

Nuclear PDFs

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OUTLINE

- ① INTRODUCTION
- ② REVIEW OF GLOBAL ANALYSES OF NUCLEAR PDFs
- ③ NUCLEAR EFFECTS IN νA DIS
- ④ THE NUCLEAR GLUON DISTRIBUTION

Introduction

- Information on **hadronic structure**
- **Initial state** for hard processes in collisions involving hadrons
 - Deep inelastic scattering (DIS): $\ell A, \nu A$
 - Drell-Yan (DY): $A + B \rightarrow \ell^+ + \ell^-$
 - Jets, Photons, Hadrons at large p_T ; Heavy Quarks; ...
in $pA, AA, (\gamma A, eA)$ collisions
- Provide **nuclear corrections** for global analyses of **proton PDFs** in a **flexible way**

THEORETICAL BASIS: FACTORIZATION

- Factorization theorems
 - provide (field theoretical) **definitions of universal PDFs**
 - make the formalism **predictive**
 - make a statement about the **error**
- **PDFs** and predictions for **observables+uncertainties** refer to this **standard pQCD framework**
- There might be breaking of QCD factorization, deviations from **DGLAP** evolution — in particular in a nuclear environment

Still need solid understanding of standard framework to establish deviations!

In the nuclear case, consider factorization as a **working assumption** to be tested phenomenologically

Review of global analyses of nuclear PDFs

NPDFs FROM ℓA DIS AND DY DATA

Global analyses of NPDF by four groups:

- **HKN'07** [[PRC76\(2007\)065207](#)]
LO, NLO, error PDFs, $\chi^2/dof = 1.2$
- **EPS'09** [[JHEP0904\(2009\)065](#)]
LO, NLO, error PDFs, $\chi^2/dof = 0.8$
Use also inclusive π^0 data at midrap. from $d + Au$ and $p + p$ coll. at RHIC \rightarrow gluon
- **DS'04** [[PRD69\(2004\)074028](#)]
fi rst NLO analysis, 'semi-global', no error PDFs, $\chi^2/dof = 0.76$
- **nCTEQ** [[PRD80\(2009\)094004](#)]
NLO, same data as HKN'07 (up to cuts), no error PDFs (so far), $\chi^2/dof = 0.95$, official release soon

Table from [Hirai et al., arXiv:0909.2329](#)

| | R | Nucleus | Experiment | EPS09 | HKN07 | DS04 | |
|-----------|--------|-----------|-------------|------------|-------|------|---|
| DIS | A/D | D/p | NMC | | o | | |
| | | 4He | SLAC E139 | o | o | o | |
| | | | NMC95 | o (5) | o | o | |
| | | Li | NMC95 | o | o | | |
| | | Be | SLAC E139 | o | o | o | |
| | | | C | EMC-88, 90 | | o | |
| | | NMC 95 | | o | o | o | |
| | | SLAC E139 | | o | o | o | |
| | | FNAL-E665 | | | o | | |
| | | N | BCDMS 85 | | o | | |
| | | | HERMES 03 | | o | | |
| | | Al | SLAC E49 | | o | | |
| | | | SLAC E139 | o | o | o | |
| | | Ca | EMC 90 | | o | | |
| | | | NMC 95 | o | o | o | |
| | | | SLAC E139 | o | o | o | |
| | | | FNAL-E665 | | o | | |
| | | Fe | SLAC E87 | | o | | |
| | | | SLAC E139 | o (15) | o | o | |
| | | | SLAC E140 | | o | | |
| | | | BCDMS 87 | | o | | |
| | | Cu | EMC 93 | o | o | | |
| | | Kr | HERMES 03 | | o | | |
| | | Ag | SLAC E139 | o | o | o | |
| | | Sn | EMC 88 | | o | | |
| | | Au | SLAC E139 | o | o | o | |
| | | | SLAC E140 | | o | | |
| | | Pb | FNAL-E665 | | o | | |
| | | A/C | Be | NMC 96 | o | o | o |
| | | | Al | NMC 96 | o | o | o |
| Ca | NMC 95 | | | o | | | |
| | NMC 96 | | o | o | o | | |
| Fe | NMC 96 | | o | o | o | | |
| Sn | NMC 96 | | o (10) | o | o | | |
| Pb | NMC 96 | | o | o | o | | |
| A/Li | C | NMC 95 | o | o | | | |
| | Ca | NMC 95 | o | o | | | |
| DY | A/D | C | | o | o | o | |
| | | Ca | FNAL-E772 | o (15) | o | o | |
| | | Fe | | o (15) | o | o | |
| | | W | | o (10) | o | o | |
| | | Fe | | o | o | | |
| A/Be | W | FNAL E866 | o | o | | | |
| | W | | o | o | | | |
| π pro | dA/pp | Au | RHIC-PHENIX | o (20) | | | |

WHAT ARE THE DIFFERENCES?

Main differences:

- **Choice of data sets** (see previous table)
 - **Parametrization of input distributions**
 - **Assumptions on PDFs**
 - Data less constraining than in proton case → need to make more assumptions (otherwise flat directions in χ^2 function and fits don't converge)
 - Assumptions replace uncertainty! → error bands (of a single fit) underestimate true uncertainties
-

Consequences?

- **Use different sets of NPDFs to scan over assumptions**
- Include more data sets → allows to relax assumptions
- New ideas to handle flat directions?
- Neural Network NPDFs?

WHAT ARE THE DIFFERENCES?

Further differences:

- **Heavy flavor schemes**

- **DS'04**: 3-Fixed Flavor Number Scheme (3-FFNS) → **no charm PDF**
- **HKN'07, EPS'09, nCTEQ**: Variable Flavor Number Schemes (VFNS)

→ Beware of comparing 'apples with oranges'!

