Physics in the XXIst century

Johann Collot
Laboratoire de Physique Subatomique
et de Cosmologie de Grenoble

Université Grenoble Alpes, CNRS/IN2P3

http://lpsc.in2p3.fr/collot

collot@in2p3.fr

Richard Feynman (1918-1988)

If you can't explain your grand mother what you are studying, maybe it's because you don't know very well what you are studying.

Scope / Objectives

Give a very broad overview of contemporary physics :

- very brief historical view
- principles , basic concepts and theories (with as little maths as possible !!!)
- universe & particles
- matter
- quantum information
- major puzzles and challenges
- modern definitions of units (new since 2019 ...)
- physics in the landscape of science and society

Will indeed focus on what we're trying to achieve, not what we will do

Consider it as a debate, not a prophecy

Physics

According to etymology, science of nature.

In history was also considered as natural philosophy or the philosophy of nature

In modern era, science of matter, space and time.

As a corollary, physics is concerned by all natural phenomena, and vice versa all natural phenomena should obey physical laws.

If a biological phenomena is proved to break a physical law, this law needs to be reconsidered.

Ernest Rutherford : ``(natural) Science is physics, anything else is stamp collecting''

Understand : natural science always makes use of physics

Vacuum : the never ending puzzle

Was not seriously considered in western science until XVIIth century
E. Torricelli, B. Pascal, O. Von Guericke ...

Was part of Chinese science and philosophy since the very beginning

Descartes - Newton controversy : action at a distance ?

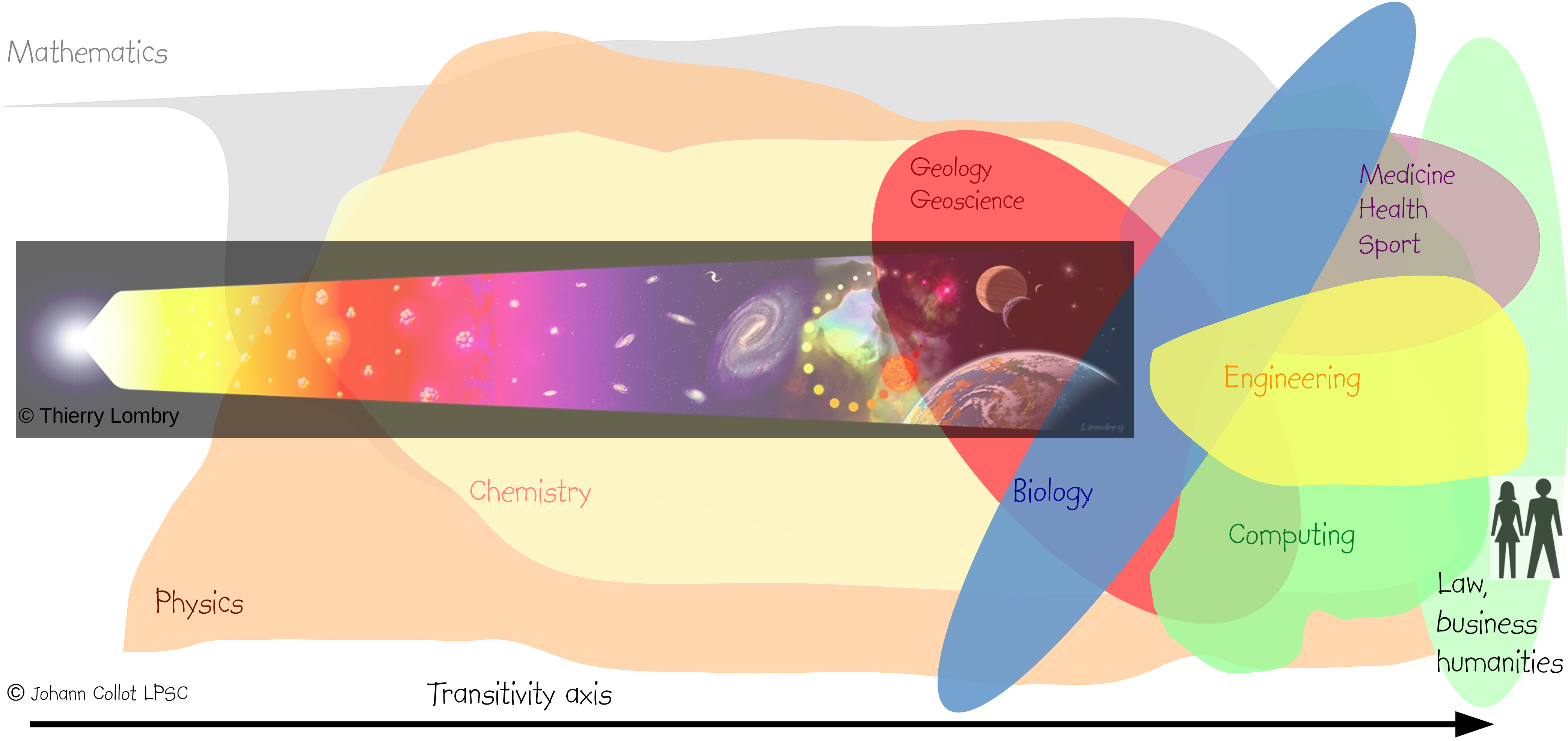
Switching on quantum mechanics, a universe can never be empty because of small scale fluctuations : the cosmological constant problem S. Weinberg, Rev. Mod. Phys. 61, 1 (1989)

Because of that, an "empty" universe could expand much faster than a matter dominated one : W. de Sitter universe

And the lowest scale universe (Planck Scale) is subjected to Quantum Gravity

In a sense, everything is potentially already there in vacuum !

Science transitive logic

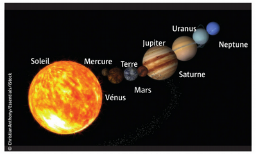


Very brief historical introduction to physics

Science & time scale

Univers age : 13.7 G years

educacion.uncomo.com



Solar system age : 5 G years

©ChristianAnthony/essential/istock

Advent of life : 3.5 G years

©Futura-Sciences

First hominini : 5 M years

ecoles.ac-rouen.fr

First homo-sapiens : 200,000-100,000 years

Genetic Literacy Project

Civilisation rise : 30,000 years

Chauvet Cave in France

Ancient Greece culmination : 2500 years

The School of Athens - Raphael's Fresco

Advent of modern science : 400 years

Isaac Newton painted by Jean Leon Huens

Legacy examples from ancient civilisations

- Sexagesimal (base 60) numeral system for measuring time, angles and geographic coordinates from Sumerians (3rd millenium BCE : Before Common Era)
- 24 hours/day from ancient Egypt
- Magnetism - Compass from China (300 BCE)
- Seismometer from China : Zhang Heng in 132 CE
- Paper (first century BCE) and printing from China (woodblock printing 220 CE and movable type 1040 CE)
- Trigonometry invented in India by Aryabhata (476-550 CE) (also relativity of motion, Earth proper rotation around its axis !)
- Hindu-Arabic numerals derived from Hindu decimal numerals (500 CE)
- Zero from India and south-eastern Asia (around 600-700 CE)
- Algebra first introduced by Al Khwarizmi (780-850 CE)

Chinese versus European technology in the XVth century

Zheng He admiral's 100m-long junk that crossed the Indian Ocean in 1413
6000 km direct crossing. 1000 passengers / junk

C. Columbus' Santa Maria

Science development

Scientific progress has been a very diffuse and global process, carried out in parallel and very often through the fruitful exchanges among different civilizations in the world.

The true history of all of this is not sufficiently known and many who actively contributed are not credited for their input to the advance of science.

The same is true today. Science makes progress not only because a few make remarkable contributions, but rather as millions (if not billions) of people test and improve it every day, and thereafter make it ever more solid.

Ancient era

Democritus (460-370 Before Common Era) : Atomic theory of the universe - To explain how a scent propagate in atmosphere - Also, discoverer of causality
But ideas largely dismissed until XIXth century

Plato (425-347 BCE) : Mathematics should be the basis of natural philosophy.
Postulated that uniform circular motion should be the geometrical model for planetary motion.

Aristotle (384-322 BCE) : Concepts and knowledge should be based on perception.
Father of ancient and medieval natural philosophy.

Eudoxus of Cnidus (390-337 BCE) : First geocentric model of planetary system.

Aristarchus of Samos (310-230 BCE) : First heliocentric model of planetary system.
But largely dismissed by others.

Universe according to Aristotle (384-322 BCE)

aether , place of perfection and eternal uniform circular motion

Moon

Sublunary sphere : place of 4 elements

air

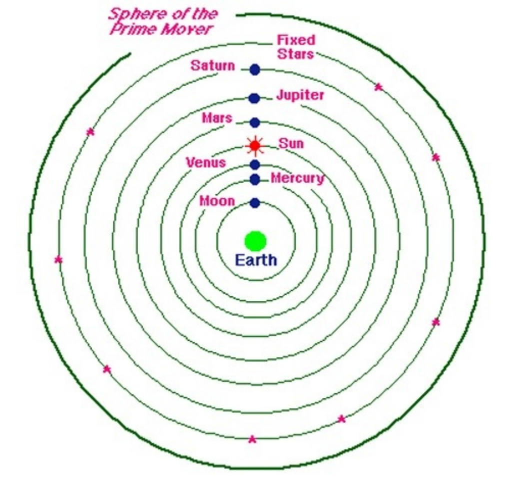
water

Earth

fire

heavy

light



Aristotle's Universe

Natural motion in sublunar sphere

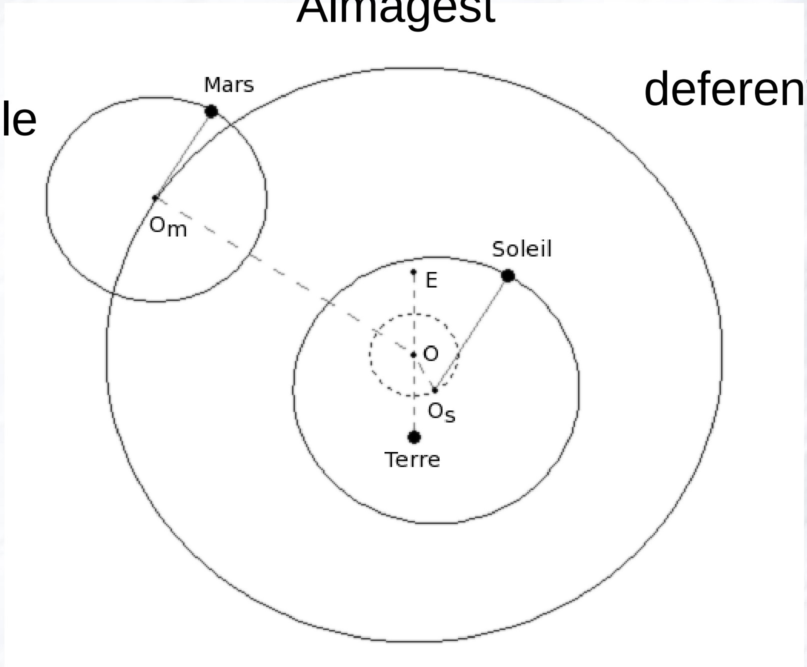
Everything in sublunary sphere was made of 4 elements

Today, the 4 elements read solid, liquid, gaz, radiation

Claudius Ptolemy

100 - 170 Common Era (CE)

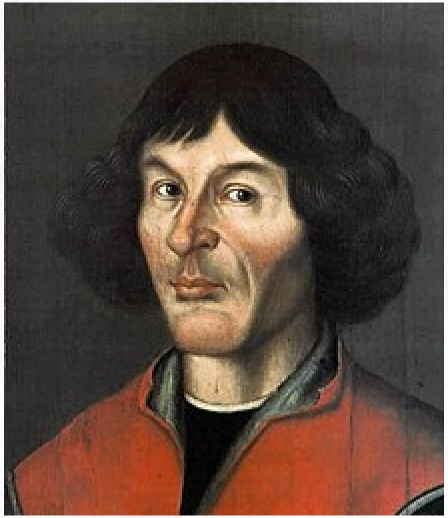
Almagest



A complicated model based on (too) many circular motions ! A real Swiss clock , but inaccurate at the end, not matching the experimental data and no underlying principle

Towards the scientific revolution

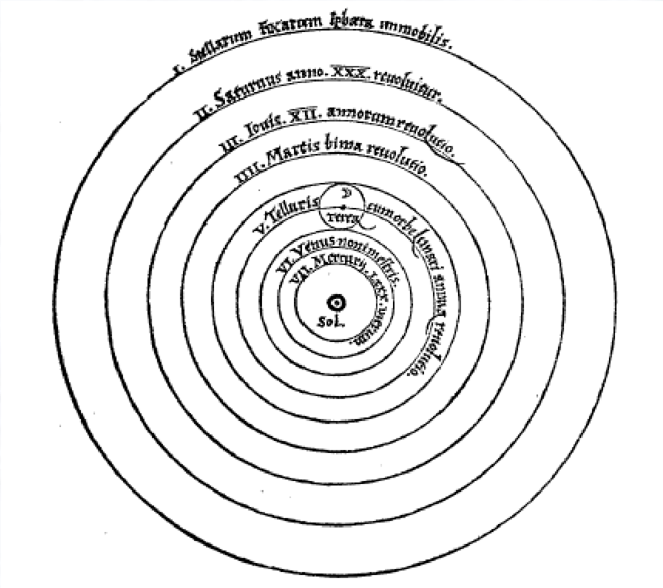
Nicolaus Copernicus



1473-1543

Motivated by the desire to simplify the model and personal observations, But still based on the uniform circular motion paradigm. Kept it confidential until the dusk of his life !

