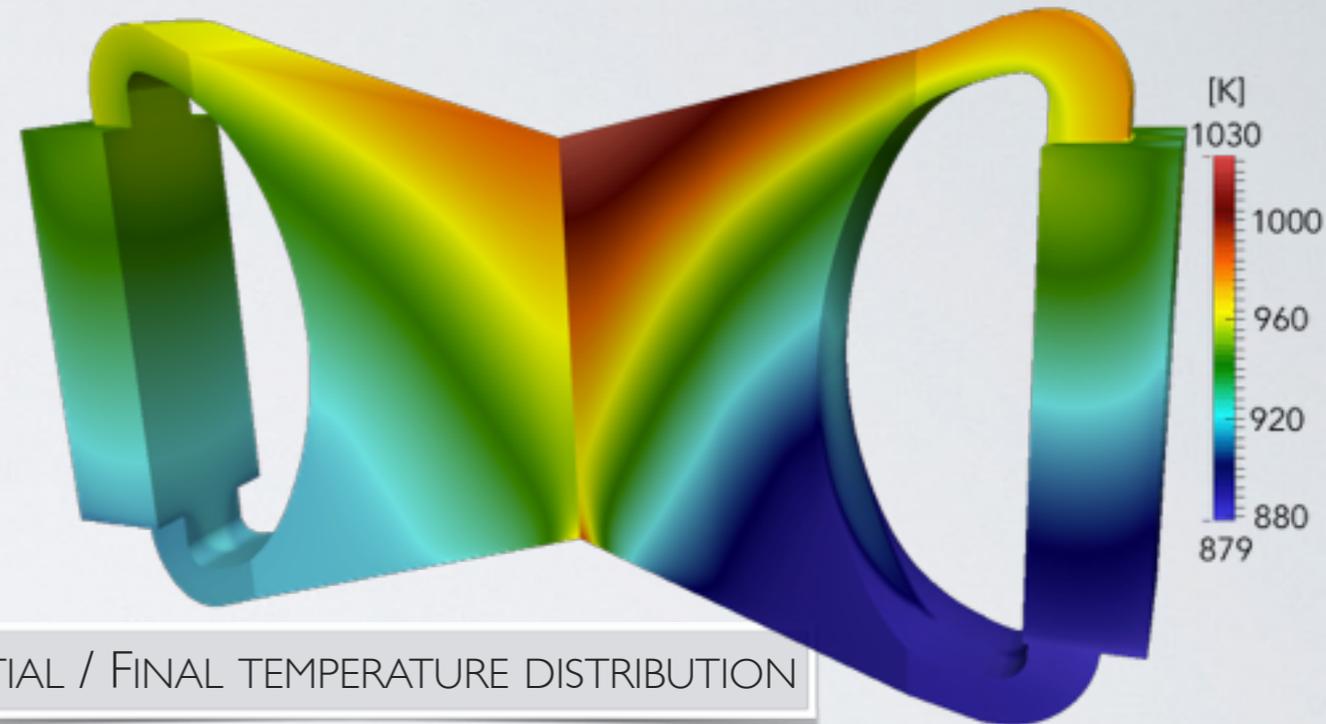
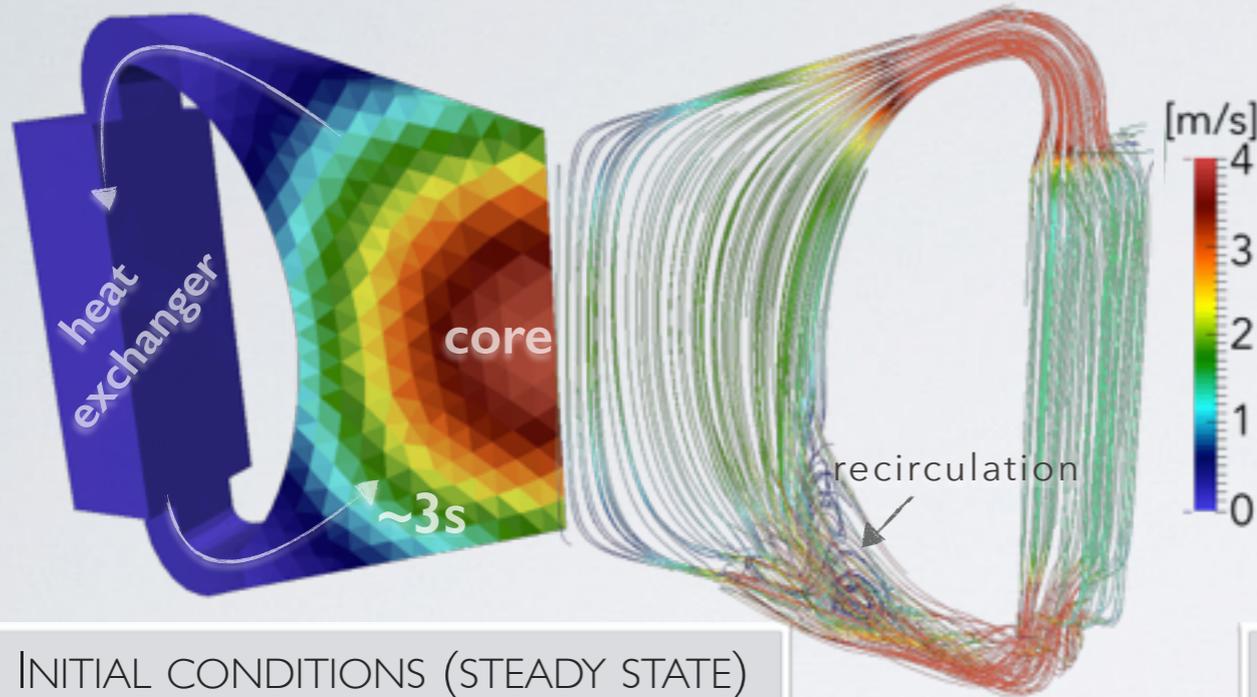