- **Parameters and other**

- Input scale Q_0 , $\alpha_s(M_Z)$, m_c , m_b
- Evolution in n -space (**DS**) and x -space (**HKN, EPS, nCTEQ**)
- Target Mass Corrections (TMC)
see, e.g., [**IS et al., JPG35(2008)053101**; **Qiu, Accardi, JHEP0807(2008)090**]

Connected to GRV'98 proton PDFs $f_i^p(x, Q)$:

- $Q_0^2 = 0.4 \text{ GeV}^2$ (NLO), $Q_0^2 = 0.26 \text{ GeV}^2$ (LO), m_c, m_b, α_s as in GRV'98
- 3-Fixed flavor scheme (no charm PDF)
- strange PDF dynamically generated, i.e., $s^p(x, Q_0^2) = 0$

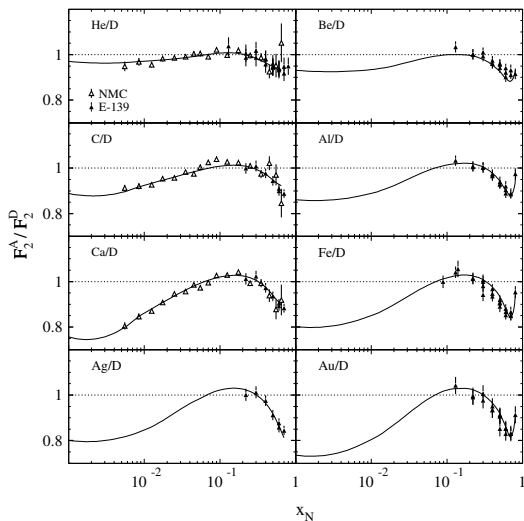
Parametrization of input distributions:

- PDFs for bound protons inside nucleus A : $f_i^{p/A}(x, Q)$
- Convolution relation:
$$f_i^{p/A}(x_N, Q_0^2) = \int_{x_N}^A \frac{dy}{y} W_i(y, A, Z) f_i^p(x_N/y, Q_0^2)$$
- Weight functions W_v (valence), W_s (sea), W_g (gluon). For example:

$$W_v(y, A, Z) = A[a_v \delta(1 - \epsilon_v - y) + (1 - a_v) \delta(1 - \epsilon_{v'} - y)] + n_v (y/A)^{\alpha_v} (1 - y/A)^{\beta_v} + n_s (y/A)^{\alpha_s} (1 - y/A)^{\beta_s}$$

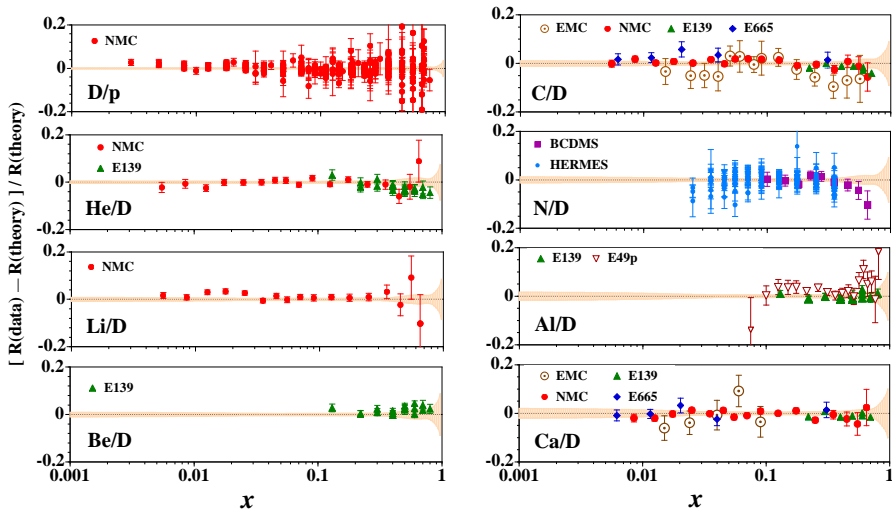
- Note:
 - Convolution simple product in Mellin moment space: very elegant
 - Ansatz valid for $0 < x_N < A!$
 - The x -space approaches (HKN, EPS, nCTEQ) are restricted to $0 < x_N < 1$
 - However, the DS'04 PDF grids apparently are restricted to $0 < x_N < 1$ (and the momentum sum rule integrates to unity in this range)

Excellent fit to a **restricted** data set (420 points): $\chi^2/\text{dof} = 0.75$



- LO, NLO, error PDFs
- Related to MRST'98 proton PDF: $Q_0^2 = 1 \text{ GeV}^2$
- Uses multiplicative ansatz: $f_i^{p/A}(x_N, Q_0^2) = R_i(x_N, Q_0, A, Z) f_i^p(x_N, Q_0^2)$
- Weight factor: $R_i(x, A, Z) = 1 + (1 - \frac{1}{A^\alpha}) \frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1-x)^{\beta_i}}$ ($i = u_v, d_v, \bar{q}, g$)
- neglects region $x_N > 1$
- includes all current DIS & DY data sets, in particular deuterium data
- uses Hessian method to produce error PDFs

- Reasonable fits: $\chi^2/dof = 1.2$



- LO, NLO, error PDFs
- Related to CTEQ6.1M proton PDF: $Q_0 = 1.3 \text{ GeV}$

- Uses multiplicative ansatz: $f_i^{p/A}(x_N, Q_0^2) = R_i(x_N, Q_0, A, Z) f_i^p(x_N, Q_0^2)$

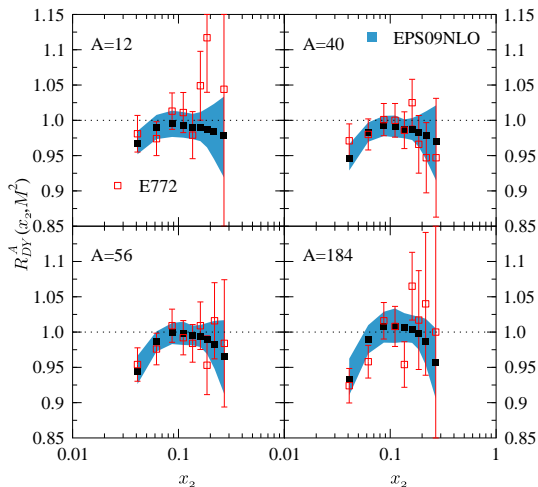
- Weight factor is a piecewise defined function:

$$R_i(x, A, Z) = \begin{cases} a_0 + (a_1 + a_2 x)(e^{-x} - e^{-x_a}) & x \leq x_a \\ b_0 + b_1 x + b_2 x^2 + b_3 x^3 & x_a \leq x \leq x_e \\ c_0 + (c_1 - c_2 x)(1 - x)^{-\beta} & x_e \leq x \leq 1 \end{cases}$$

where the parameters $a_i, b_i, c_i, \beta, x_a, x_e$ are A -dependent

- neglects region $x_N > 1$
- includes π^0 RHIC data with a weight 20 to constrain gluon
- uses Hessian method to produce error PDFs

- Excellent fit: $\chi^2/dof = 0.8$
- Show here, as an example, comparison with DY data



Work in collaboration with:

- People from LPSC Grenoble: [K. Kovarik](#), [J. Y. Yu](#), [T. Stavreva](#), [IS](#)
 - CTEQ-members: [F. Olness \(SMU\)](#), [J. Owens \(FSU\)](#), [J. Morfin \(FNAL\)](#), [C. Keppel \(JLAB\)](#)
-

- The results shown in the following are from [IS, Yu, Kovarik, Keppel, Morfin, Olness, Owens, PRD80\(2009\)094004](#)
- A first set of nCTEQ nuclear PDFs will be released in the near future

Framework as in CTEQ6M proton fit:

- **Same functional form** for **bound proton PDFs** inside a nucleus A as for free proton PDFs (restrict x to $0 < x < 1$):

$$x f_k^{p/A}(x, Q_0) = c_0 x^{c_1} (1-x)^{c_2} e^{c_3 x} (1 + e^{c_4 x})^{c_5}, \quad k = u_v, d_v, g, \bar{u} + \bar{d}, s, \bar{s},$$

$$\bar{d}(x, Q_0)/\bar{u}(x, Q_0) = c_0 x^{c_1} (1-x)^{c_2} + (1 + c_3 x)(1-x)^{c_4}$$

(bound neutron PDFs $f_k^{n/A}$ by isospin symmetry)

- **A -dependent fit parameters:** (reduces to free proton parameters $c_{k,0}$ for $A = 1$)

$$c_k \rightarrow c_k(A) \equiv c_{k,0} + c_{k,1}(1 - A^{-c_{k,2}}), \quad k = 1, \dots, 5$$

- **PDFs for a nucleus** (A, Z) : $f_i^{(A,Z)}(x, Q) = \frac{Z}{A} f_i^{p/A}(x, Q) + \frac{A-Z}{A} f_i^{n/A}(x, Q)$
- **Input parameters:** $Q_0 = m_c = 1.3 \text{ GeV}$, $m_b = 4.5 \text{ GeV}$, $\alpha_s^{\overline{\text{MS}}}(M_Z) = 0.118$
- **Heavy quark treatment:** ACOT scheme
- **Standard DIS-cuts:** $Q > 2 \text{ GeV}$, $W > 3.5 \text{ GeV}$

EXPERIMENTAL INPUT

Use same data as HKN'07 (up to cuts)

- DIS F_2^A/F_2^D data sets: 862 points (before cuts)
- DIS $F_2^A/F_2^{A'}$ data sets: 297 points (before cuts)
- DY data sets $\sigma_{DY}^{pA}/\sigma_{DY}^{pA'}$: 92 points (before cuts)

Table from [Hirai et al., arXiv:0909.2329](https://arxiv.org/abs/0909.2329)

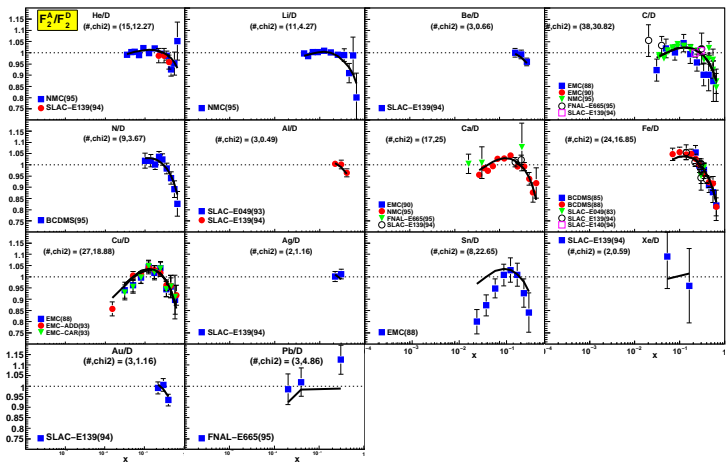
| | R | Nucleus | Experiment | EPS09 | HKN07 | DS04 |
|-----------|----------|-----------|-------------|--------|-------|------|
| DIS | A/D | D/p | NMC | | 0 | |
| | | 4He | SLAC E139 | 0 | 0 | 0 |
| | | | NMC95 | 0 (5) | 0 | 0 |
| | | Li | NMC95 | 0 | 0 | |
| | | Be | SLAC E139 | 0 | 0 | 0 |
| | | | EMC-88, 90 | | 0 | |
| | | C | NMC 95 | 0 | 0 | 0 |
| | | | SLAC E139 | 0 | 0 | 0 |
| | | FNAL-E665 | | 0 | | |
| | | N | BCDMS 85 | | 0 | |
| | | | HERMES 03 | | 0 | |
| | | Al | SLAC E49 | | 0 | |
| | | | SLAC E139 | 0 | 0 | 0 |
| | | Ca | EMC 90 | | 0 | |
| | | | NMC 95 | 0 | 0 | 0 |
| | | | SLAC E139 | 0 | 0 | 0 |
| | | | FNAL-E665 | | 0 | |
| | | Fe | SLAC E87 | | 0 | |
| | | | SLAC E139 | 0 (15) | 0 | 0 |
| | | | SLAC E140 | | 0 | |
| | BCDMS 87 | | | 0 | | |
| | Cu | EMC 93 | 0 | 0 | | |
| | Kr | HERMES 03 | | 0 | | |
| | Ag | SLAC E139 | 0 | 0 | 0 | |
| | Sn | EMC 88 | | 0 | | |
| | Au | SLAC E139 | 0 | 0 | 0 | |
| | | SLAC E140 | | 0 | | |
| | Pb | FNAL-E665 | | 0 | | |
| | A/C | Be | NMC 96 | 0 | 0 | 0 |
| | | Al | NMC 96 | 0 | 0 | 0 |
| | | Ca | NMC 95 | | 0 | |
| | | | NMC 96 | 0 | 0 | 0 |
| | | Fe | NMC 96 | 0 | 0 | 0 |
| Sn | | NMC 96 | 0 (10) | 0 | 0 | |
| Pb | | NMC 96 | 0 | 0 | 0 | |
| A/Li | | C | NMC 95 | 0 | 0 | |
| | Ca | NMC 95 | 0 | 0 | | |
| DY | A/D | C | | 0 | 0 | 0 |
| | | Ca | FNAL-E772 | 0 (15) | 0 | 0 |
| | | Fe | | 0 (15) | 0 | 0 |
| | | W | | 0 (10) | 0 | 0 |
| | A/Be | Fe | FNAL E866 | | 0 | 0 |
| W | | | | 0 | 0 | |
| π pro | dA/pp | Au | RHIC-PHENIX | 0 (20) | | |