Geocentrism



On the revolutions of the celestial spheres
Published in 1542 in his last year !

Tycho Brahe

1546 - 1601

In 1576, he observed a supernova with no parallax, hence located much beyond Moon. But in Aristotelian science, space beyond Moon is perfect and immutable.

Tycho realized that this was wrong ! But if this was wrong, maybe other assertions in Aristotelian science could be wrong.

In 1577, he observed a comet crossing the crystalline spheres holding Mercury & Venus ! Hence, these spheres (orbia) could not be made of matter as claimed in Aristotelian science.

He also pointed out that observations made by Ptolemy and Copernicus were inaccurate.

Tycho Brahe II

Supported by Frederick II of Denmark, he spent 20 years on Ven Island to improve the measurements of planet trajectories and in particular of Mars.

He built the first modern observatory : Uraniborg , with a library, students, technicians. Denmark spent 0.8% of its GNP during 20 years in Uraniborg.

With naked eyes, he managed to measure planets' apparent angles with a precision of 36 arc-seconds : 0.35 mm at 2 m !

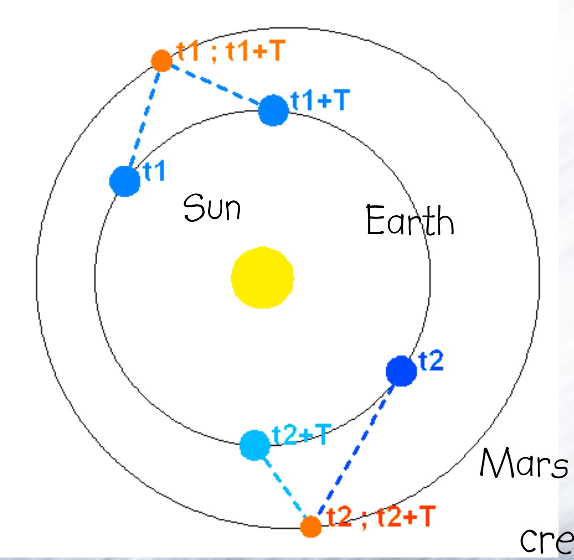
In 1599, he settled in Prague, where he hired Johannes Kepler in 1600 with the mission to compute the planet trajectories

Johannes Kepler

Kepler thought he could do it in a few weeks. It will cost him six years instead !

1571 - 1630

From Brahe's legacy, Kepler benefited of 10 years of Mars orbit data with an angle precision of one arc-minute.



credit : ASM

First Kepler law : 1609

The orbits of planets are ellipses with the Sun located at one of the two focuses.

After 2000 years, end of the uniform circular motion paradigm. Advent of modern science.

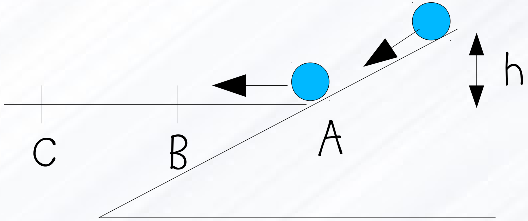
Galileo Galilei (1564-1642)

Portrait by Justus Sustermans

Father of experimental (terrestrial) mechanics and experimental methods

Was the first scientist to understand that there was a need to isolate a phenomenon by comparing orders of magnitude and try to study it experimentally : i.e. devising model & carrying out measurements

Gravity, Free fall , instantaneous velocity , principle of relativity, inertia.



Measuring instantaneous free fall velocity :

$$v=\sqrt{2gh}$$

Classical physics

Isaac Newton (1643-1727)

- From Kepler's and Galileo's findings, he built the first coherent and "universal" theory of physics, which is still "usable" to describe the motion of objects in many situations (moderate speed, week gravitational field and macroscopic size)
- The mathematical principles of natural philosophy published in 1687 mark the beginning of modern science, what we now call classical physics. Modern physics rather denotes XXth century advances (Quantum mechanics, relativity …)
- It also marked the beginning of massive applications : clocks, navigation, mechanical and (thermal) machines ; of the industrial revolution…

Industrial revolution, thermodynamics, statistical mechanics, atomic theory..

- Work of many, among whom Sadi Carnot (1796-1822), James Joules (1818-1889), Julius Robert von Mayer (1814-1878), Rudolph Clausius (1822-1888), Lord Kelvin (1824-1907) , Ludwig Boltzmann (1844-1906) ...

- Led to :
  - first principle : Total energy of isolated systems is conserved
  - second principle : Entropy , $S = k \ln(\Omega)$ is always growing for isolated systems
  - absolute-zero temperature : Kelvin scale , based on Carnot efficiency, is not reachable in a finite number of operations (in other words cannot be reached !)

James Maxwell (1831 - 1879)

Second force unification : electricity, magnetism and optics are different phenomena of same physical underlying reality

But also application of field theory to physics ! Vacuum is populated by electromagnetic fields (B and E)

In fact the notion of field (field lines) was guessed by Michael Faraday who had very little mathematical skills.

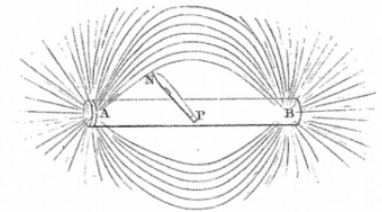


Fig. 25.

Maxwell predicted electromagnetic waves that were discovered by H. Hertz (1857 - 1894) in 1888.

Legend says Hertz had no idea what kind of applications could be made of EM waves !

Analytical or theoretical mechanics

Joseph-Louis Lagrange (1736-1813)

William Hamilton (1805-1865)

They both contributed to reformulate classical mechanics into its more powerful form that we now use, based on energetic concepts (kinetic energy, potential energy ...) and more general coordinates ...

They introduced the Lagrangian and the Hamiltonian of a system ... Very powerful concepts that we use in quantum mechanics and in field theory.

Led to synchronous stationary action principle, one of the most fundamental postulate of physics :

$$\delta(S)=0=\delta\left(\int_{t_1}^{t_2} L\,dt\right)$$

Along a trajectory, S (action) is stationary.

Towards modern physics

Light and relativity

- Classical mechanics prevailed as the physical science paradigm until the beginning of the XIXth century. Our infinite, steady state universe had a Euclidian geometry and was endowed with an absolute reference system : absolute coordinate frame + absolute time

- In 1801, Thomas Young's double-slit experiment marked the beginning of wave optics. Light was then modeled as waves that propagate in a new medium : aether, that filled the universe, as light is observed to reach us from distant stars. The absolute reference system was bound to aether

- A few decades later, Michael Faraday postulated the existence of the electric and the magnetic fields. Taking this as a starting point, James Maxwell edified the theory of the electromagnetic fields : Maxwell equations. Electricity, magnetism, optics were now seen as different manifestations of a common physical reality. Aether bore the electromagnetic fields. Light was one species of a more general concept : electromagnetic waves, which were discovered by Heinrich Hertz in 1888.

Light and relativity

- But the electromagnetic waves appeared to propagate at a constant velocity in vacuum (aether) irrespective of the velocities of their source or observer. This became one of the biggest scientific puzzles of the end of XIXth century.

- Some even thought that classical mechanics – a pure mathematically-based science - that once unified the terrestrial and astronomical mechanics, could even be considered as a different discipline from physics.

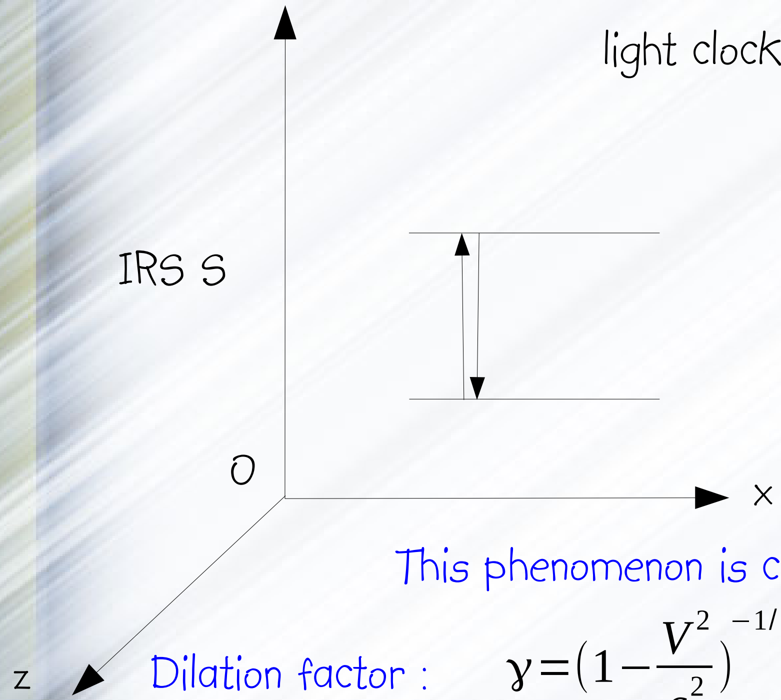
- The puzzle was progressively elucidated by Lorentz, Fitzgerald, Poincaré … and finally by the young Albert Einstein in 1905 in a famous article called : On the electrodynamics of moving bodies published in the German review Annalen der Physik.

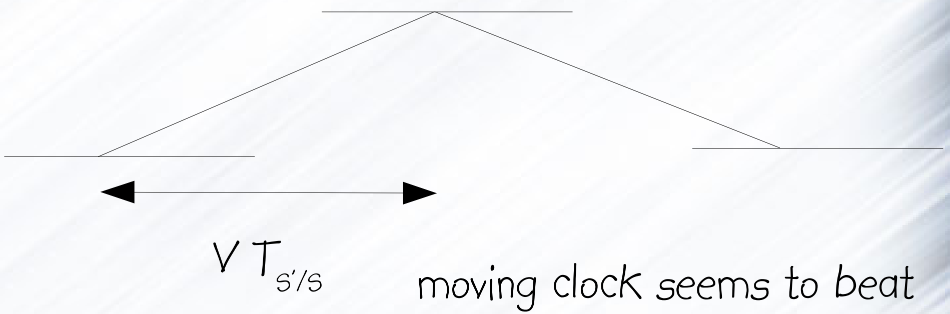
Special Relativity

- Based on two principles :
  - All physical laws are the same in all inertial (non-accelerated) reference systems (called principle of relativity).
  - The modulus of the limit (light) velocity in vacuum observed in an inertial reference system is universal and does not depend upon the state of motion of the source nor of the observer. (linked to causality principle, cause and effect should be ordered in time)

Special relativity

light clock between two perfect mirrors





moving clock seems to beat slower, and hence moving time is bigger

This phenomenon is called time dilation.