INITIAL / FINAL TEMPERATURE DISTRIBUTION



## II. APPLICATION CASES: INSTANTANEOUS OVER COOLING TRANSIENT

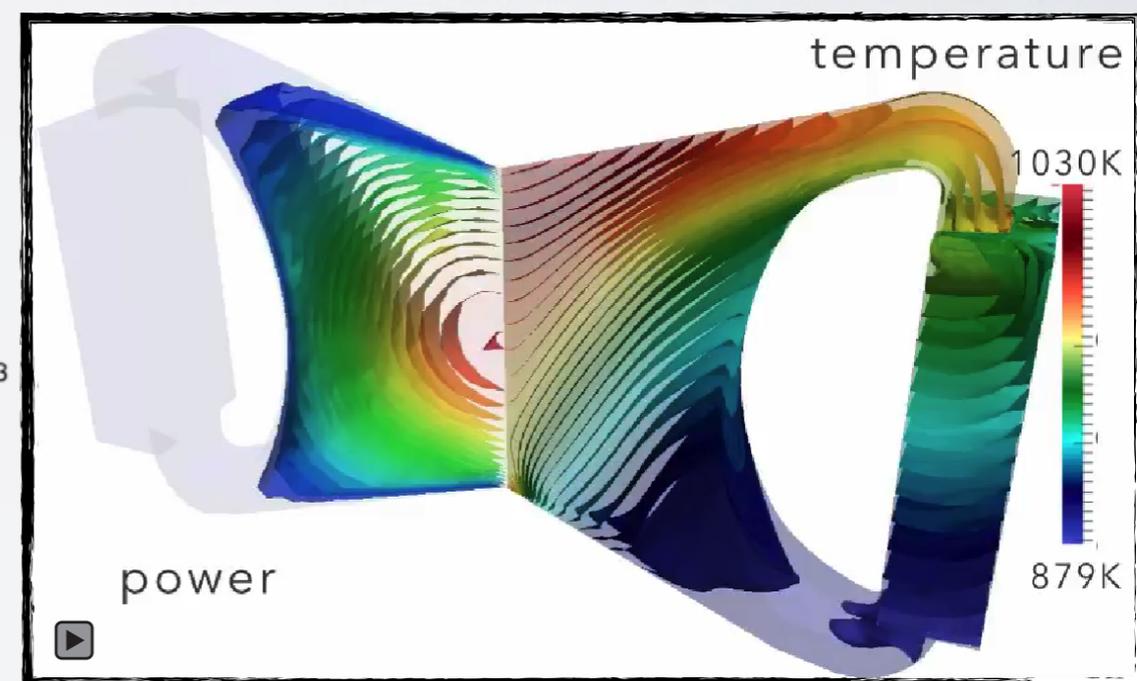