RESULTS: DECU3 FIT

- 708 (1233) data points after (before) cuts
- 32 free parameters; 675 d.o.f.
- Overall $\chi^2/\text{d.o.f.} = 0.95$
- individually:
 - for F_2^A/F_2^D : $\chi^2/\text{pt} = 0.92$
 - for $F_2^A/F_2^{A'}$: $\chi^2/\text{pt} = 0.69$
 - for DY: $\chi^2/\text{pt} = 1.08$
- **Our simple approach works!**

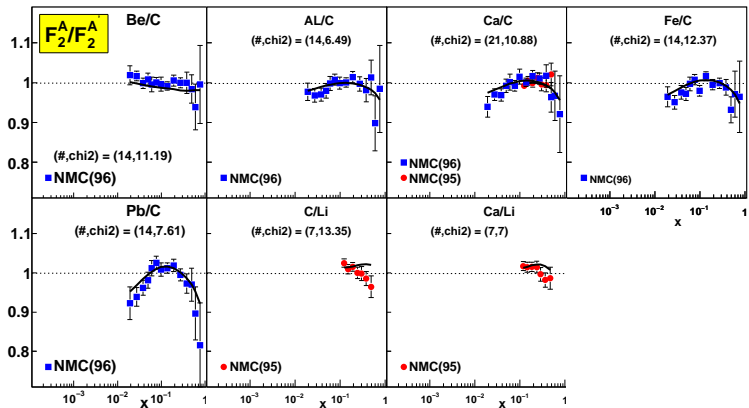
RESULTS: DE CUT3 FIT

DIS DATA VS x



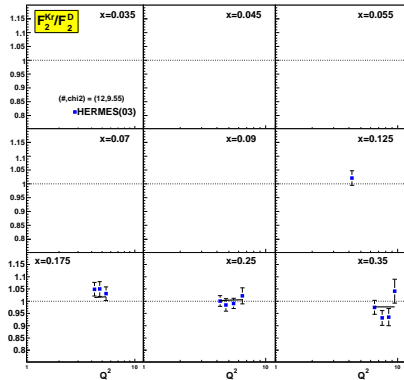
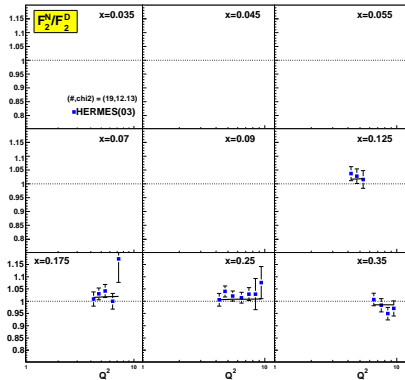
RESULTS: DECVT3 FIT

DIS DATA VS X



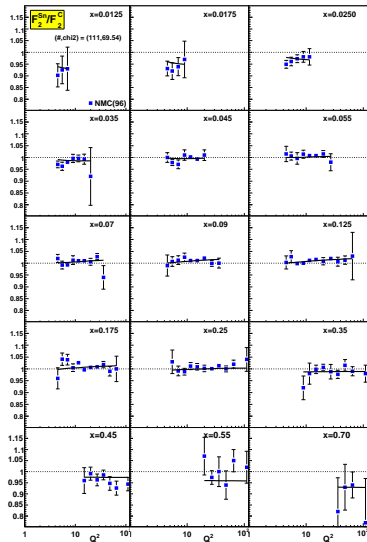
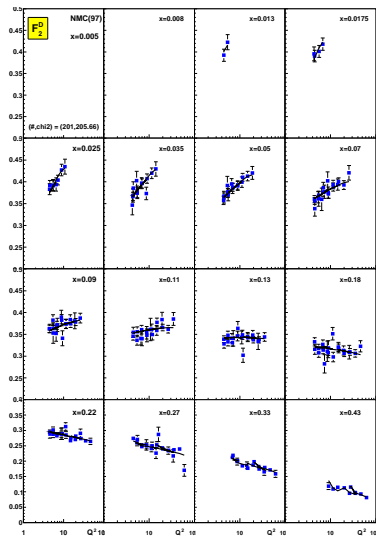
RESULTS: DE CUT3 FIT

HERMES DATA VS Q^2



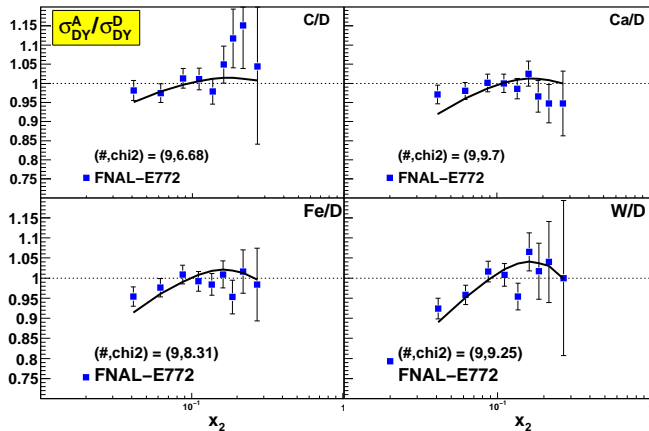
RESULTS: DECut3 FIT

NMC DATA FOR D AND Sn/C vs Q^2



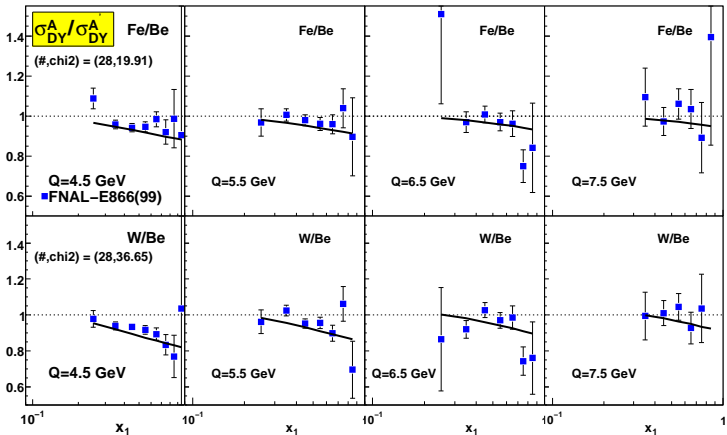
RESULTS: DECVT3 FIT

DRELL-YAN DATA



RESULTS: DECut3 FIT

DRELL-YAN DATA



CONCLUSIONS I

- Factorization works well for ℓA DIS and DY data
- Nuclear CTEQ PDFs will be released in the near future

Nuclear effects in νA DIS

I.S., Yu, Keppel, Morfi n, Olness, Owens, PRD77(2008)054013

I.S., Yu, Kovarik, Keppel, Morfi n, Olness, Owens, PRD80(2009)094004

Kovarik, Yu, Keppel, Morfi n, Olness, Owens, I.S., Stavreva, work nearing completion

WHY NEUTRINO DIS?