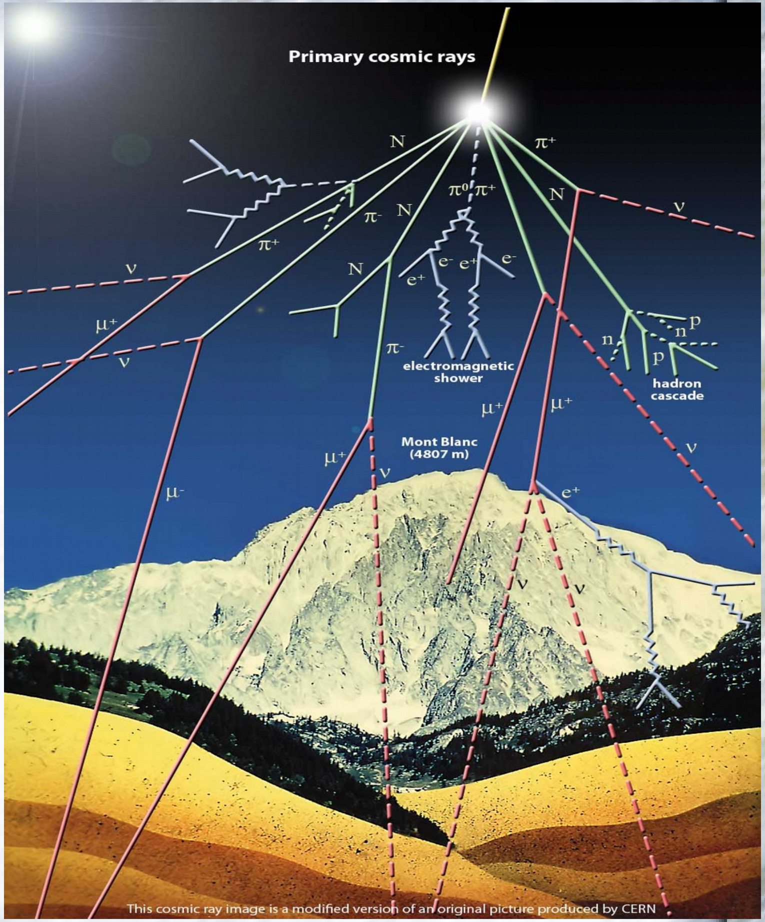
Dilation factor : $\gamma = \left(1 - \frac{V^2}{c^2}\right)^{-1/2}$

Special relativity

In collisions of cosmic rays that take place in the external layer of the atmosphere (>10 km from the ground), short-lived muons are created. Their lifetime is around 2 μs when measured at rest. Most of these muons carry a big kinetic energy. But even travelling at a speed close to that of light, these muons could only cross around 600 m.
However for an *observer bound to the earth surface* (10 km away), their lifetime is dilated by a big factor that depends upon their travelling velocity. So high-velocity muons reach the ground.

For an *observer bound to a muon*, at rest w.r.t. it, this muon only lives 2 μs. But it now *sees* the atmosphere being contracted by a γ factor. So it also reaches the ground.

All in all, *both observers see the same physics phenomenon* although their relative perception of time and space differs.



Minkowski Spacetime

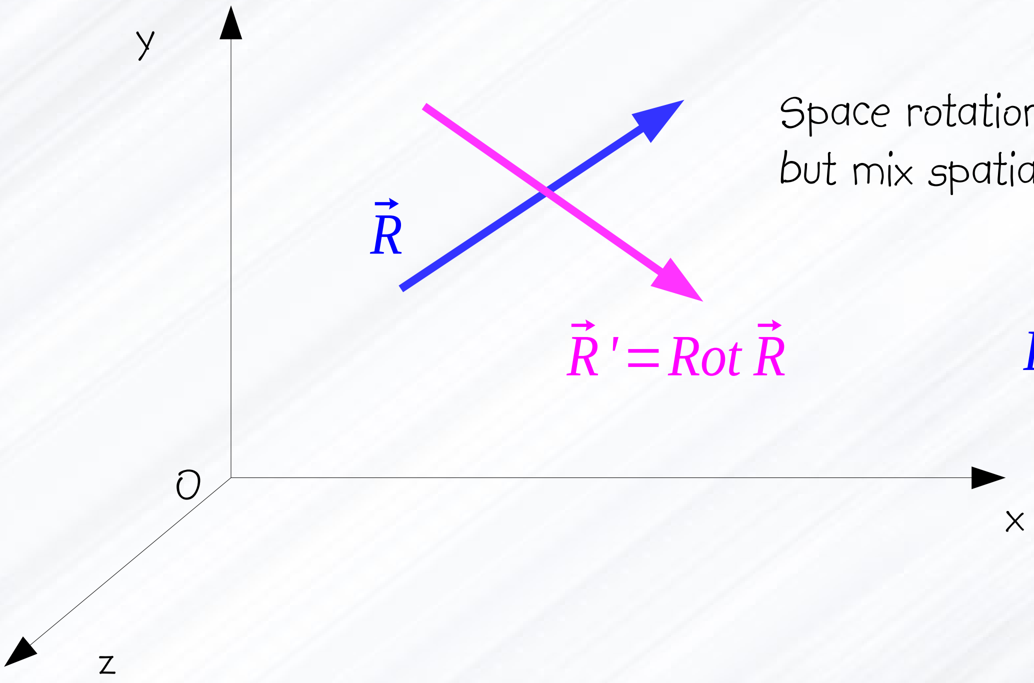
Hermann Minkowski's opening words during the 80th Assembly of German Natural Scientists and Physicians in September 1908.

The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. Their tendency is a radical one. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve independence.

As a corollary, a point in spacetime is localized by 4 coordinates : (ct, x, y, z) and it's called an event.

PS : H. Minkowski was Einstein's math professor in Zürich ! Legend says Einstein skipped Minkowski's lectures ...

Living in a 3-dimensional space



$\vec{R}$

$\vec{R}' = Rot\,\vec{R}$

Space rotations keep distance/length constant but mix spatial coordinates ...

$$R = \sqrt{x^2 + y^2 + z^2} = R' = cte$$

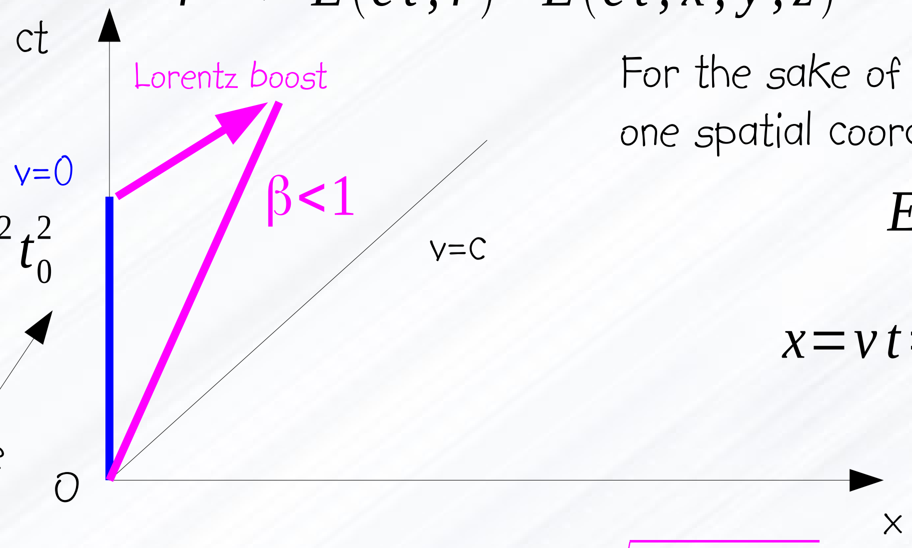
Living in a 4-dimensional space

$$\vec{r} \Rightarrow E(ct,\vec{r}) = E(ct,x,y,z)$$ event, position in 4-D space

For the sake of simplicity, we will only consider one spatial coordinate : x

$$E(ct,x,0,0)$$

$$x = vt = \frac{v}{c}ct = \beta ct \Rightarrow ct = \frac{x}{\beta}$$



$$E^2 = c^2 t_0^2$$

Proper time

$$E^2 = \sqrt{(ct)^2 - x^2} = c^2 t_0^2$$

Minus otherwise cannot be made constant as both coordinates are bigger

Living in a 4-dimensional space

The modulus of a 3-D (classical space) vector is left invariant under space rotations that mix spatial dimensions.

It is the same for a 4-D Minkowski vector, but how to mix (rotate) time and spatial dimensions ?

The answer is that a Lorentz boost mixes time and space, while keeping the modulus of the 4-D vector (4-vector) invariant.

So to go from classical physical quantities to relativistic quantities, one needs to extend classical 3-vectors to 4-vectors (or tensors) !

Also physical events are within light cone ($\beta < 1$), therefore proper time is always > 0 then causality is always enforced (cause precedes effect). Democritus was right !

Living in a 4-dimensional space

Example : 4-velocity $\vec{v} \Rightarrow V(\gamma c, \gamma \vec{v})$ $\gamma = (1 - \frac{v^2}{c^2})^{-1/2}$

$$V^2 = \gamma^2 (c^2 - v^2) = \gamma^2 c^2 (1 - \beta^2) = c^2 \text{ with } \beta = \frac{V}{c}$$

The modulus of all four-velocities is equal to c which is a constant. In SR, all objects travel with a velocity c but in a four-dimensional space.

Living in a 4-dimensional space

Example : 4-momentum $\vec{p}=m\vec{V} \Rightarrow P=mV$ $\gamma=\left(1-\frac{v^2}{c^2}\right)^{-1/2}$

$P=(m\gamma c, m\gamma\vec{v})$ $P^2=m^2V^2=m^2c^2$ Modulus of P constant if m and c are constant

$P=(E/c,\vec{p})$ $E=\gamma mc^2$ E called total energy

$$P^2=m^2c^2=E^2/c^2-\vec{p}^2 \Rightarrow E^2=\vec{p}^2c^2+m^2c^4$$

Mass may be converted into motion and motion into mass (e.g. in Nuclear reactors, particle colliders ....)

Travelling at a speed approaching c towards the center of the Milky Way .

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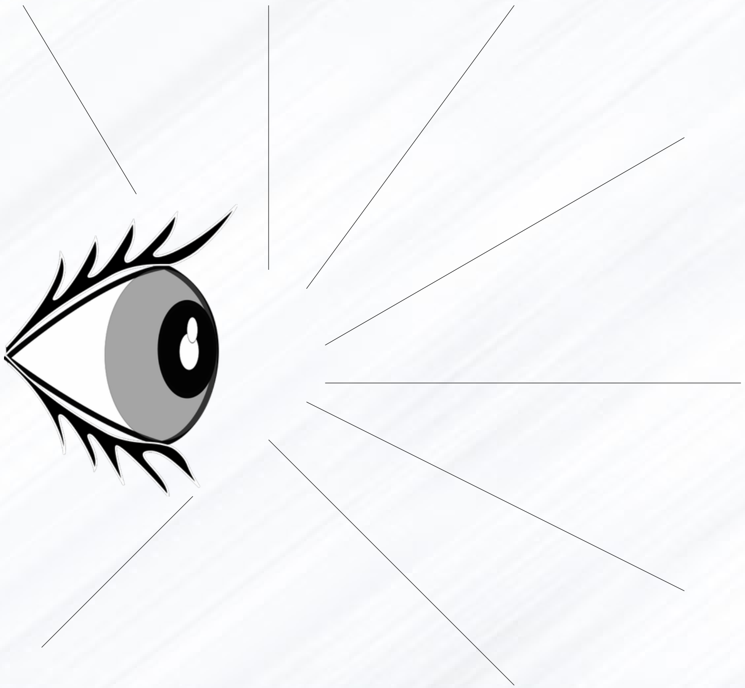
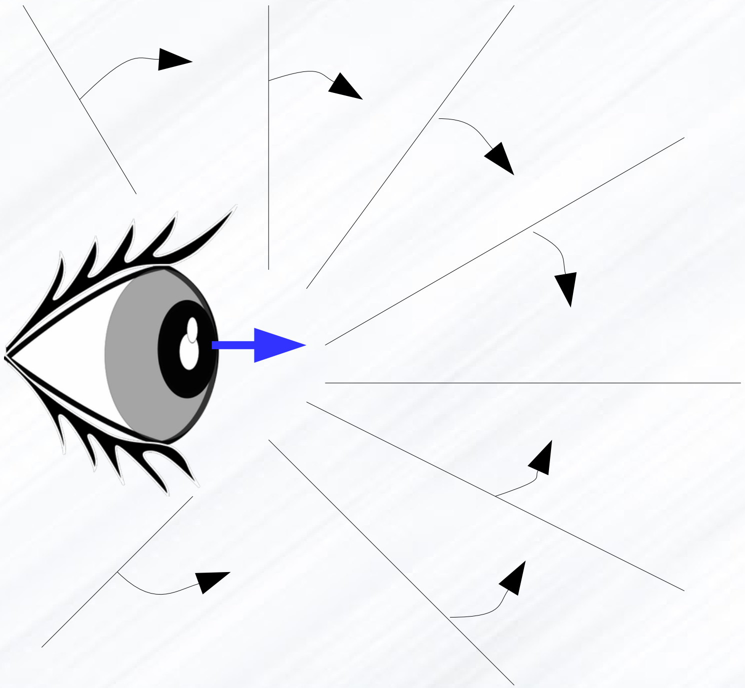
Relativistic light aberration



observer at rest
with respect to
light source.