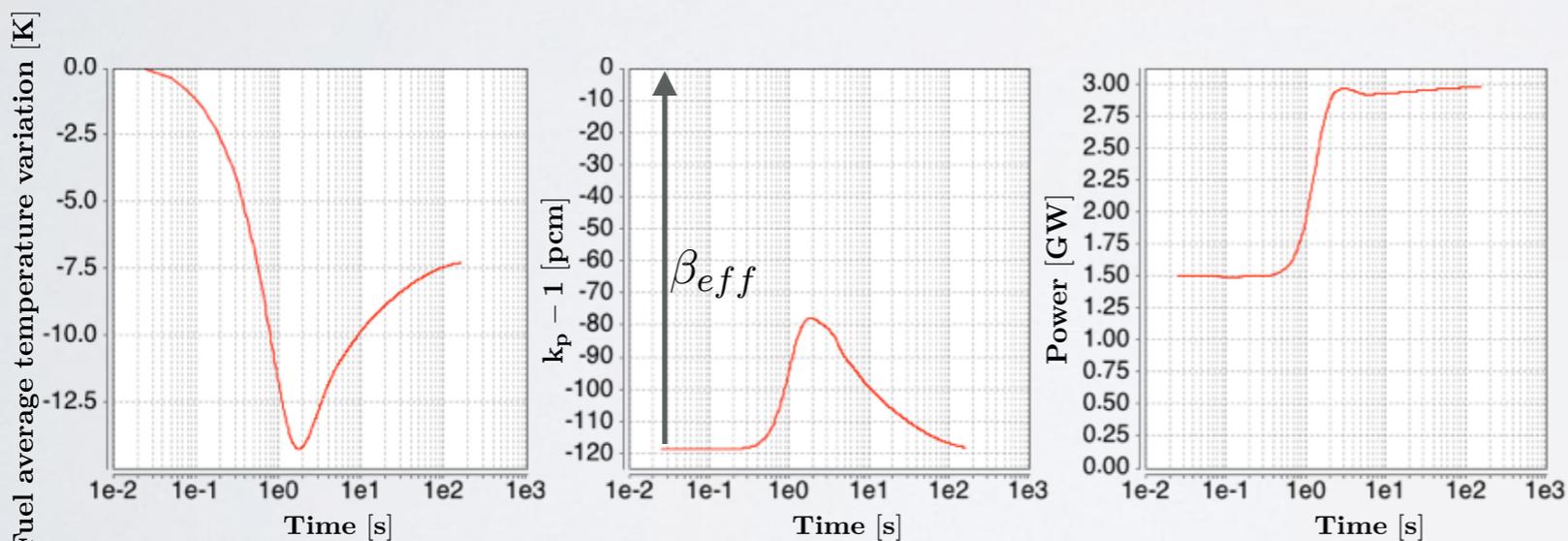
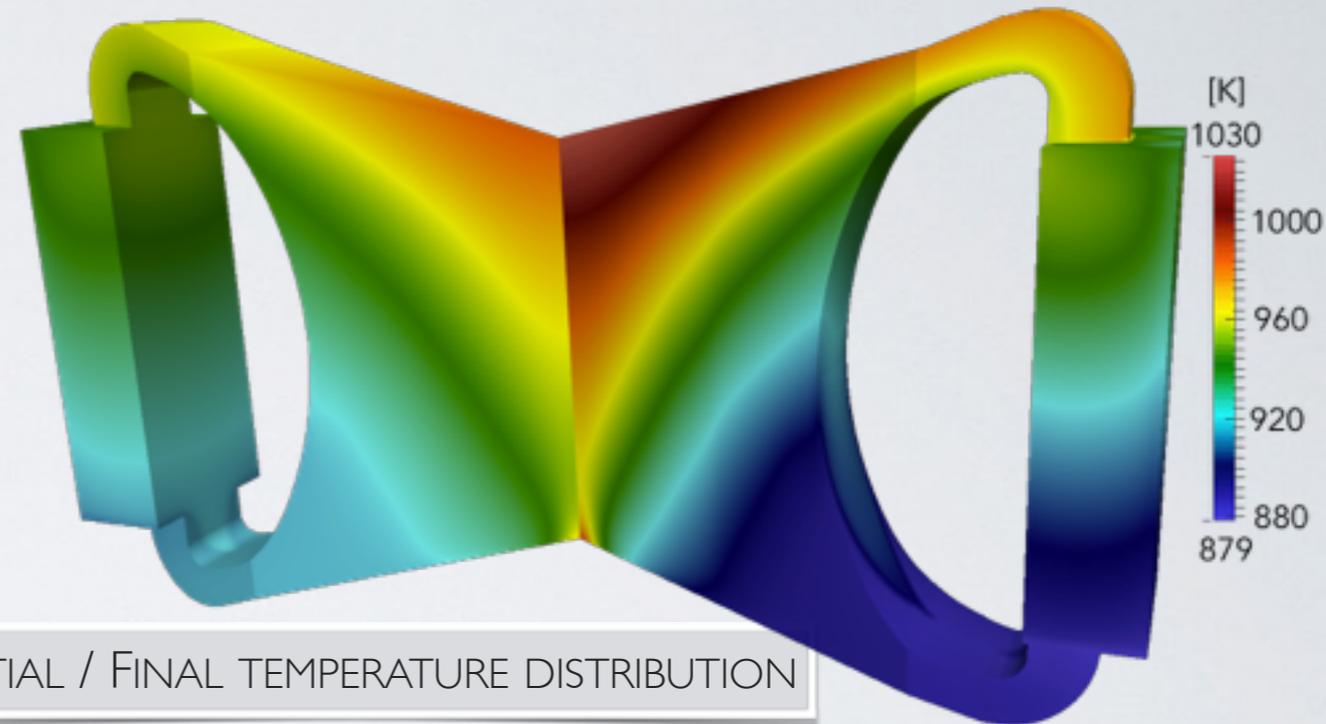
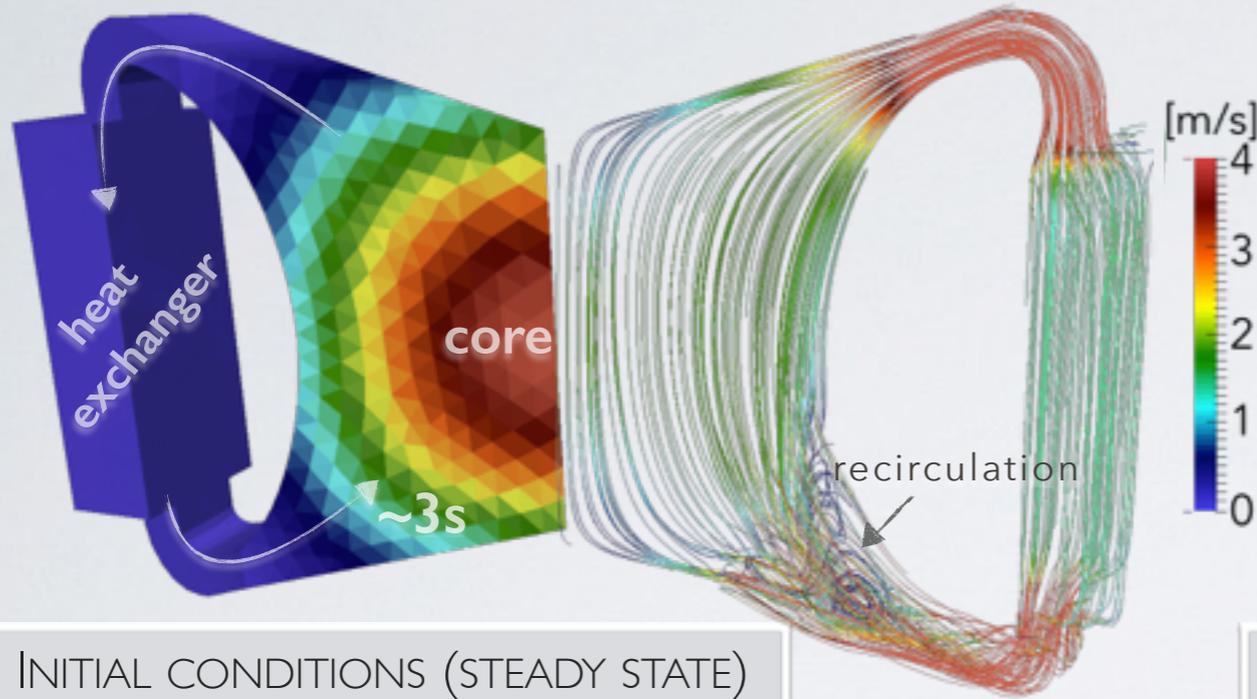
Intermediate fluid temperature variation in the heat exchanger