- **Flavor separation:**
Neutrino sfs depend on different combinations of PDFs
- **Dimuon production:**
 - Main source of information on the strange sea
 - Large uncertainty on $s(x, Q^2)$ has significant influence on the W and Z benchmark processes at the LHC
- **Data interesting for proton PDF and NPDF**
- **For proton PDF: need nuclear corrections**
- **EW precision measurements:**
Paschos-Wolfenstein analysis: extraction of $\sin^2 \theta_W$
- **LBL precision neutrino experiments:**
Need good understanding of neutrino–nucleus cross sections

NUCLEAR CORRECTION FACTORS

Be \mathcal{O} an **observable** calculable in the parton model

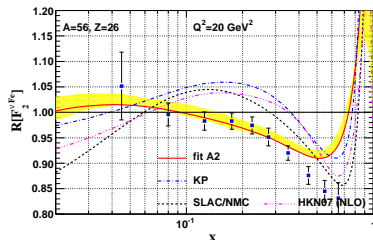
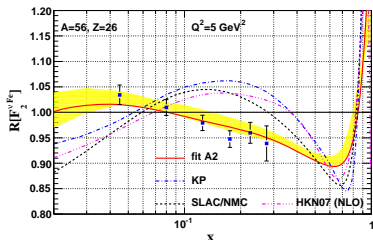
Define **nuclear correction factor**:

$$R[\mathcal{O}] := \frac{\mathcal{O}[\text{nPDF}]}{\mathcal{O}[\text{PDF}]} \quad \text{or for data} \quad R[\mathcal{O}] := \frac{\mathcal{O}^{\text{exp}}}{\mathcal{O}[\text{PDF}]}$$

- Factor needed to correct data to the free nucleon level
- Note: different observables \Rightarrow different correction factors
- In particular, correction factor for $F_3^{\nu A}$ could be quite different from $F_2^{\nu A}$!
- Also $R[F_2^{\ell A}]$, $R[F_2^{\nu A}]$, $R[F_2^{\bar{\nu} A}]$, $R[d^2\sigma^{\nu A}/dx dQ^2]$, ... are all (more or less) different **even for universal nPDFs**

Note: the term “nuclear effects” is less precise and (mis-)used in the literature for a lot of different things

NUCLEAR CORRECTION FACTOR $R[F_2^{\nu Fe}]$



- Are nuclear corrections in charged-lepton and neutrino DIS different?
- Obviously the PDFs from fits to $\ell A + \text{DY}$ data do not describe the NuTeV νFe DIS data.
- A global analysis of $\ell A + \text{DY} + \nu A$ data confirms this result! (see backup slides for a detailed account)

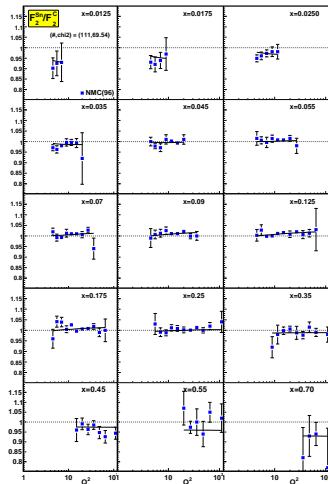
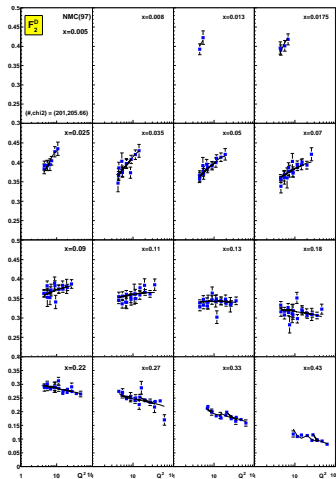
Note: $x_{\min} = 0.02$ in these figures.

CONCLUSIONS II

- We observe different nuclear effects in $\ell A+DY$ data as opposed to NuTeV νFe data
- These are precision effects relevant for precision observables
- Paukkunen and Salgado come to different conclusions in a recent paper
- The main reason for the different results is that Paukkunen and Salgado use uncorrelated systematic errors, whereas we take into account the full error correlation matrix
- For more details see backup slides and a publication in preparation

The nuclear gluon distribution

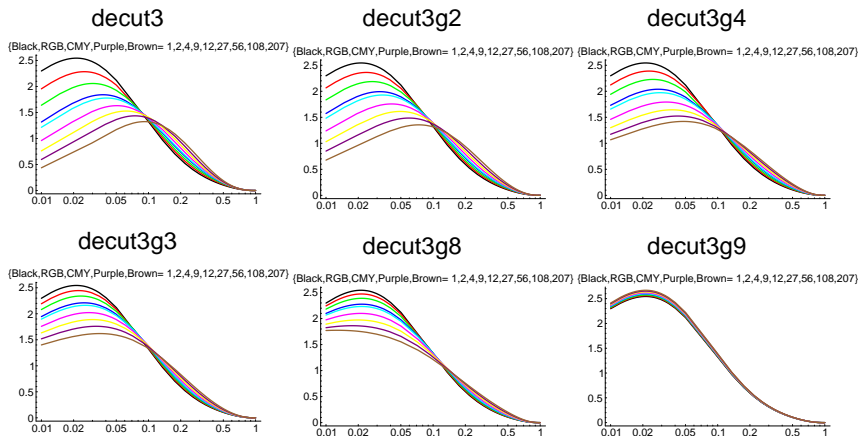
$g^A(x, Q^2)$ WEAKLY CONSTRAINED BY Q^2 -DEPENDENCE OF NMC DATA



- $x \sim 0.01 \dots 0.4$, $Q^2 \sim 10 \dots 100 \text{ GeV}^2$

THE NUCLEAR GLUON DISTRIBUTION

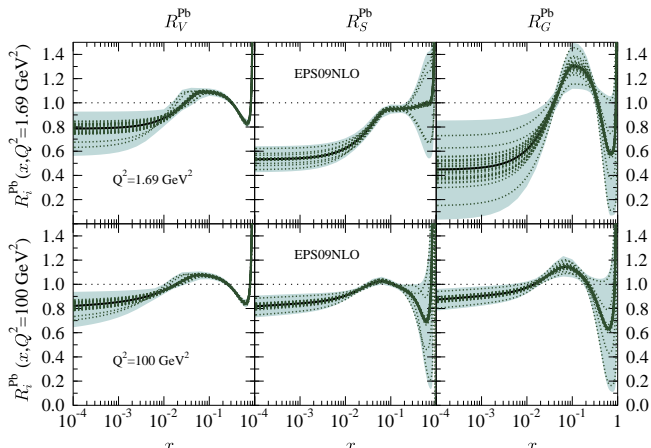
A series of **equally good fits** ($\chi^2/pt \simeq 0.9$) to $\ell A+DY$ data with different gluons