Relativistic light aberration

Observer moving at a high velocity with respect to light source

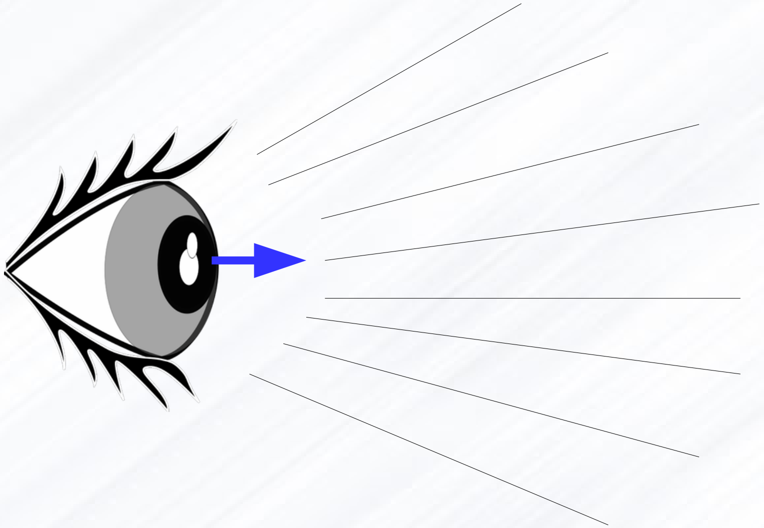


Light always travels at c whatever the angle. So as longitudinal component increases, transverse component is reduced, and then the angle diminishes. Similar to running against rain.

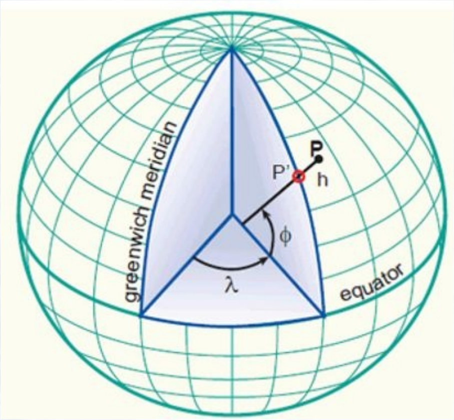
Relativistic light aberration

Observer moving at a high velocity with respect to light source

Light always travels at c whatever the angle. So as longitudinal component increases, transverse component is reduced, and then the angle diminishes. Similar to running against rain.

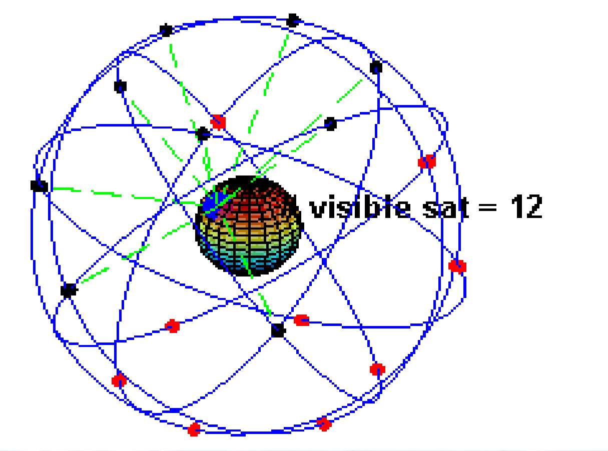


Global Positioning System (GPS)



To localize an event near the earth surface, we need to measure in a given IRS the time at which it happened, its latitude, its longitude and its altitude : (t, h, λ, φ) in geodesic coordinates (ellipsoid).

The global positioning system directly uses the limit velocity universality principle to determine the event coordinates.



A constellation of satellites orbit at a distance of 20 200 km from the earth surface. Their orbits and number are set so as to guarantee that each terrestrial point simultaneously views a minimum of 4 satellites.

5 terrestrial stations are in charge of precisely localizing all satellites and of synchronizing their atomic clocks.

Global Positioning System (GPS)

Every 30 s, each GPS satellite broadcasts electromagnetic wave signals (L1= 1575.42 MHz , L2=1227.6 MHz) containing t_i , the emission time, and $\vec{r}_i$, its position coordinates at the emission time.

A terrestrial GPS receiver simultaneously records a set of at least 4 equations w.r.t. at least 4 satellites :

$$d_i = |\vec{r} - \vec{r}_i| = c(t - t_i)$$

c=299 792 458 m/s

where : $\vec{r}$ and t are the unknown time and coordinates of the receiver.

Since there's at least 4 independent equations, the GPS receiver computer can determine the 4 unknown quantities : t, altitude, latitude, longitude

We will also see later, that the synchronization of the satellite clocks is regularly needed (many times per day) to correct the relativistic time drift.

Exercise

Traveling at a velocity of 0.98 c, how much time would it take to reach Proxima Centauri, the nearest star (apart from the Sun) located at 4.2 light-years from the Earth.

What would then be the time elapsed on the Earth ?

Solution

Longitudinal dimension gets contracted by :

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}=\frac{1}{\sqrt{1-0.98^2}}=5$$

$$t=4.2/5/0.98\simeq 0.86\,y$$

Traveling time as observed from the Earth :

$$t=0.86\,x\,5.0\simeq 4.29\,y$$

Traveling time much shorter for the traveler !
But need to develop the technology to reach 0.98 c !

Reasons that call for a generalization of relativity

The principle of relativity as stated so far is only valid in inertial reference systems : i.e.: non-accelerated frames. This implies that at least one system of that sort exists, i.e. a system which is totally isolated, or in other words which is subjected to no forces. If we consider that gravity is an infinite range force, this may not be the case in any place of our universe. On top of that it is ethically unacceptable to admit that the form of the physics laws would differ if the reference frame were to be accelerated.

Electromagnetic forces are correctly described in special relativity. They stem from the Maxwell equations which may easily be made Lorentz invariant. But this is not the case of the Newtonian gravitation force which is a classical mechanics concept. After 1905, Einstein tried to include gravity into special relativity but his findings contradicted the law of free fall in a gravitation field : in particular the vertical free fall speed would depend upon the horizontal speed of the body. This was one more reason to correct/extend special relativity.

Einstein paid a little more attention to Galileo Galilei's free fall law.

General principle of relativity

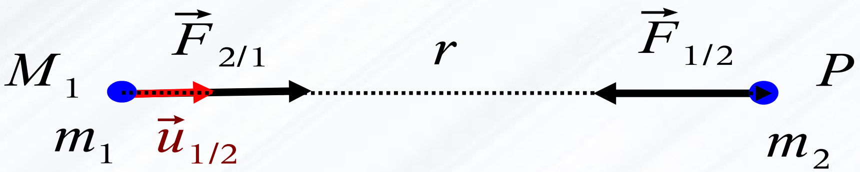
All systems of reference are equivalent with respect to the formulation of the fundamental laws of physics.

At first sight, this may appear in contradiction with our immediate observations when we are *subjected* to acceleration, as we clearly feel when an acceleration is exerted or not, i.e. when a lift *starts* or *stops* : the higher the acceleration, the bigger our feeling is.

But Einstein understood that contrary to our first impressions, this *observation* precisely confirmed the principle of general relativity.

Gravitational field

Between two point-like masses m_1 and m_2 located at points M_1 and P separated by a distance r :



$$\vec{F}_{1/2} = -G\frac{m_1 m_2}{r^2}\vec{u}_{1/2}$$

$G = 6.6743\ 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$

$\vec{g}(P) = -G\frac{m_1}{r^2}\vec{u}_{1/2}$ gravitational field – Property of space induced by m_1 located at M_1

$[g] = m\,s^{-2}$

$\vec{F}_{1/2} = m_2\vec{g}(P)$ If g is produced by a general mass distribution (distribution of point-like masses), its expression differs from the formula given above, but this relation remains correct.

The gravitational field is a more general concept than the gravitation force, like the electric and magnetic fields in electromagnetism. When a gravitational perturbation is induced, it propagates through the gravitational field at the limit velocity with respect to an IRS.

Equivalence Principle

If gravitation is the only interaction exerted on m_2 : $\vec{F}_{1/2} = m_2 \vec{g}(P) = m_2 \vec{a}(P) \Rightarrow \vec{g}(P) = \vec{a}(P)$

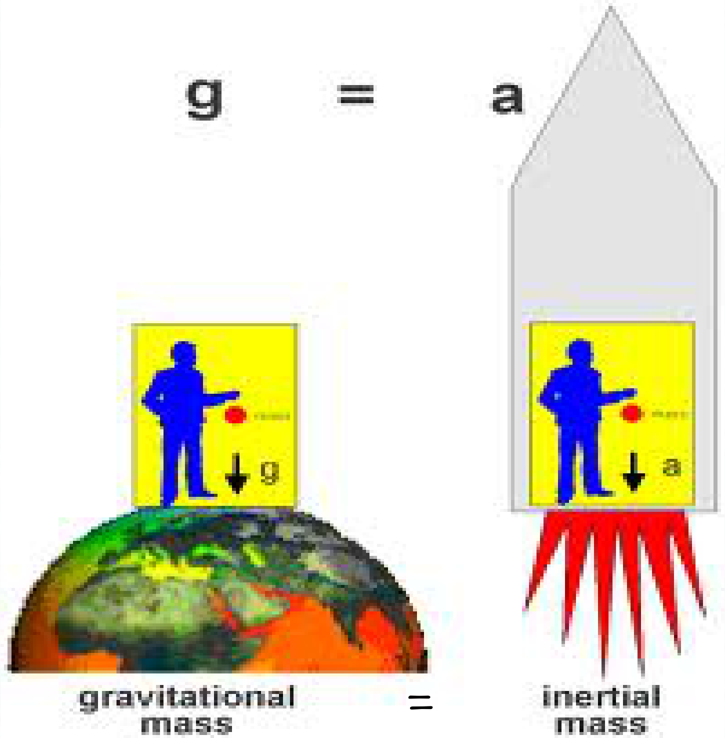
A gravitation field is locally equivalent to an acceleration field (Equivalence Principle of Einstein)
if the gravitational mass is equal to the inertial mass (experimentally verified with great precision) (Equivalence Principle of Newton or Weak Equivalence principle).

In a closed and local (small) laboratory, no experiments can be performed that will distinguish between the effects of a gravitational field and the effects due to an acceleration field with respect to an inertial reference frame.

Conclusion : Going from an IRS to an accelerated frame is like adding gravity !

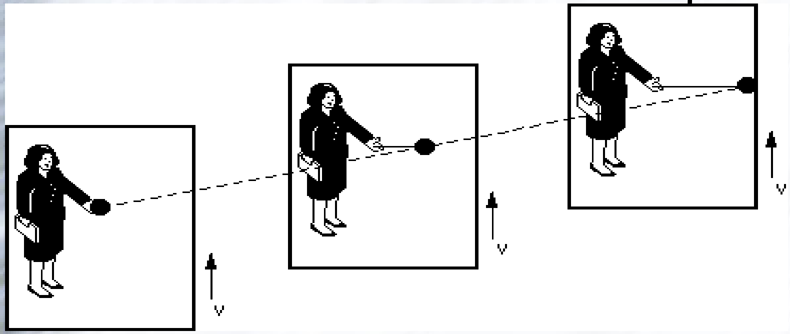
Inversely, removing gravity, e.g. in a local free falling laboratory is like going to an inertial reference frame.

Equivalence Principle

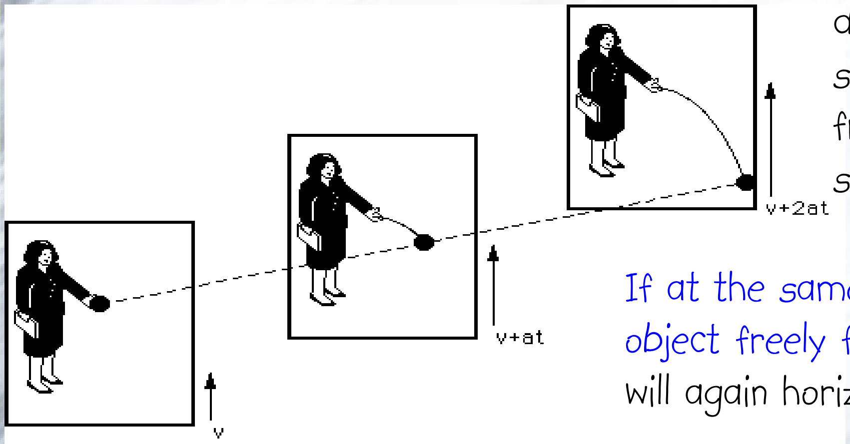


The rocket accelerates
with respect to an
Inertial Reference System

Free fall revisited in the context of the Equivalence Principle



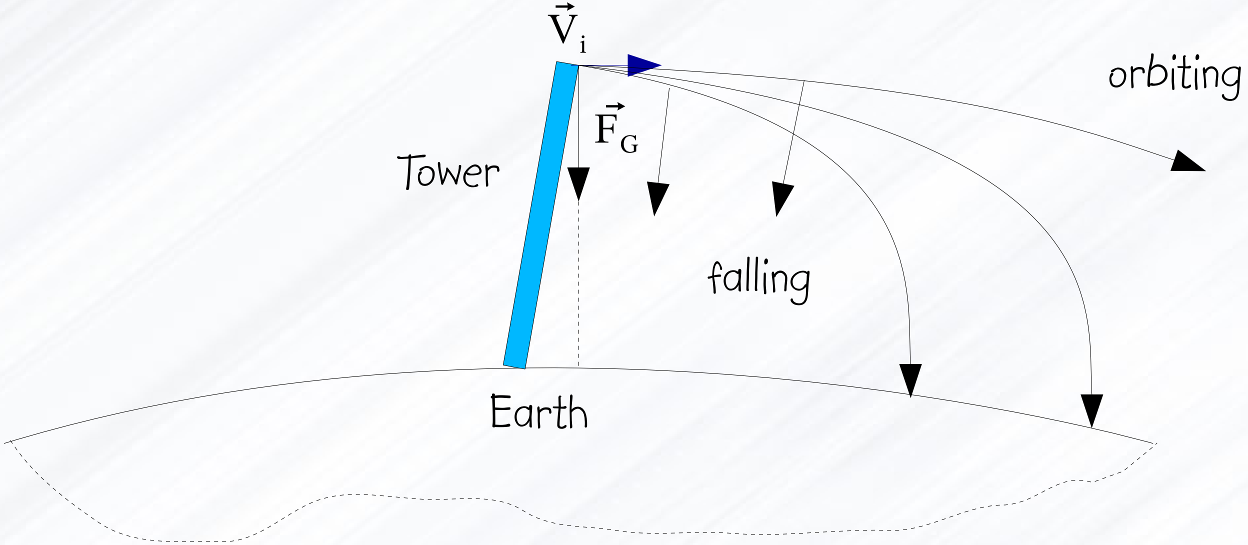
The ball is horizontally thrown in an IRS moving at the constant speed v. Observed inside, it moves horizontally and hits the opposite wall. An external observer can see the ball moving along a straight and slanted line.