- Good behavior of the neutronics - T&H response of the MSFR
- TFM-OpenFOAM coupling operational
- High precision & low computational cost

## II. APPLICATION CASES: INSTANTANEOUS OVER COOLING TRANSIENT

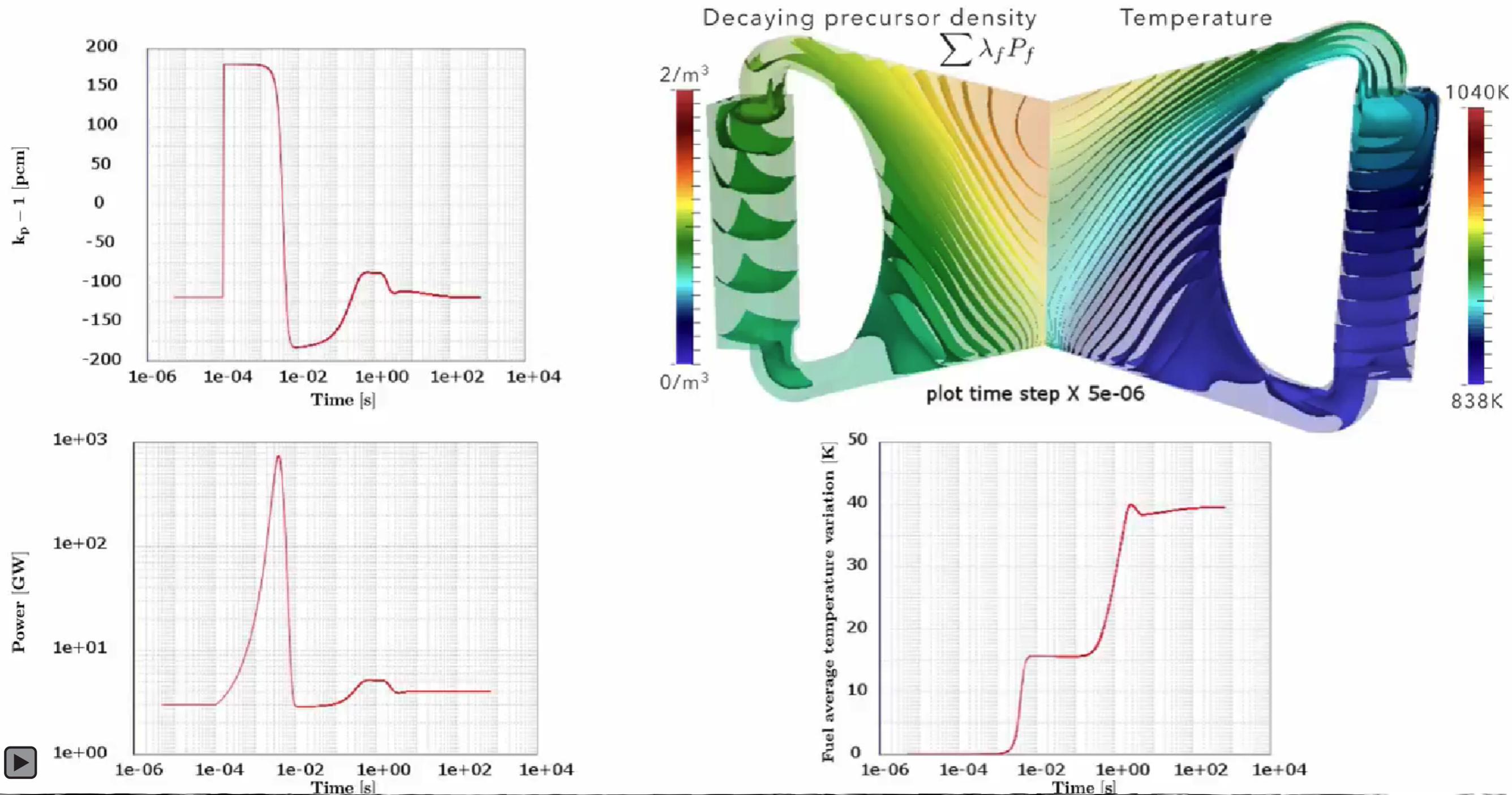