Shown are the gluon distributions at the scale $Q_0 = 1.3 \text{ GeV}$ for different A vs x

GLUON UNCERTAINTY IN EPS'09

- EPS'09 also uses RHIC data for inclusive pion production to constrain the gluon
- This involves fragmentation functions $D_i^\pi(z, \mu^2)$ into pions
- Large uncertainties! Still some of the gluons of the decut3g series lie outside the error band of R_G^{Pb}



NEED HARD PROBES IN pA TO CONSTRAIN NPDFS

Hard probes in pp , $p\bar{p}$ to constrain proton PDFs:

- Tevatron inclusive jet data \rightarrow gluon
- Lepton pair production \rightarrow sea quarks
- Vector boson production \rightarrow sea quarks (less useful due to high scale)

Other interesting processes:

- Prompt photon production [see Arleo, Gousset] \rightarrow gluon
- Heavy quark production \rightarrow gluon?
- $\gamma + j \rightarrow$ gluon
- $\gamma + j_Q$ (see talk by T. Stavreva) \rightarrow gluon, charm
- $\gamma + J/\Psi$ (see talk by M. Machado)
- Quarkonium production?

Backup slides

Analysis of νA , ℓA and DY data

Kovarik, Yu, Keppel, Morfin, Olness, Owens, Schienbein, Stavreva, work nearing completion

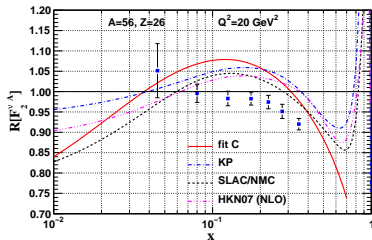
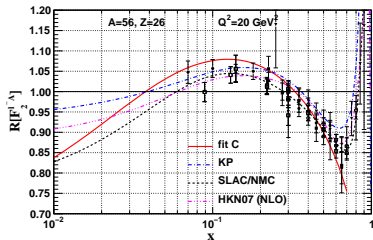
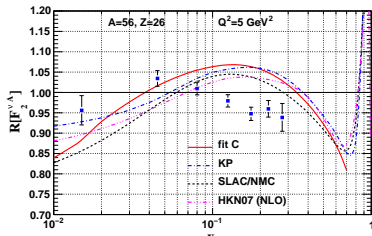
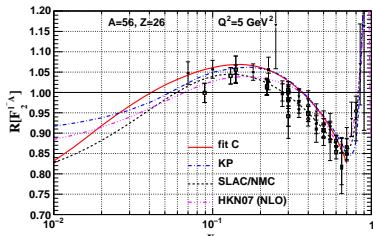
COMBINING ℓA DIS, DY AND νA DIS DATA

- ℓA and DY data sets as before
- 8 Neutrino data sets
 - NuTeV cross section data: νFe , $\bar{\nu} Fe$
 - CHORUS cross section data: νPb , $\bar{\nu} Pb$
 - NuTeV dimuon data: νFe , $\bar{\nu} Fe$
 - CCFR dimuon data: νFe , $\bar{\nu} Fe$
- Problem: Neutrino data sets have much higher statistics. Systematically study fits with different weights.

| Weight | Fit name | ℓ data | χ^2 (/pt) | ν data | χ^2 (/pt) | total χ^2 (/pt) |
|--------------|----------|-------------|----------------|------------|----------------|----------------------|
| $w = 0$ | decut3 | 708 | 639 (0.90) | - | - | 639 (0.90) |
| $w = 1/7$ | glofac1a | 708 | 645 (0.91) | 3134 | 4710 (1.50) | 5355 (1.39) |
| $w = 1/4$ | glofac1c | 708 | 654 (0.92) | 3134 | 4501 (1.43) | 5155 (1.34) |
| $w = 1/2$ | glofac1b | 708 | 680 (0.96) | 3134 | 4405 (1.40) | 5085 (1.32) |
| $w = 1$ | global2b | 708 | 736 (1.04) | 3134 | 4277 (1.36) | 5014 (1.30) |
| $w = \infty$ | nuanua1 | - | - | 3134 | 4192 (1.33) | 4192 (1.33) |

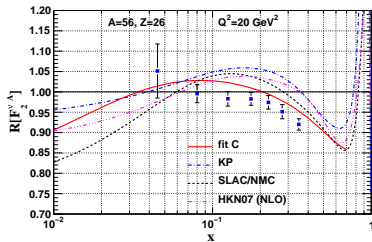
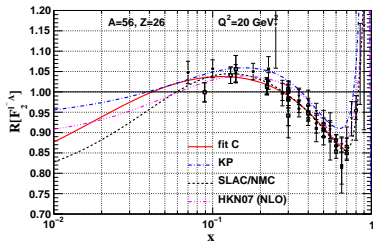
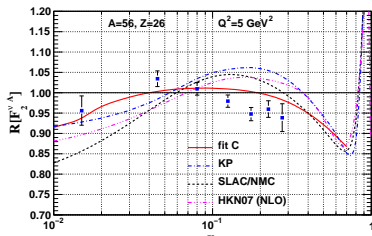
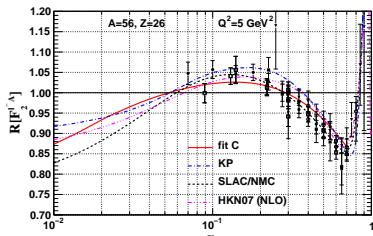
$R[F_2^{\ell Fe}]$ (LEFT) VS $R[F_2^{\nu Fe}]$ (RIGHT)

decut3 ($w = 0$)