The ball is now horizontally thrown in a uniformly accelerated reference system. Observed inside, it moves downwards along a parabola. An external observer still sees the ball moving along a straight and slanted line. The free fall is the same as in a fixed reference system subjected to a constant gravitational field.

If at the same time as the ball is released, the observer lets another object freely fall, with respect to this new reference object, the body will again horizontally move, removing gravity.

General relativity



The strong equivalence Principle

The gravitational motion of a small test body depends only on its initial position in spacetime and on its initial velocity, and not on its constitution.

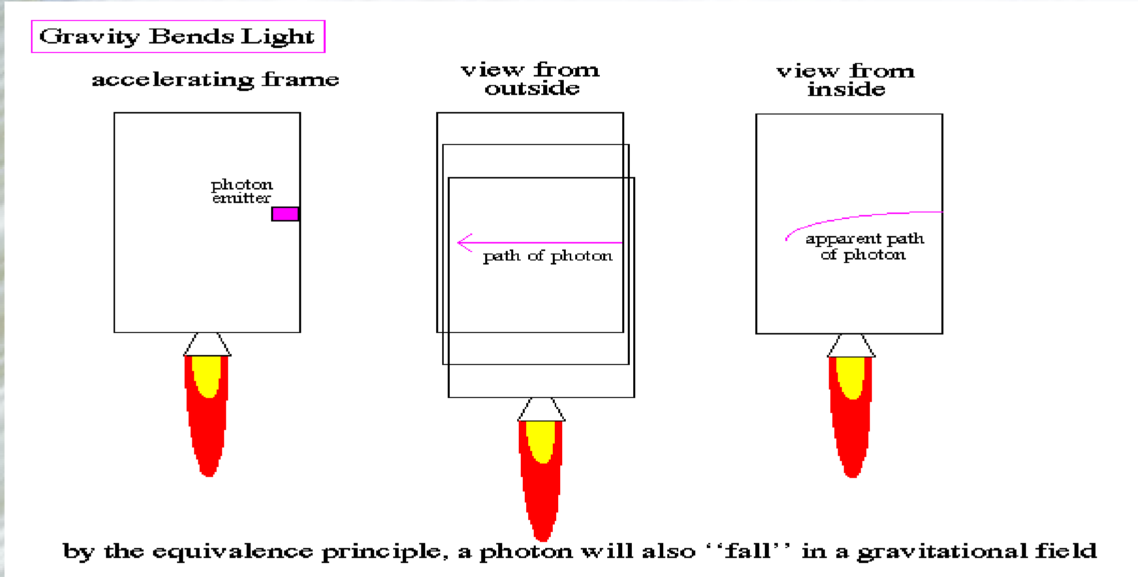
and

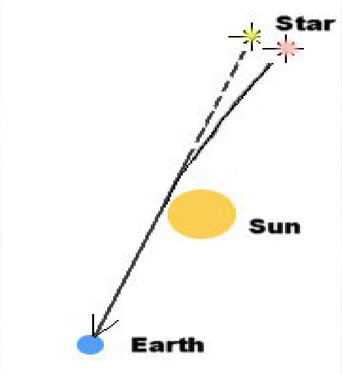
The outcome of any local experiment (be it gravitational or not) in a freely falling laboratory is independent of the velocity of the laboratory and its location in spacetime.

Einstein's theory of general relativity is the only theory of gravity that complies with the strong equivalence principle.

In other words, it says that a local freely falling RS is an inertial reference system. Then in a local freely falling reference system, the laws of special relativity apply.

A few physical consequences of the equivalence principle





Prediction of GR, angular deviation = 1.7 '' , confirmed by Arthur Eddington and Frank Dyson in 1919.

One may also notice that inside the spaceship, the velocity of light at the end of the trajectory is greater than that at the beginning. The second principle of SR is not valid anymore in an accelerated frame.

General Relativity

General principle of relativity (extension of SR principle) :

All systems of reference are equivalent with respect to the formulation of the fundamental laws of physics.

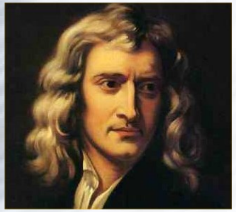
Strong equivalence principle :

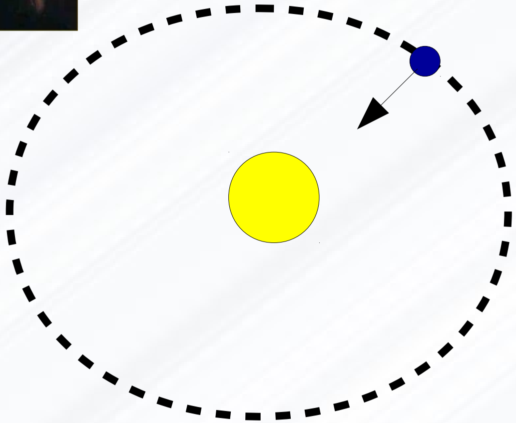
The gravitational motion of a small test body depends only on its initial position in spacetime and on its initial velocity, and not on its constitution. (Free fall, but also all trajectories)

and

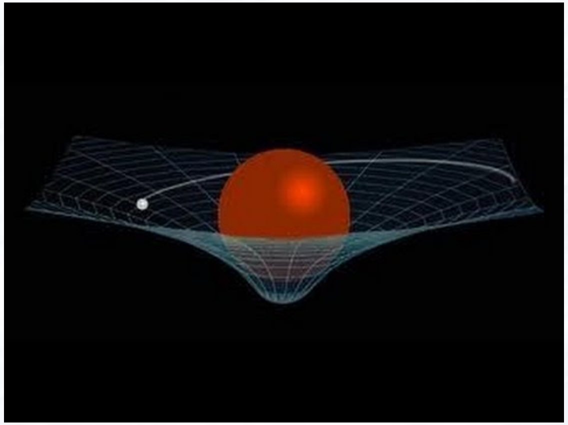
The outcome of any local experiment (be it gravitational or not) in a freely falling laboratory is independent of the velocity of the laboratory and its location in spacetime.

General relativity





According to Newton, a force is exerted on Earth by Sun.



According to Einstein, space is curved by Sun and the free motion of Earth is deflected (no gravitational apparent force anymore). The Earth is freely falling around the Sun and follows a geodesic in the Sun gravitational field. Gravitational forces are only "induced" if observed by a ref. system fixed w.r.t. Sun. They are a bit like "inertial forces" in classical mechanics.

Physics spacetime

In special relativity (SR), the invariant interval element reads :

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 = \sum_{\mu=0}^{3} \sum_{\nu=0}^{3} g_{\mu\nu} dx^\mu dx^\nu$$

Metric : to compute the interval knowing the coordinates variations. This is a pure geometrical object.

In SR, spacetime is pseudo-Euclidean (Minkowski space), the metric always reduces to :

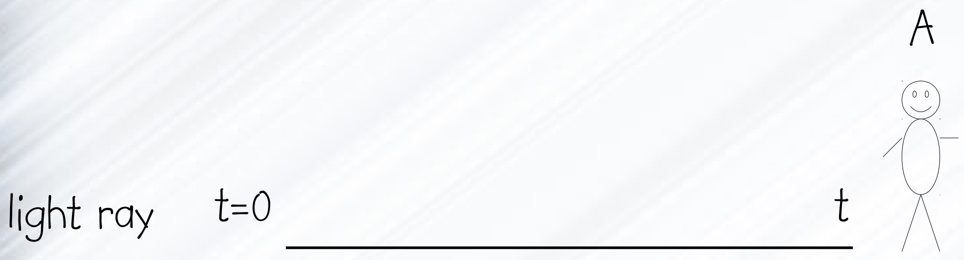
$$g = \eta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

In General Relativity, the interval expression reads the same, but Gaussian coordinates are now used instead of the Cartesian ones and the metric varies from point to point as a function of the mass content of the Universe. Spacetime is now a pseudo-Riemannian manifold with an intrinsic curvature which is the manifestation of gravity. Gravity is not anymore a force but a pure geometrical phenomenon (hence Galileo's law of free fall). If the mass distribution evolves, so do the Universe and its geometry. Forces are replaced (induced) by the variation of the metric when observed in a non freely-falling frame.

Exercises

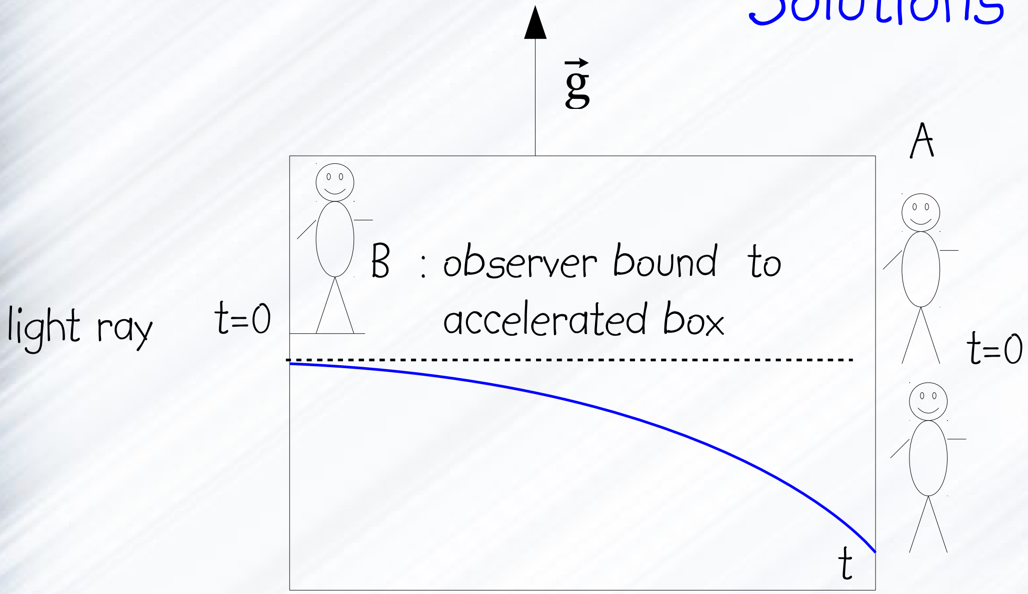
- Using the principles of general relativity , show that light gets deflected in a gravitational field (for the sake of simplicity consider a constant gravitational field)
- Still using the principles of GR, show that two identical clocks immersed in a gravitational field, and located at two different elevations separated by h, tick at a different pace. In a weak and constant field, whose intensity is g, the ratio of the two clock periods reads : $T_B \simeq (1 - \frac{g\,h}{c^2}) T_A$ where A and B refer to the upper and lower clocks, respectively.