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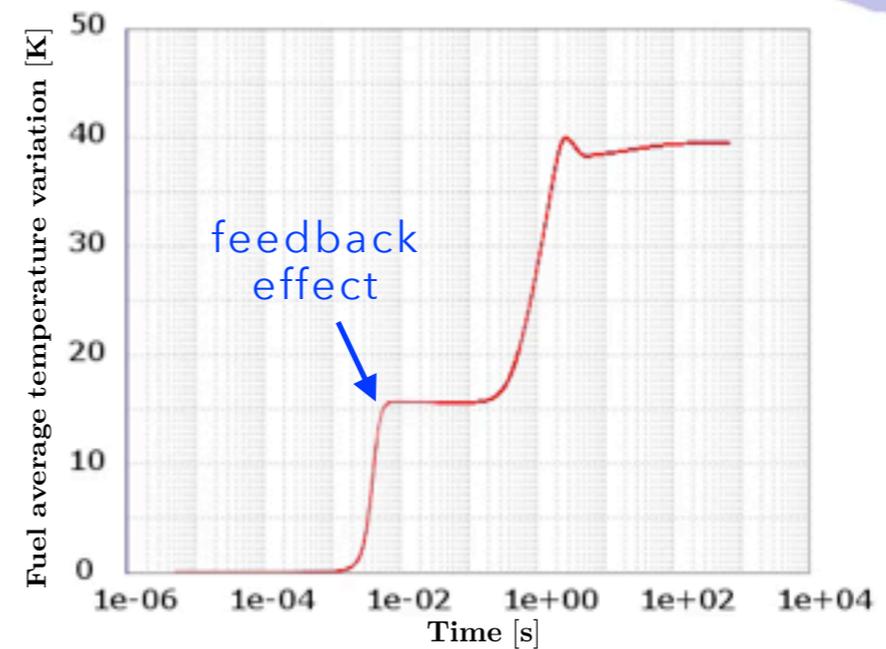
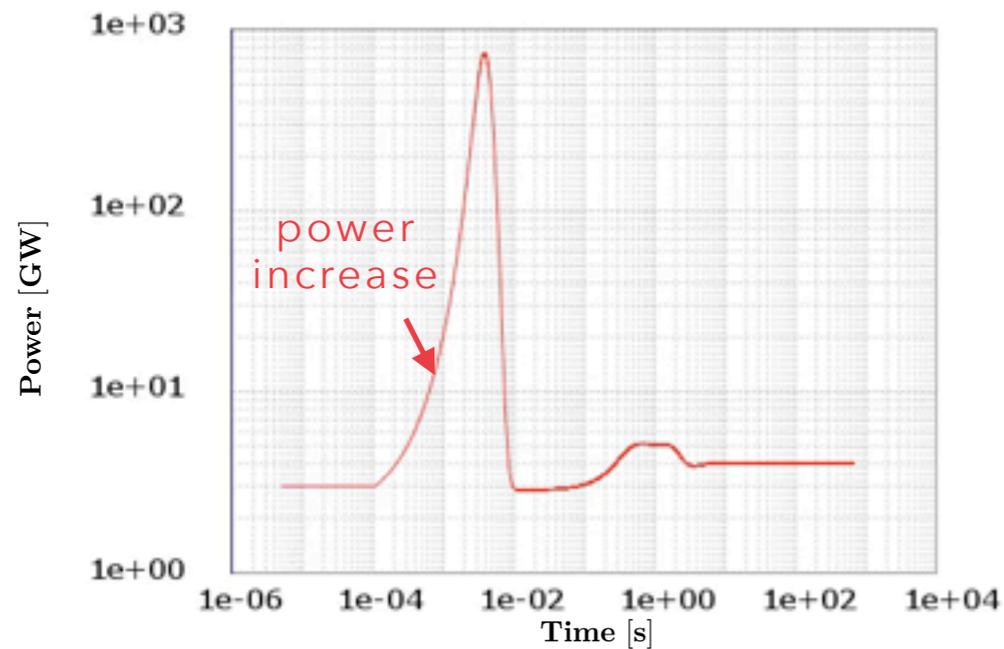
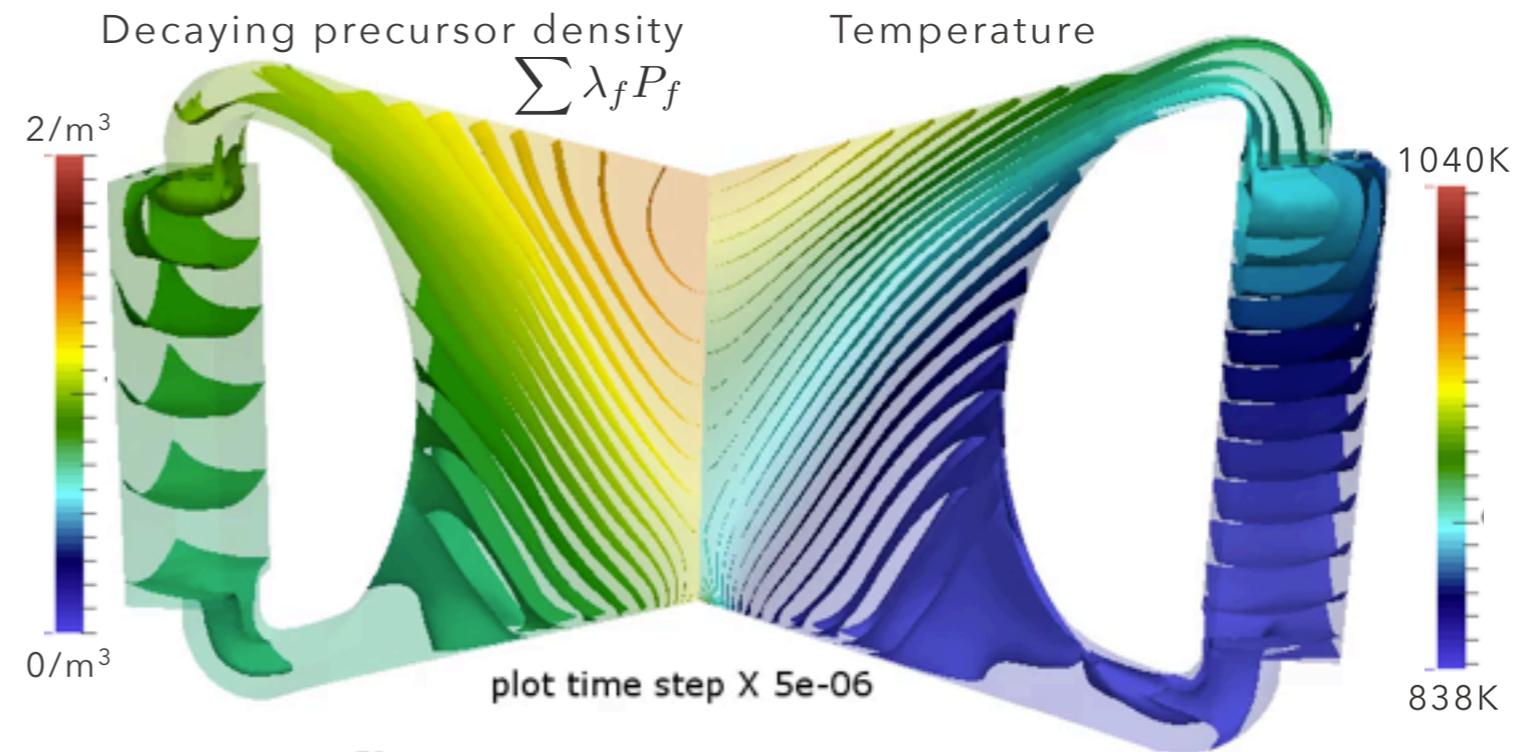
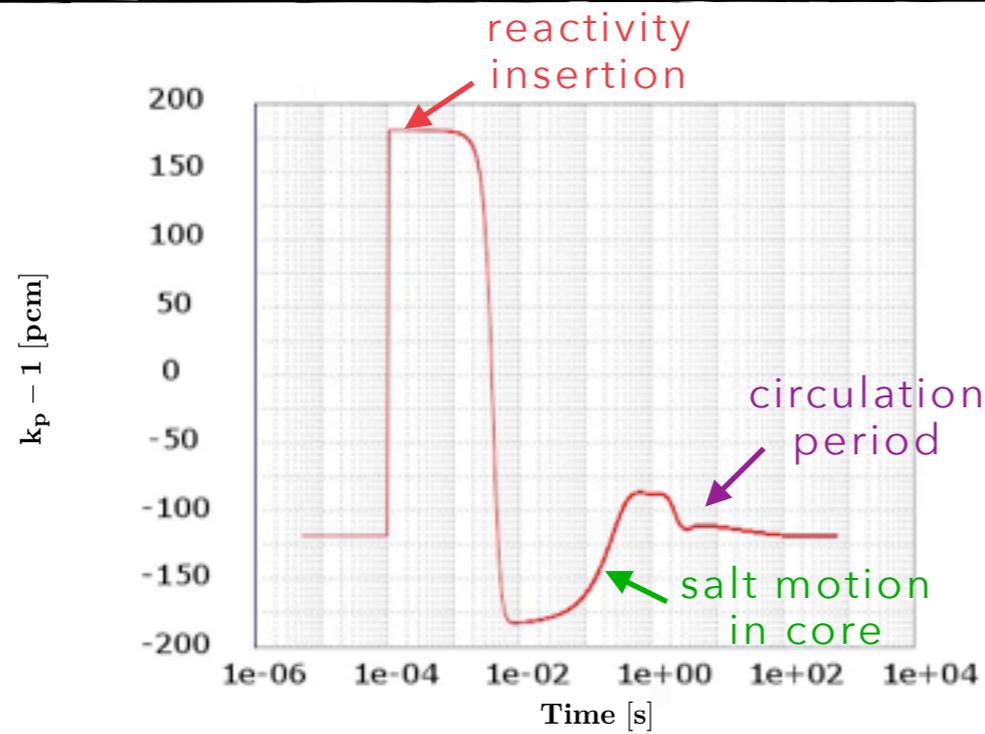
## II. APPLICATION CASES: INSTANTANEOUS REACTIVITY INSERTION OF 300 pcm