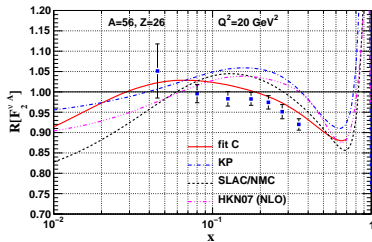
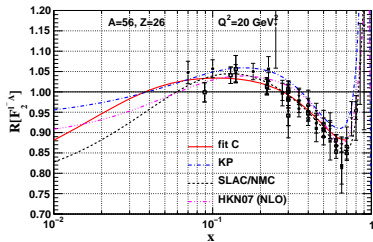
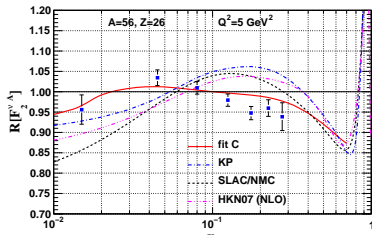
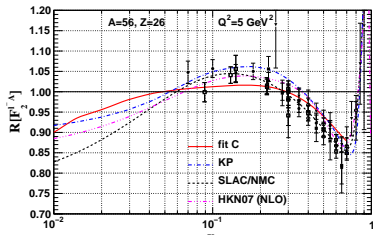
$R[F_2^{\ell Fe}]$ (LEFT) VS $R[F_2^{\nu Fe}]$ (RIGHT)

glofac1a ($w = 1/7$)



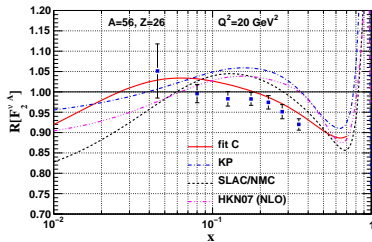
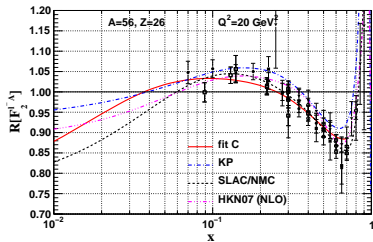
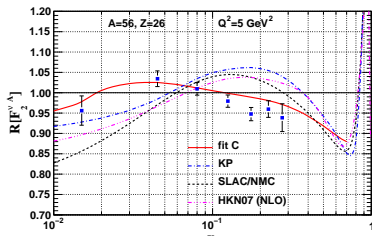
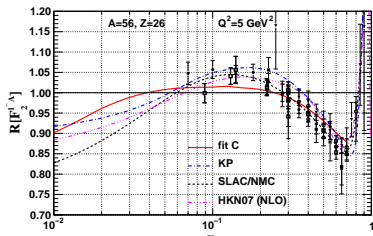
$R[F_2^{\ell Fe}]$ (LEFT) VS $R[F_2^{\nu Fe}]$ (RIGHT)

glofac1c ($w = 1/4$)



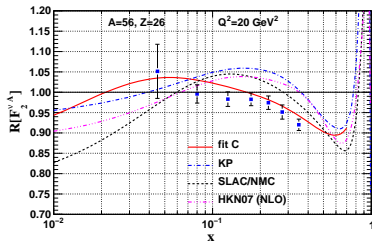
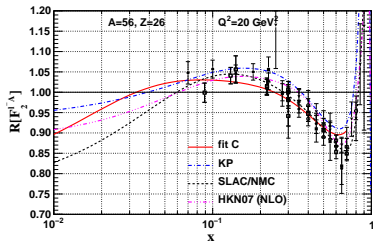
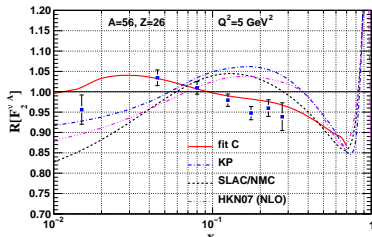
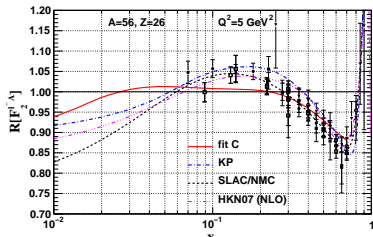
$R[F_2^{\ell Fe}]$ (LEFT) VS $R[F_2^{\nu Fe}]$ (RIGHT)

glofac1b ($w = 1/2$)



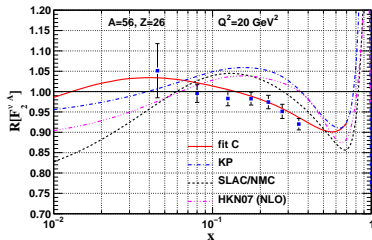
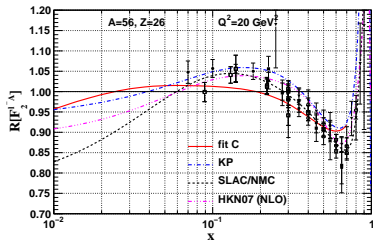
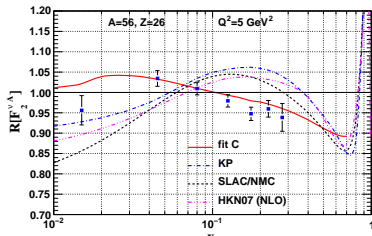
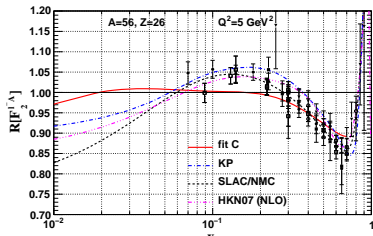
$R[F_2^{\ell Fe}]$ (LEFT) VS $R[F_2^{\nu Fe}]$ (RIGHT)

global2b ($w = 1$)



$R[F_2^{\ell Fe}]$ (LEFT) VS $R[F_2^{\nu Fe}]$ (RIGHT)

nuanua1 ($w = \infty$)



IS THERE A REASONABLE COMPROMISE FIT?

| Weight | Fit name | ℓ data | χ^2 (/pt) | ν data | χ^2 (/pt) | total χ^2 (/pt) |
|--------------|----------|-------------|----------------|------------|----------------|----------------------|
| $w = 0$ | decut3 | 708 | 639 (0.90) | - | - | 639 (0.90) |
| $w = 1/7$ | glofac1a | 708 | 645 (0.91) | 3134 | 4710 (1.50) | 5355 (1.39) |
| $w = 1/4$ | glofac1c | 708 | 654 (0.92) | 3134 | 4501 (1.43) | 5155 (1.34) |
| $w = 1/2$ | glofac1b | 708 | 680 (0.96) | 3134 | 4405 (1.40) | 5085 (1.32) |
| $w = 1$ | global2b | 708 | 736 (1.04) | 3134 | 4277 (1.36) | 5014 (1.30) |
| $w = \infty$ | nuanua1 | - | - | 3134 | 4192 (1.33) | 4192 (1.33) |