Solutions



A : free falling observer
Inertial observer

Solutions

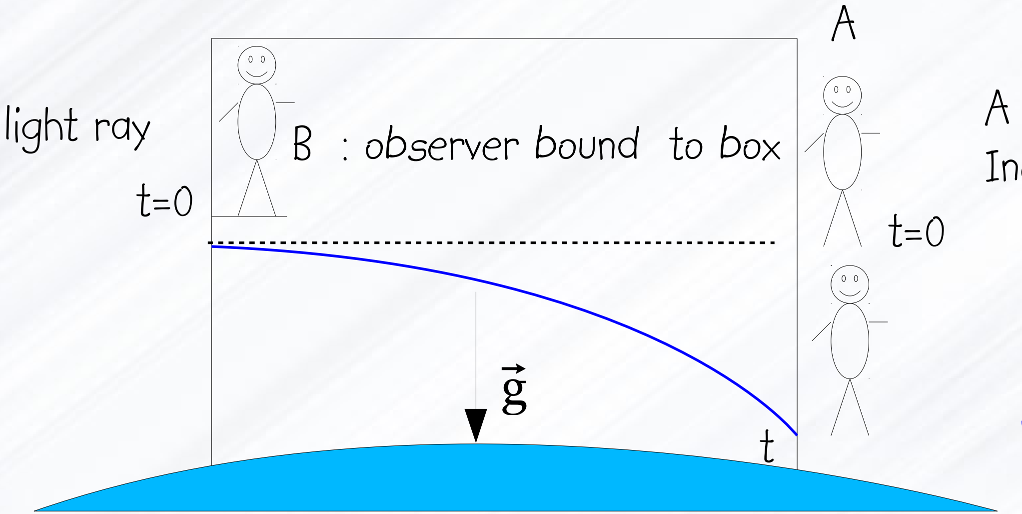


A : free falling observer
Inertial observer

Light being bent downwards for B
while still going straight for A

Also, with respect to B, light becomes faster than c. But B is not an inertial observer.

Solutions

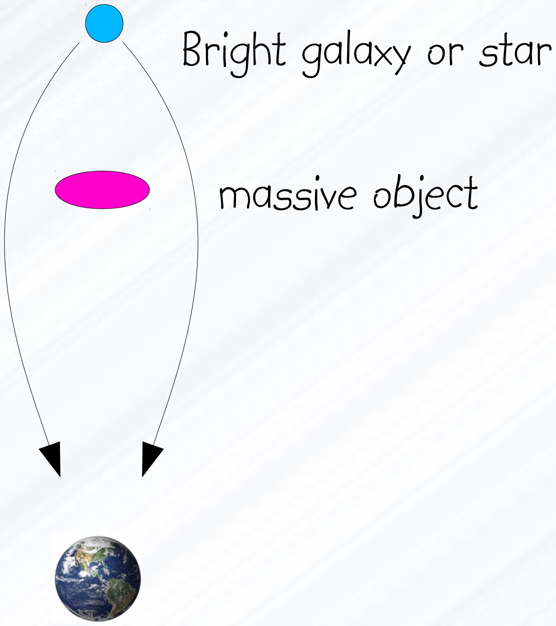


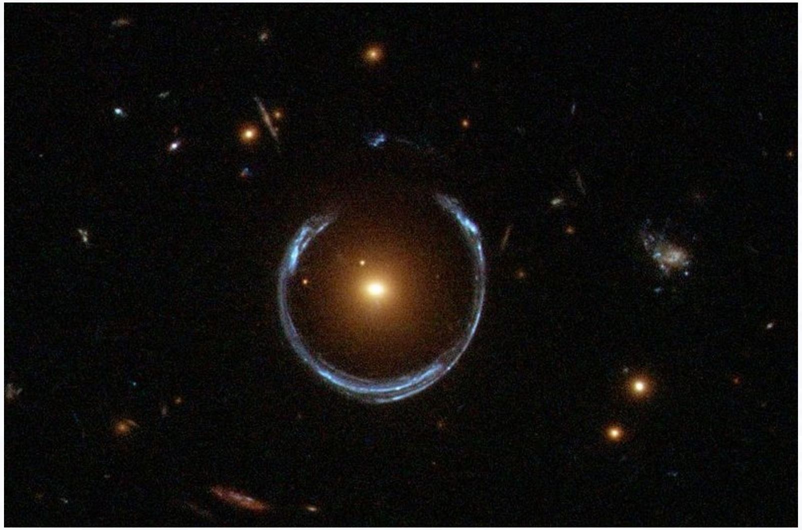
A : free falling observer
Inertial observer

light being bent downwards for B
while still going straight for A

According to equivalence principal, same as box being fixed and subjected to gravitational field.
Conclusion, gravitational field bends light towards massive object.

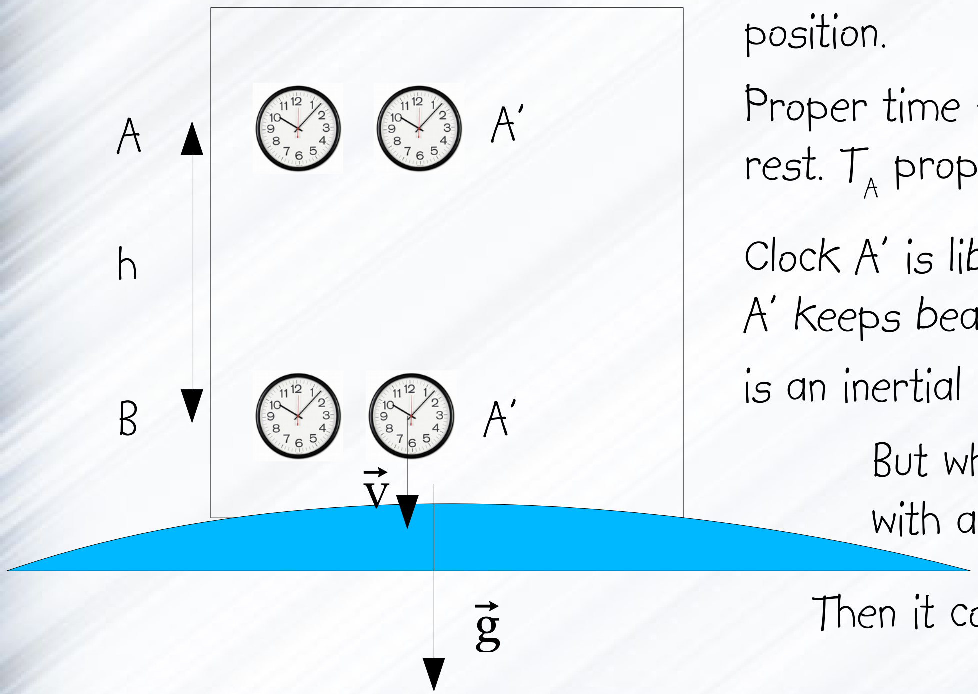
Gravitational lensing : Einstein's ring





Observation by Huble Space Telescope
called Horseshoe Einstein's ring

Solutions : gravitational time dilation



A' is a duplication of A and synchronized to A at same position.

Proper time = time as seen by a clock observed at rest. T_A proper time period of clock A = $T_{A'}$

Clock A' is liberated, then freely falling.
A' keeps beating with same proper time period T_A as it is an inertial observer. So it remains in pace with A.

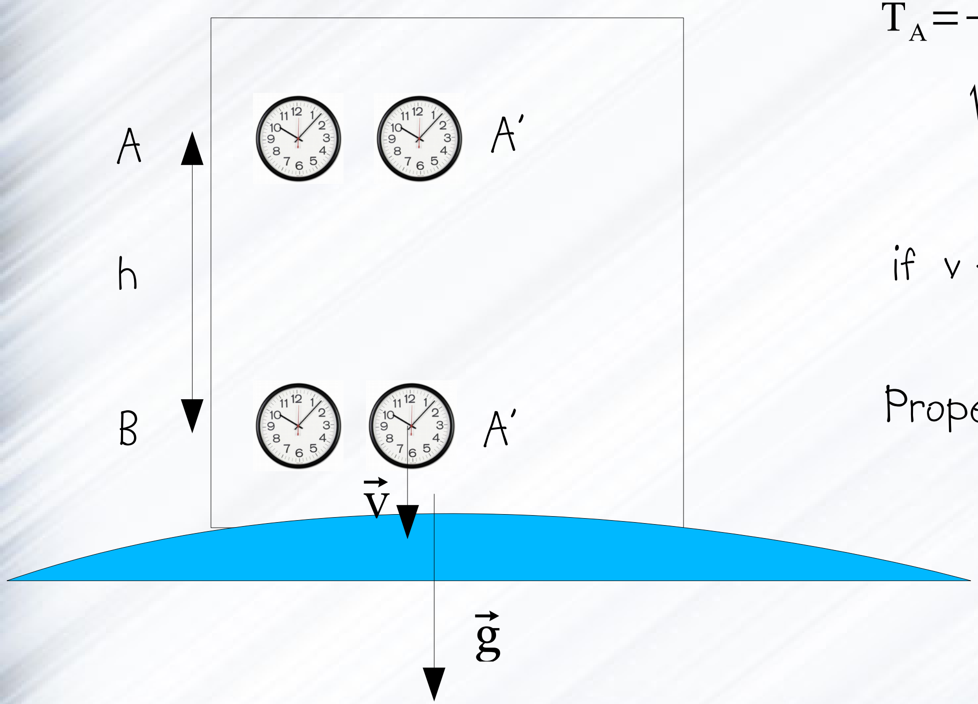
But when passing next to B, it sees clock B moving with a relative velocity :

$$v = \sqrt{2gh}$$

Then it concludes that :

$$T_A = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} T_B$$

Solutions : gravitational time dilation



$$T_A = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} T_B$$

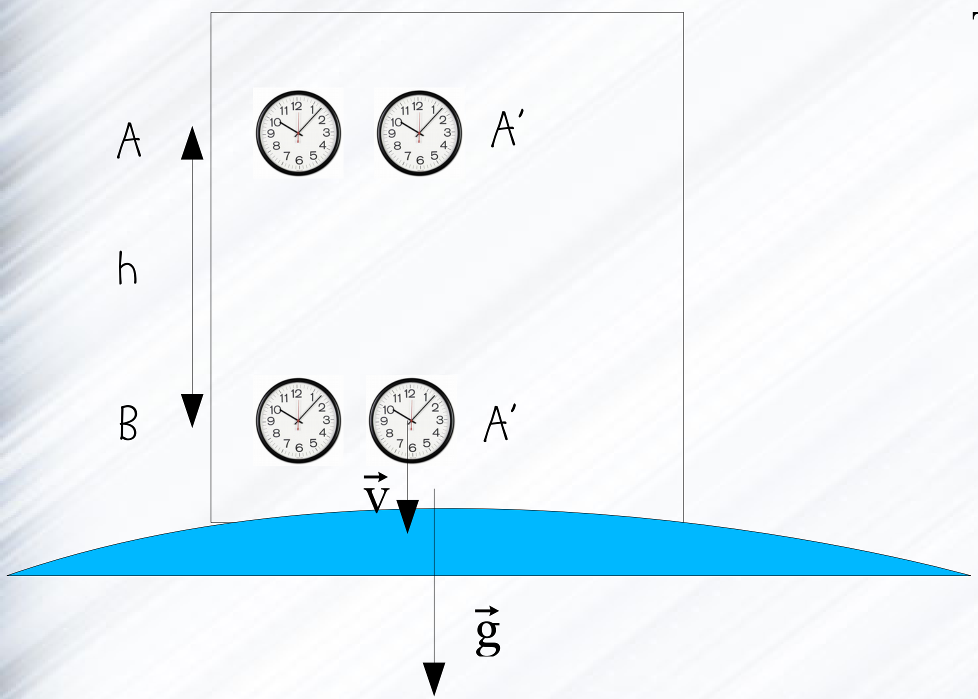
if $v \ll c$:

$$T_B = \sqrt{1-\frac{2gh}{c^2}}\, T_A \simeq \left(1-\frac{gh}{c^2}\right) T_A$$

Proper time period of B appears shorter to A

On Earth, if h = 1 km and g ≈ 10 m s^{-2}, B lags by 9.6 ns per day with respect to A (could be measured with atomic clocks)