### Instantaneous & unrealistic reactivity insertion



## II. APPLICATION CASES: INSTANTANEOUS REACTIVITY INSERTION OF 300 pcm

### Instantaneous & unrealistic reactivity insertion

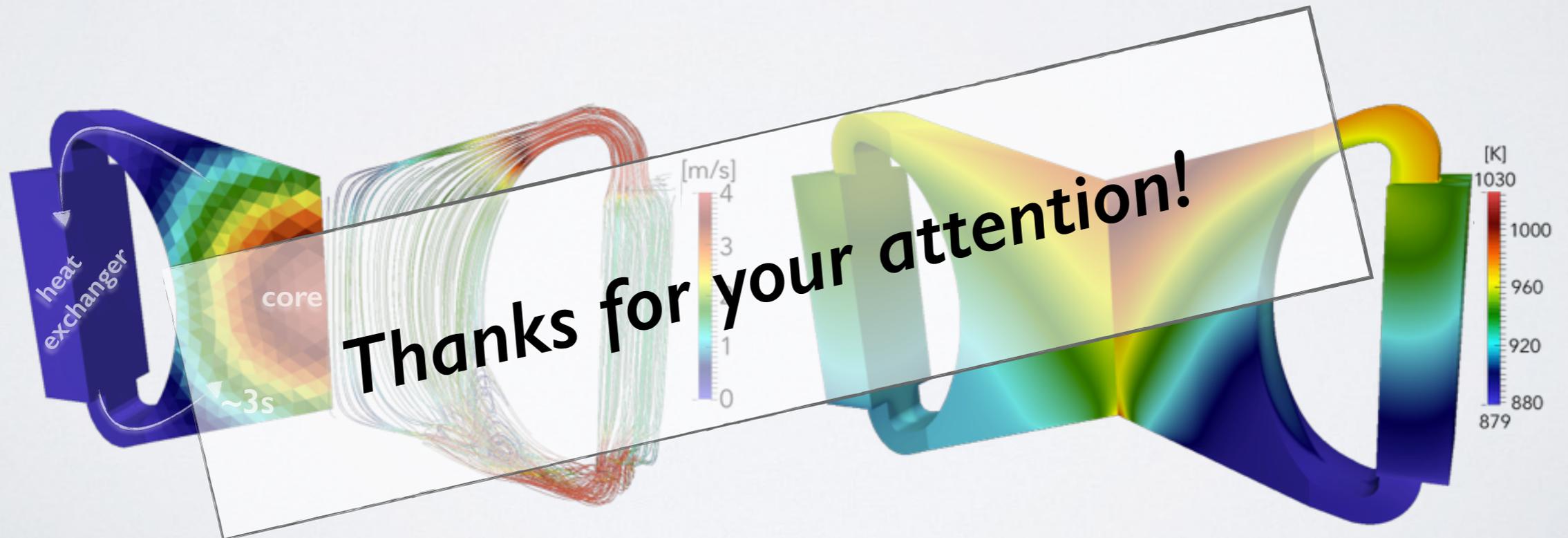


### Conclusions

- TFM-OpenFOAM coupling operational: high precision & low computational cost
- Implementation of the transient fission matrices calculation in the SERPENT code
- Good results for kinetics parameters and transient calculations

### Future work

- Compare the model to a dedicated numerical benchmark
- Include the sensibility to the crossed cells in the fission matrices
- Application of the TFM approach on different nuclear systems



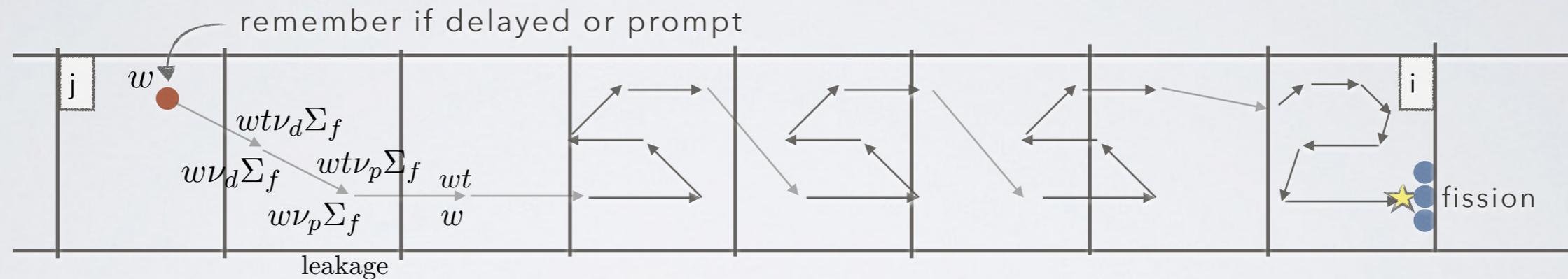
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## BACKUP SLIDES

# BACKUP SLIDES

## SERPENT ESTIMATION OF THE MATRICES:

During a classical critical calculation:



Simple explicite implementation: summing the neutron production of the fission events normalized by the neutron creation amount (prompt and delayed).

*Trouble: extremely slow convergence*

Better implicite implementation (*this work*): integration of the fission neutron production and absorption at each interaction (« delta tracking on »)

*Advantage: much more events per neutron history, improved statistics*

Advantages of the matrices estimation in a critical calculation:

- Utilisation of the correct emission spectrum
- Utilisation of the correct source neutron distribution inside the elementary volume ( $j$ )

# BACKUP SLIDES

## EFFECTIVE FRACTION OF DELAYED NEUTRON $\beta_{eff}$ CALCULATION:

We create the prompt and delay matrix operator:  
+ Eigenvalue & Eigenvector

$$\underline{\underline{\tilde{G}_{all}}} = \begin{pmatrix} \underline{\underline{\tilde{G}_{\chi_p \nu_p}}} & \underline{\underline{\tilde{G}_{\chi_d \nu_p}}} \\ \underline{\underline{\tilde{G}_{\chi_p \nu_d}}} & \underline{\underline{\tilde{G}_{\chi_d \nu_d}}} \end{pmatrix} \rightarrow k_{eff} \ \& \ \mathbf{N} = (\mathbf{N}_p \ \mathbf{N}_d)$$

Its importance:  
transpose matrix and Eigenvector

$$\underline{\underline{\tilde{G}_{all}^{adj}}} \rightarrow \mathbf{N}^* = (\mathbf{N}_p^* \ \mathbf{N}_d^*)$$