Gravitational redshift



$$T_B \simeq \left(1 - \frac{g\,h}{c^2}\right) T_A$$

$$c\,T_B \simeq \left(1 - \frac{g\,h}{c^2}\right) c\,T_A$$

$$\lambda_B \simeq \left(1 - \frac{g\,h}{c^2}\right) \lambda_A$$

light wavelength

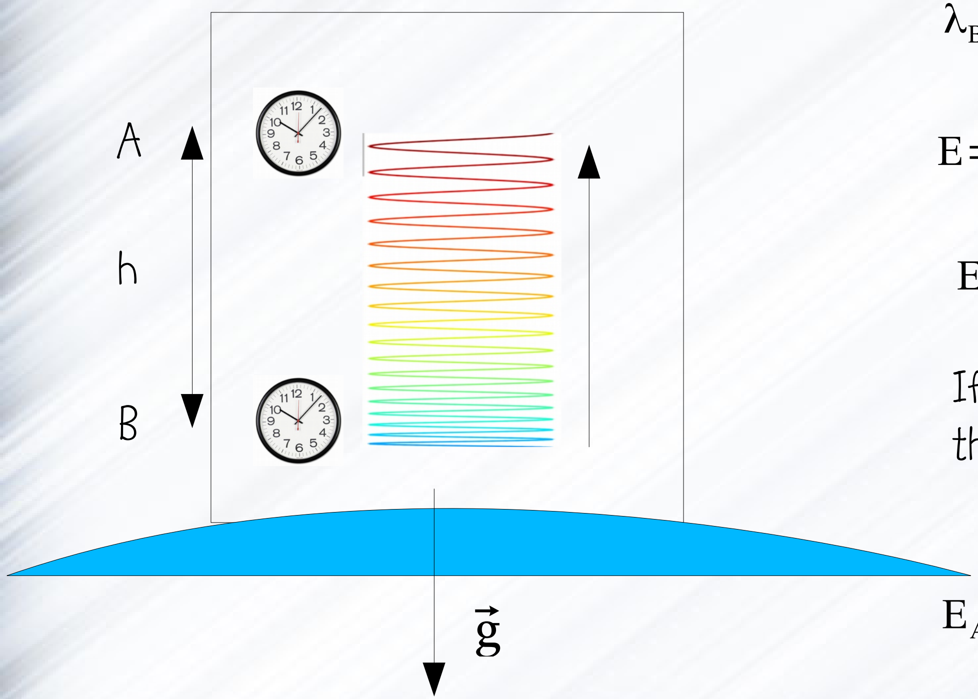
$$\lambda_A \simeq \left(1 + \frac{g\,h}{c^2}\right) \lambda_B$$

$$\frac{\Delta\lambda}{\lambda} \simeq \frac{g\,h}{c^2}$$

Light wavelength gets larger as it espaces massive object : gravitational redshift

Checked at 0.007 % (PRL 45(26) 2081-2084, 1980) and even better in GPS

Gravitational redshift : where's the energy gone ?



$$\lambda_B = (1 - \frac{g\,h}{c^2})\,\lambda_A$$

$$E = \frac{h\,c}{\lambda}$$ Photon energy

$$E_A = (1 - \frac{g\,h}{c^2})\,E_B$$

If one defines, the "gravitational mass" of the photon as : $m_G = \frac{E}{c^2}$

$$E_A = E_B - \frac{E_B}{c^2}\,g\,h$$

Gravitational potential energy

When going up, part of the photon energy is turned into gravitational potential energy.

Back to GPS

In the Earth gravitational field, the proper time elementary interval of a clock orbiting at the distance r from the Earth center is approximately given by :

$$dt_0 \simeq \left(1 - \frac{GM}{c^2 r} - \frac{v^2}{2c^2}\right) dt$$

Earth mass

elementary time interval at r = infinity

gravitational correction

Special relativity correction due to the orbiting velocity v

Remembering that the GPS satellites orbit at 20400 Km from the Earth surface, one can compute that the gravitational correction implies that the clocks located on the Earth lags by 48 μs behind the satellite clocks per day while the SR correction produces a delay of 7.8 μs per day of the satellite clocks with respect to the Earth clocks. The net correction is a delay of the ground clocks of 38.1 μs per day.

GPS atomic clocks need frequent synchronizations but their frequency is tuned slightly lower than the value of the ground clocks (10.22999999543 MHz instead of 10.23 MHz) to correct most of the relativistic delay.

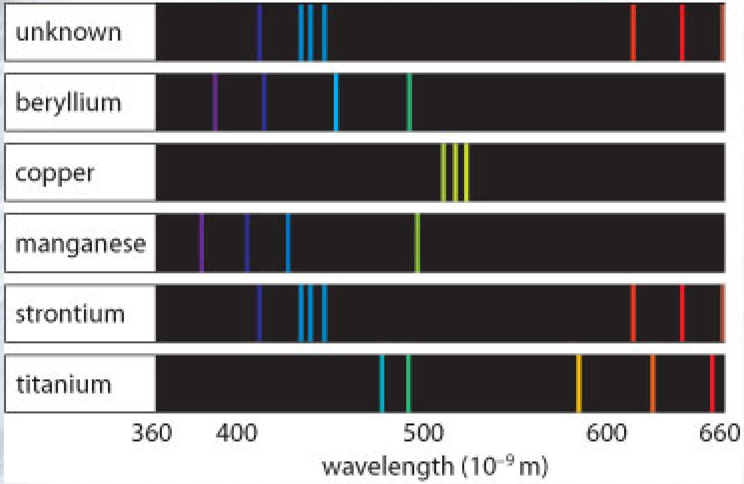
We wish to speak in some way about the structure of atoms, but we cannot speak about atoms in ordinary language.

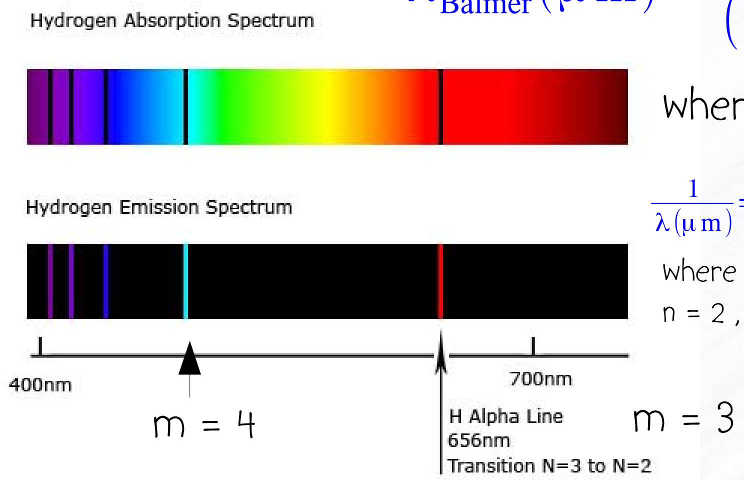
Werner Heisenberg (1901 - 1976)

Advent of quantum mechanics

XVIIIth - XIXth centuries: atomic spectra of gaseous elements are measured as a set of discrete lines (Thomas Melvill, Johan Herschel, Gustave Kirchhoff, Robert Bunsen, William Wollaston, Joseph von Fraunhofer, Johann Balmer, Friedrich Paschen...)

$$\lambda_{Balmer}(\mu m) = \frac{0.365 * m^2}{(m^2 - 4)}$$





where : m > 2

$$\frac{1}{\lambda(\mu m)} = \frac{4}{0.365}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)$$

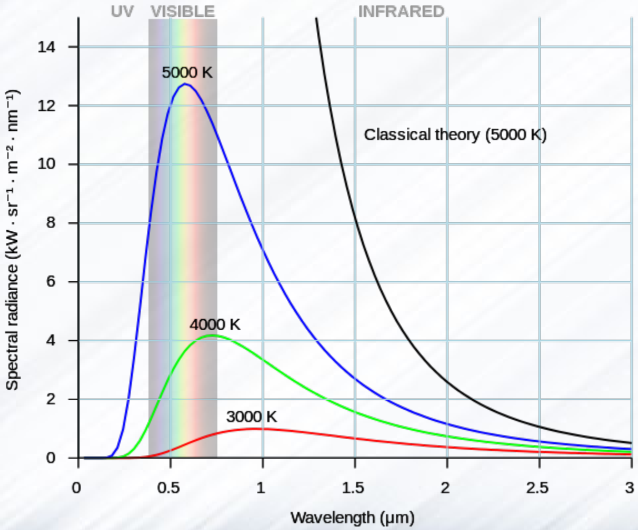
where n >0 and m > n

n = 2 , Balmer series

First observation of discrete spectra as soon as 1752 by Th. Melvill !

Advent of quantum mechanics

End of XIXth - Beginning of XXth century : problem of black-body spectral emittance. (John Rayleigh, James Jeans, Max Planck, Albert Einstein, Satyendra Bose, Jozef Stefan, Ludwig Boltzmann....)



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Electromagnetic radiation is quantized : made of individual quanta, later on called photons (1920s)

Frequency

$$E = h\nu$$

Planck constant = 6.626 10^{-34} J s (quantum of action)

Advent of quantum mechanics

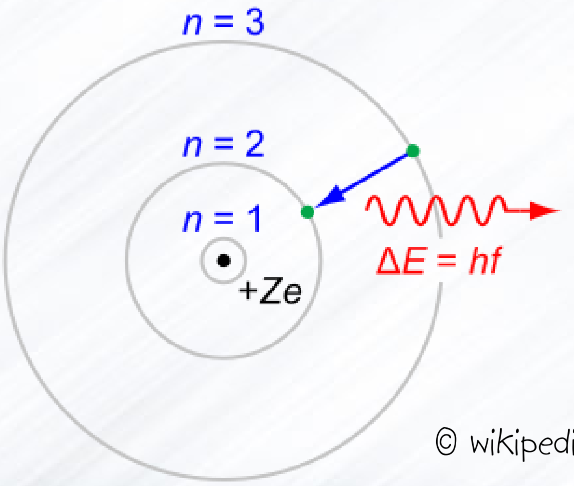
Beginning of XXth century : development of atom model as solar-like system (Ernest Rutherford, Joseph John Thomson, Robert Millikan, Heinrich Hertz, Albert Einstein, Wilhelm Röntgen, William Crookes, Niels Bohr, Hans Geiger, James Franck, Gustav Hertz)

Standing (stationary) states of electrons orbiting around a very small and heavy nucleus :

For hydrogen : $$E_n = \frac{E_1}{n^2} = \frac{-13.6\,\mathrm{eV}}{n^2}$$

Only transitions between stationary states are possible (Niels Bohr) : $$h\nu = E_1\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

1913



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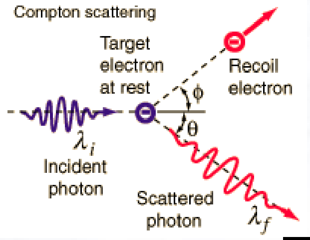
Advent of quantum mechanics

Beginning of XX^th^ century : wave mechanics (Arthur Compton, Louis de Broglie, Erwin Schrödinger, George Paget Thomson, Clinton Davisson, Lester Germer)

Photons may behave like particles : they bear momentum

$$E = h\nu \Rightarrow p = \frac{h}{\lambda}$$



Particles (electrons, ...) may behave like waves :

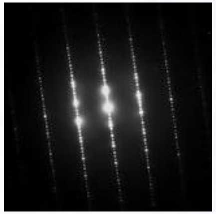
$$p = \frac{h}{\lambda}$$

Electron orbits are stationary waves :

$$2\pi r = n\lambda \Rightarrow L = pr = n\hbar$$

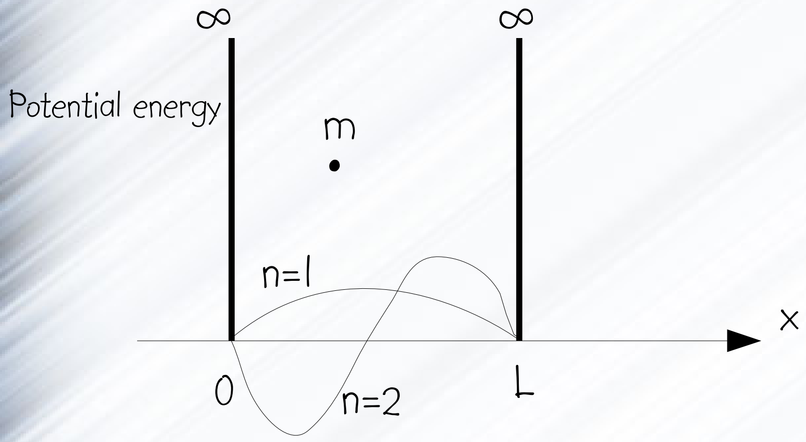
Wave is solution of Schrödinger equation where : $\hbar = \frac{h}{2\pi}$

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t) = \left(\frac{-\hbar^2}{2m}\nabla^2 + V(\vec{r})\right)\psi(\vec{r},t)$$



Electron diffraction

Example : particle in infinite potential box



$$p = \frac{h}{\lambda} = \frac{h\,n}{2L}$$

de Broglie's wavelength equation

Particle of mass m trapped in a box with infinite potential walls

Particle cannot touched the wall !