Finally, we can calculate the physical and effective fractions of delayed neutrons:

$$\beta_0 = \frac{\overset{\text{delayed}}{\sum \mathbf{N}_d}}{\underset{\text{total}}{\sum \mathbf{N}}} = \frac{k_{eff} \cdot \sum (\mathbf{N}_d)}{k_{eff} \cdot \sum (\mathbf{N})} = \frac{\sum \left( \underline{\underline{G_{\chi_p \nu_d}}} \mathbf{N}_p + \underline{\underline{G_{\chi_d \nu_d}}} \mathbf{N}_d \right)}{\sum \left( \underline{\underline{G_{all}}} \mathbf{N} \right)}$$

$$\beta_{eff} = \frac{\mathbf{N}_d^* \mathbf{N}_d}{\mathbf{N}^* \mathbf{N}} = \frac{\mathbf{N}_d^* \left( \underline{\underline{G_{\chi_p \nu_d}}} \mathbf{N}_p + \underline{\underline{G_{\chi_d \nu_d}}} \mathbf{N}_d \right)}{\mathbf{N}^* \underline{\underline{G_{all}}} \mathbf{N}}$$

importance  
weighting

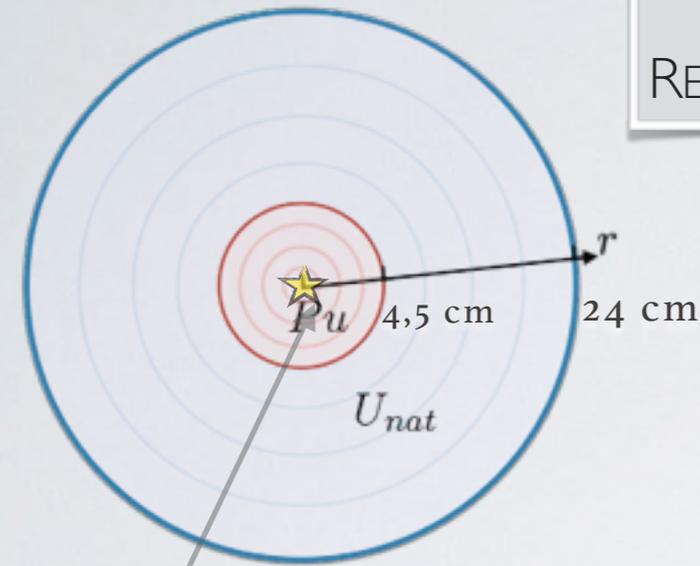


classic formulation:

$$\beta_{eff} = \frac{\int \psi^* \chi_d \nu_d \Sigma_f \psi \ dE \ d\Omega \ dE' \ d\Omega' \ dr}{\int \psi^* \chi \nu \Sigma_f \psi \ dE \ d\Omega \ dE' \ d\Omega' \ dr}$$

# BACKUP SLIDES

## BENCH CASE



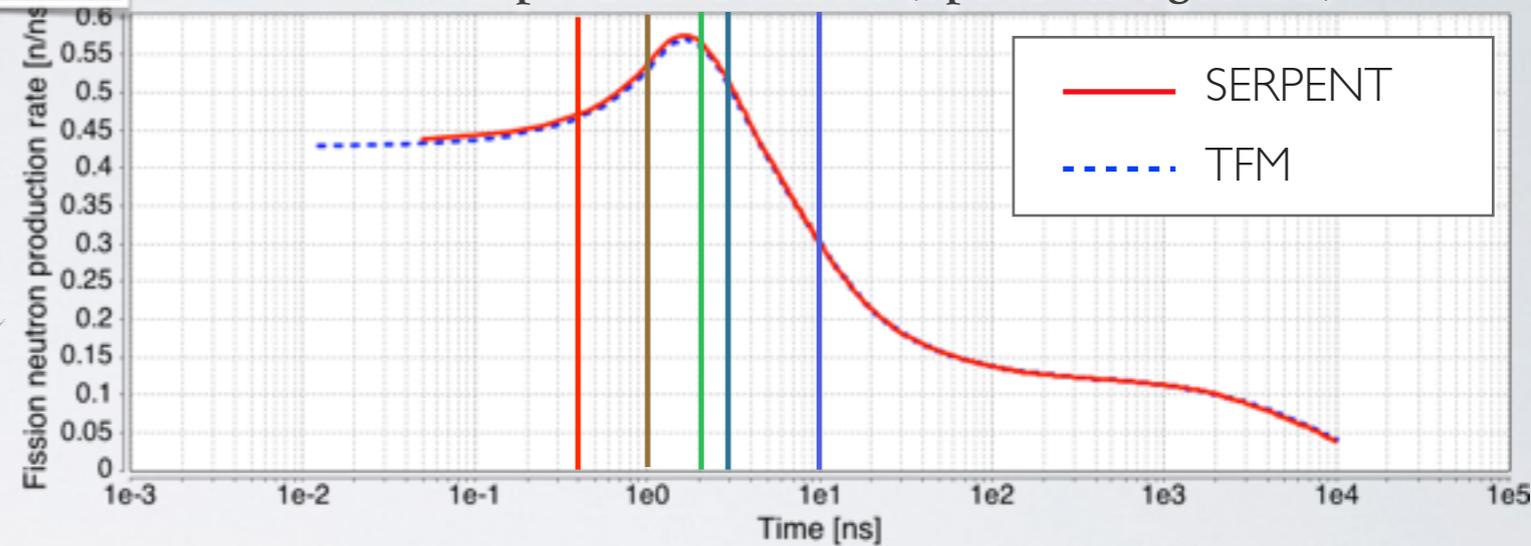
FLATTOP EXPERIMENT  
REFERENCE CODE: SERPENT

Flattop subjected to a neutron burst release

TFM EQUATION & IMPLEMENTATION TO CHECK:

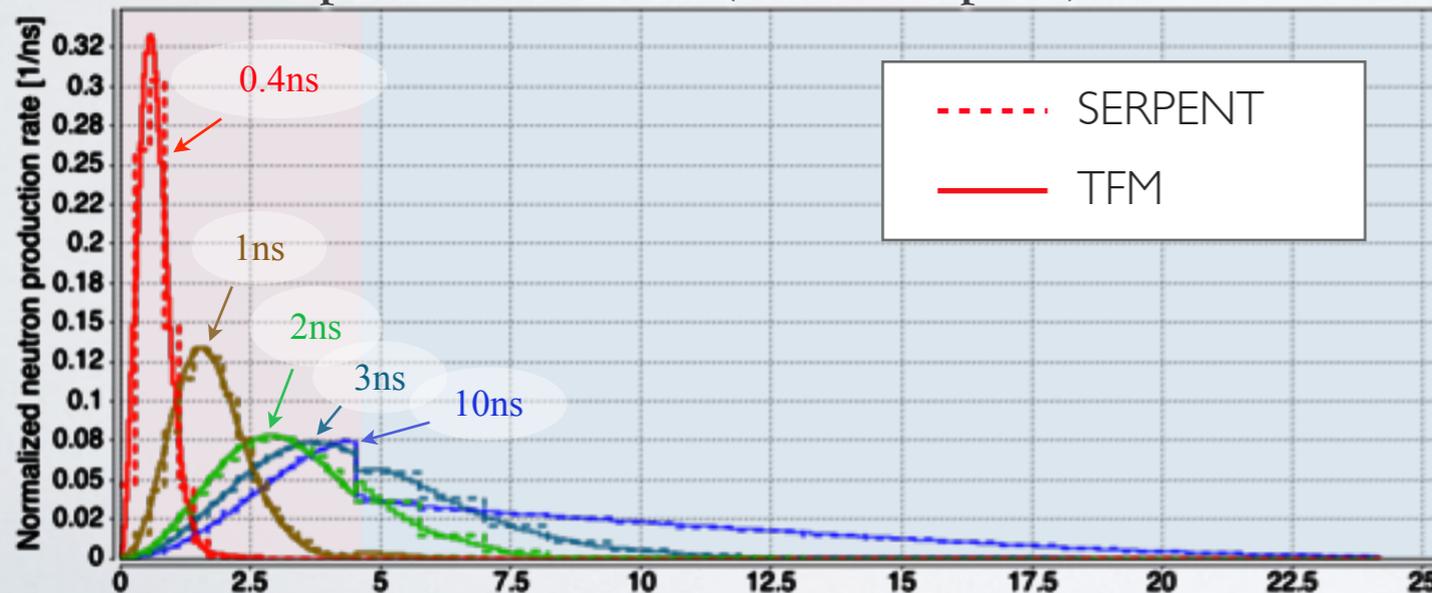
$$S(\mathbf{r}, t) = \left\langle G_{\chi_p \nu_p}(t' - t, \mathbf{r}', \mathbf{r}) \middle| S(\mathbf{r}', t') \right\rangle$$

## Temporal evolution (space integrated)



● Same evolution behavior of the neutron population through time

## Spatial evolution (time sampled)



● Good agreement of the spatial neutron propagation

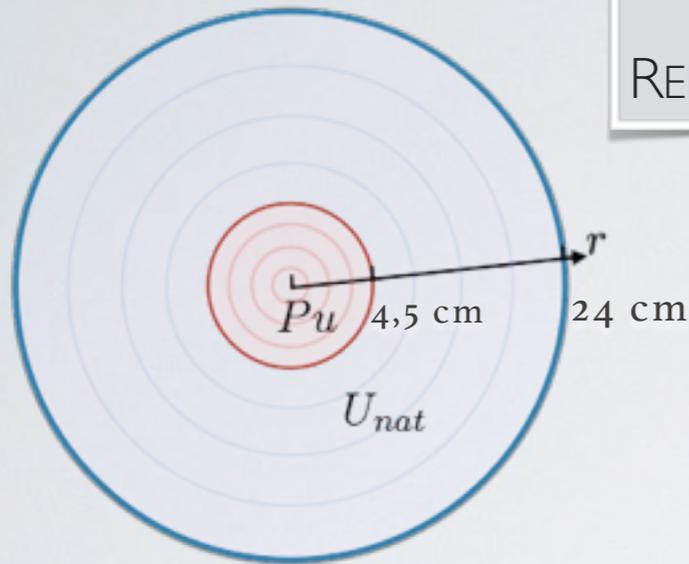
- The neutron burst is limited to the Pu area:  $k_p \gg 1$
- The neutron burst reaches the  $U_{nat}$  area:  $k_p \ll 1$

The neutron distribution tends to the equilibrium's one:  $k_p \sim 0.997$

# BACKUP SLIDES

## BENCH CASE

FLATTOP EXPERIMENT  
REFERENCE CODE : SERPENT



### Flattop kinetics calculated parameters:

- effective fraction of delayed neutron:

$$\beta_{eff}$$

- effective generation time:

$$\Lambda_{eff} = \frac{l_{eff}}{k_p}$$

### Experimental observable:

$$\alpha_{Rossi} = -\frac{\beta_{eff}}{\Lambda_{eff}}$$

method	$\beta_{eff}$	$\Lambda_{eff}$	$\alpha_{Rossi}$
TFM (this work)	$275 \pm 4 \text{ pcm}$	$13.351 \pm 0.03 \text{ ns}$	$0.206 \pm 0.004 \mu\text{s}^{-1}$
SERPENT IFP	$274 \pm 2 \text{ pcm}$	$13.24 \pm 0.02 \text{ ns}$	$0.207 \pm 0.002 \mu\text{s}^{-1}$
Experiment	-	-	$0.214 \pm 0.005 \mu\text{s}^{-1}$

- good agreement between TFM and SERPENT...
- ... and with the experimental measurements!

# BACKUP SLIDES

Reynolds Average Navier Stokes (RANS) equations:

Mass equation

$$\nabla \cdot (\overline{\mathbf{u}}) = 0$$

velocity  
↙

Momentum equation

$$\frac{\partial(\overline{\mathbf{u}})}{\partial t} + \nabla \cdot (\overline{\mathbf{u}} \otimes \overline{\mathbf{u}}) = -\frac{1}{\rho_0} \nabla \cdot \left( \overline{p} + \frac{2}{3} \overline{k} \right) + \nabla \cdot \left( \nu_{eff} \left( \frac{1}{2} (\nabla(\overline{\mathbf{u}}) + \nabla(\overline{\mathbf{u}})^t) - \frac{2}{3} \nabla \cdot (\overline{\mathbf{u}}) \underline{\underline{Id}} \right) \right) + \mathbf{g} \left( 1 + \beta_{boyancy} (\overline{T} - T_0) \right)$$

pressure  
↙

turbulent energy  
(provided by the turbulence model)  
↙

Energy equation

$$\frac{\partial \overline{T}}{\partial t} + \nabla \cdot (\overline{T} \overline{\mathbf{u}}) = \kappa_{eff} \Delta(\overline{T}) + S_{external}$$

temperature  
↙