If particle represented by a wave, solutions are :

$$\sin\left(\frac{\pi n}{L} x\right) \Rightarrow \lambda = \frac{2L}{n} \text{ where } n > 0$$

$$E_n = \frac{p^2}{2m} = \frac{h^2 n^2}{8L^2 m}$$

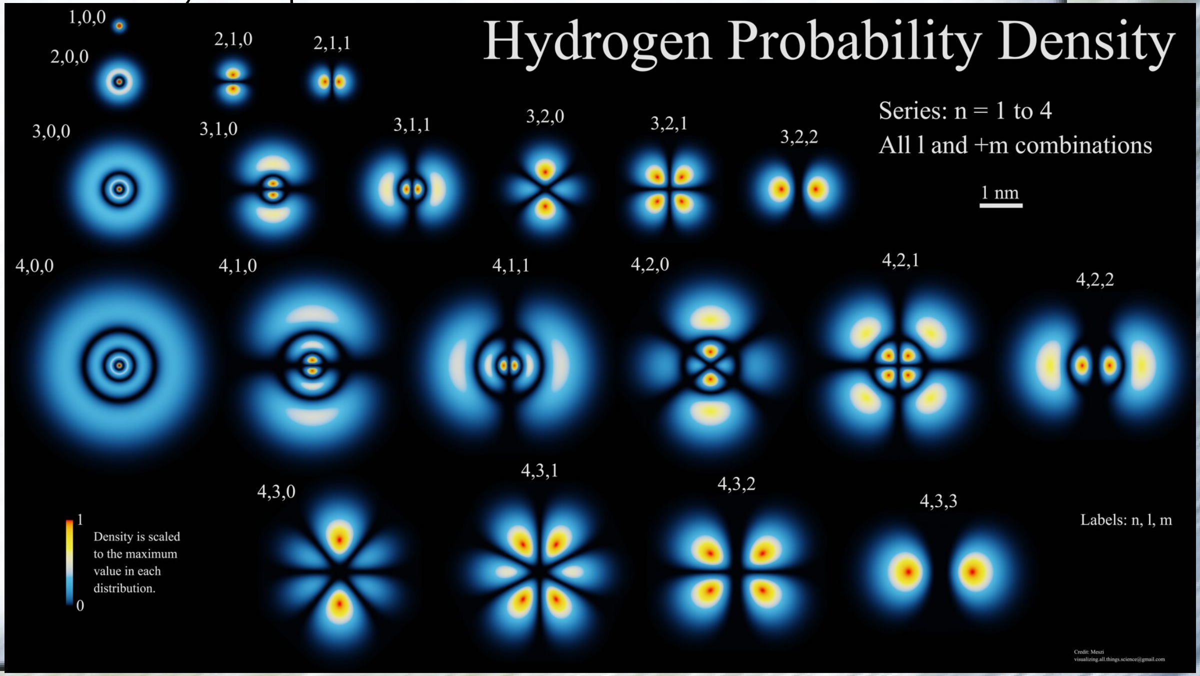
Kinetic energy is quantized !

Quantum mechanics

- Probabilistic interpretation of the wave function : Max Born in 1926
- Wave function is a complex probability amplitude

For hydrogen atom :

$|\psi(\vec{r})|^2$ is the probability to find an electron at position r



visualizing.all.things.science@gmail.com

Quantum mechanics

Niels Bohr, Erwin Schrödinger, Werner Heisenberg, Max Born, P.A.M Dirac (1920-1930)

The postulates : Copenhagen interpretation

- The state of a physical system is represented by a ket (state) $|\psi(t)\rangle$ in the space of states (Hilbert space)
- Every observable attribute of a system is described by an operator that acts on the system ket.
- The possible result of an observable A measurement is one of the eigenvalues of the corresponding operator A.
- When a measurement of an observable A is made on a state $|\psi\rangle$, the probability of obtaining an eigenvalue a_n is given by the squared modulus of the projection of $|\psi\rangle$ on the eigenstate $|a_n\rangle$, $|\langle a_n|\psi\rangle|^2$.
- Immediately after the measurement of an observable A has yielded a value a_n, the state of the system is the eigenstate $|a_n\rangle$.
- The time evolution of the state of a quantum system is described by the Schrödinger equation :

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = H(t)\,|\psi(t)\rangle$$

where H is the hamiltonian operator associated to the total energy of the system

Schrödinger equation

$$T + V = E$$

Total energy

Kinetic energy

Potential energy

Need to transform this into a differential wave equation

$$\frac{\vec{P}^2}{2m} + V = E$$

$$E \rightarrow i\hbar\frac{\partial}{\partial t} \qquad \vec{p} \rightarrow -i\hbar\vec{\nabla}$$

Correspondence principle

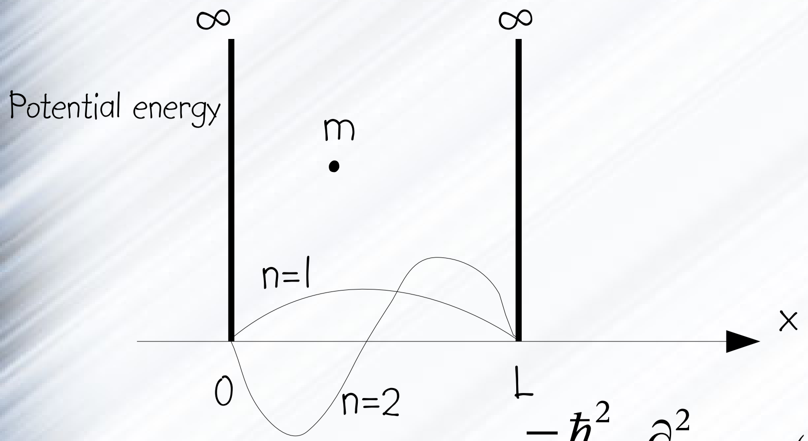
$$i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t) = \left(\frac{-\hbar^2}{2m}\nabla^2 + V(\vec{r})\right)\psi(\vec{r},t)$$

In bra-ket notation :

$$i\hbar\frac{d}{dt}|\psi(t)> = H(t)\ |\psi(t)>$$

Hamiltonian

Example : particle in infinite potential box



$$\sin\left(\frac{\pi n}{L}x\right) \Rightarrow \lambda = \frac{2L}{n} \text{ where } n>0$$

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\sin\left(\frac{\pi n}{L}x\right) = \frac{\pi^2 n^2 \hbar^2}{2L^2 m}\sin\left(\frac{\pi n}{L}x\right) = \frac{n^2 h^2}{8L^2 m}\sin\left(\frac{\pi n}{L}x\right)$$

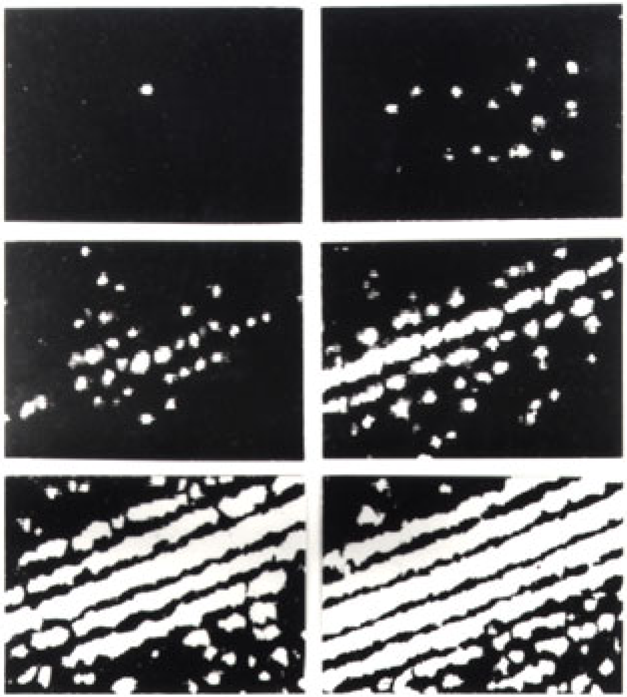
$$E_n = \frac{p^2}{2m} = \frac{h^2 n^2}{8L^2 m}$$

$$\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\sin\left(\frac{\pi n}{L}x\right) = E_n \sin\left(\frac{\pi n}{L}x\right)$$

$$\psi(\vec{r},t)_n = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi n}{L}x\right)e^{-i\frac{E_n}{\hbar}t}$$

Quantum mechanics : physical consequences

Quanta feature wave-particle duality



Modern single electron double-slit experiment

Here electrons are detected one by one, like particles, but each should be considered as having gone through both slits at the same time, like waves.

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Quantum mechanics : physical consequences

Heisenberg uncertainty inequalities (arise as a consequence, not a principle)

The fundamental cause of what is called quantum fluctuations .

At any time, both the position and the momentum of a quantum system can't be known with an infinite precision. (Heisenberg uncertainty inequalities)

$$\Delta x \Delta p_x \geqslant \frac{\hbar}{2} \qquad \hbar = \frac{h}{2\pi}$$

At any position, both the time and the energy of a quantum system can't be known with an infinite precision. (time-energy uncertainty inequality)

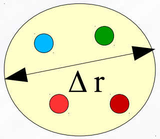
$$\Delta t \Delta E \geqslant \frac{\hbar}{2}$$

Exercises

- Using Heisenberg uncertainty inequalities, try to quantify the growing difficulties to model quantum systems made of constituents.
- The radius of a proton being 0.87 fm, and knowing that in a composite model, the proton is made of 3 light quarks of negligible mass, find a rough estimate of the proton mass
- The electron mass is 511 keV. If the electron substructure exists, it must be such that $R < 10^{-18}$ m. Quantify the difficulties to build a composite model of the electron.
- In the case of infinite potential box, show that we find back classical physics when L, m or n become macroscopic.
- Find a relativistic wave equation for photons using correspondence principle

Solutions

Composite models address the substructure of compound systems of diminishing size (atoms, nuclei, nucleons, quarks ...)



Heisenberg inequalities lead to :

$$\Delta r \ \Delta p \simeq \hbar$$

$$\Delta r \ \Delta p c \simeq \hbar c$$

For particles of mass m :

$$E^2 = p^2 c^2 + m^2 c^4 \Rightarrow E \Delta E = p c \Delta pc$$

$$\frac{pc}{E} = \frac{v}{c} = \beta$$

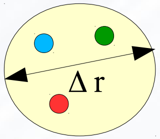
$$\Delta E \simeq \beta \frac{\hbar c}{\Delta r}$$

$$\hbar c = 197 \text{ MeV.fm}$$

Total energy is bigger and bigger calling for stronger and stronger interaction to hold structure

Solutions

Proton contains three (valence) light quarks (up, up, down) :



$$\Delta(E) \simeq \beta \frac{\hbar c}{\Delta r} \qquad \beta \simeq 1$$

$$\Delta E \simeq \frac{\hbar c}{\Delta r} = 197/0.87 = 226\,\mathrm{MeV}$$

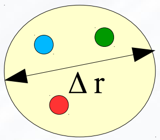
$$m c^2 \simeq 3 \Delta E = 680\,\mathrm{MeV} \qquad m_p c^2 = 938\,\mathrm{MeV}$$

But there's more constituents in a proton, and need to take into account potential energy.

Solutions

Electron substructure ?

$$\Delta r < 10^{-3}\,\mathrm{fm}$$



$$\Delta E > \beta \frac{\hbar c}{\Delta r} \qquad \beta \simeq 1$$

$$\Delta E > 197\,\mathrm{GeV}$$

$$m c^2 \simeq n\, 197\,\mathrm{GeV} \qquad m_e c^2 = 511\,\mathrm{keV}$$

Needs extremely strong interaction (binding energy) to bring the total energy back to 511 keV

Solutions

$$E_n = \frac{p^2}{2m} = \frac{h^2 n^2}{8L^2 m}$$

$$\frac{\Delta E_n}{E_n} = \frac{E_{n+1} - E_n}{E_n} = \frac{1+2n}{n^2} \rightarrow 0 \text{ when } n \rightarrow \infty$$

$$E_n(\text{MeV}) = \frac{\pi^2 (\hbar c)^2 n^2}{2mc^2 L^2}$$

1 ng -> m c^2 = 5.6 10^{20} MeV

$$E_n(\text{MeV}) = \frac{\pi^2 (\hbar c)^2 n^2}{2mc^2 L^2} = 3.4\,10^{-28}\, n^2\, \text{eV}$$

1 μm = 10^9 fm

Solutions

$E^2 = p^2c^2 + m^2c^4 = p^2c^2 \Rightarrow E^2 = p^2c^2$ For a free photon

$E \rightarrow i\hbar\frac{\partial}{\partial t} \qquad \vec{p} \rightarrow -i\hbar\vec{\nabla}$

$\frac{\partial^2}{\partial t^2}\psi(\vec{r},t) - c^2\nabla^2\psi(\vec{r},t) = 0$ Classical free light wave equation

$\psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} g(k)e^{-i(\omega t - kx)}dk$ Propagating along (0,x)

with : $\omega = kc$

$E = \hbar\omega$

Wave packet

Further reading

- On the shoulders of giants : the great works of physics and astronomy , compilation of texts edited by Stephen Hawking
- Understanding physics, David Cassidy, Gerald Holton, James Rutherford, Undergraduate texts in contemporary physics, Springer
- Naissance de la physique : de la Sicile à la Chine , Michel Soutif, EDP Sciences
- From alchemy to quarks, Sheldon L. Glashow, ITP
- La physique du XXème siècle, Michel Paty, EDP Sciences