- $w = 0$: No. Problem: $R[F_2^{\nu Fe}]$
- $w = 1/7$: No. Problem: $R[F_2^{\nu Fe}]$
- $w = 1/4, 1/2$: No.
 - $Q^2 = 5$: Undershoots $R[F_2^{\ell Fe}]$ for $x < 0.2$. Overshoots $R[F_2^{\nu Fe}]$ for $x \in [0.1, 0.3]$
 - $Q^2 = 20$: $R[F_2^{\ell Fe}]$ still ok. Overshoots $R[F_2^{\nu Fe}]$.
- $w = 1$: No. Possibly there is a compromise if more strict Q^2 cut?
 - $Q^2 = 5$: Undershoots $R[F_2^{\ell Fe}]$ for $x < 0.2$. $R[F_2^{\nu Fe}]$ ok.
 - $Q^2 = 20$: $R[F_2^{\ell Fe}]$ still ok. $R[F_2^{\nu Fe}]$ ok.
- $w = \infty$: No. Problem: $R[F_2^{\ell Fe}]$

DISCUSSION/INTERMEDIATE CONCLUSION

Discussion based on the comparison of the nuclear correction factors $R[F_2^{\ell A}]$ and $R[F_2^{\nu A}]$

- There is definitely a tension between the NuTeV and the charged lepton data
 - There is a clear dependence on the weight.
 - Theory curves for $R[F_2^{\ell A}]$ and $R[F_2^{\nu A}]$ are both shifted down with increasing weight of the neutrino data.
- Preliminary conclusion: **At the level of the (high) precision there doesn't seem to be a good compromise fit of the combined ℓA , DY and νA data.**
- However one has to be careful:
 - These are precision effects
 - For each weight, the curves have uncertainty bands not considered
 - The figures show the comparison to only few (representative) data

Consider next quantitative criterion based on χ^2

TOLERANCE CRITERION

Probability distribution for the χ^2 function

$$P_N(\chi^2) = \frac{(\chi^2)^{N/2-1} e^{-\chi^2/2}}{2^{N/2} \Gamma(N/2)}$$

Determine ξ_{50}^2 and ξ_{90}^2 (i.e. $p = 50$, $p = 90$):

$$\int_0^{\xi_p^2} d\chi^2 P_N(\chi^2) = p/100$$

Condition for compatibility of two fits:

The 2nd fit (χ_n^2) should be within the 90% C.L. region of the first fit ($\chi_{n,0}^2$)

$$\chi_n^2 / \chi_{n,0}^2 < \xi_{90}^2 / \xi_{50}^2 \quad \Leftrightarrow \quad C_{90} \equiv \frac{\Delta\chi^2}{\frac{\chi_{n,0}^2}{\xi_{50}^2} (\xi_{90}^2 - \xi_{50}^2)} < 1$$

see CTEQ'01, PRD65(2001)014012; MSTW'09, EPJC(2009)63,189-285

TOLERANCE CRITERION $C_{90} < 1$:

TOTAL χ^2 FOR A) $\ell A+DY$ DATA AND B) NEUTRINO DATA

90% tolerance condition for the **charged lepton** χ^2 and the **neutrino** χ^2

- decut3: 638.9 ± 45.6 (best fit to only charged lepton and DY data)
- nuanua1: 4192 ± 138 (best fit to only neutrino data)

Is there a compromise fit compatible to both, decut3 **and** nuanua1?

| Weight | Fit name | ℓ data | χ^2 | ν data | χ^2 | total χ^2 (/pt) |
|--------------|-----------------|-------------|----------------|------------|-------------------|----------------------|
| $w = 0$ | decut3 | 708 | 639 | - | nnnn NO | 639 (0.90) |
| $w = 1/7$ | glofac1a | 708 | 645 YES | 3134 | 4710 NO | 5355 (1.39) |
| $w = 1/4$ | glofac1c | 708 | 654 YES | 3134 | 4501 NO | 5155 (1.34) |
| $w = 1/2$ | glofac1b | 708 | 680 YES | 3134 | 4405 NO*** | 5085 (1.32) |
| $w = 1$ | global2b | 708 | 736 NO | 3134 | 4277 YES | 5014 (1.30) |
| $w = \infty$ | nuanua1 | - | nnn NO | 3134 | 4192 | 4192 (1.33) |

Observations:

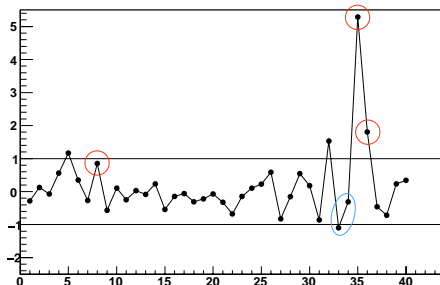
- There is no good compromise fit based on the 90% C.L. criterion.
- Our best candidate is **glofac1b** which is *marginally* compatible: $4405 - 4192 \simeq 1.5 \times 138$
- Observations in agreement with the previous conclusions based on $R[F_2^{\ell Fe}]$ and $R[F_2^{\nu Fe}]$.

Let's have a look at the tolerance criterion applied to the individual data sets!

TOLERANCE CRITERION $C_{90} < 1$:

INDIVIDUAL DATA SETS: $n = 1, \dots, 32$ VS DECUT3; $n = 33, \dots, 40$ VS NUANUA1

glofac1a ($w = 1/7$)

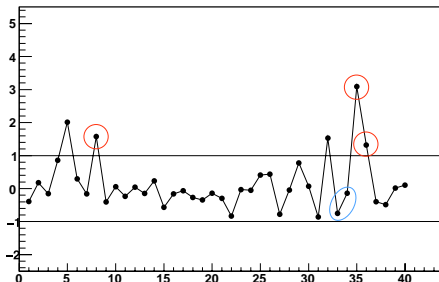


- Y-axis: C_{90} ; X-axis: Number of the data set ($n = 1, \dots, 40$)
- Important data sets:
 - $n = 8$ (red circle): Fe/D charged lepton data
 - blue ellipse: CHORUS $\nu Pb, \bar{\nu} Pb$ cross section data
 - $n = 35, 36$ (red ellipse): NuTeV $\nu Fe, \bar{\nu} Fe$ cross section data

TOLERANCE CRITERION $C_{90} < 1$:

INDIVIDUAL DATA SETS: $n = 1, \dots, 32$ VS DECUT3; $n = 33, \dots, 40$ VS NUANUA1

glofac1c ($w = 1/4$)

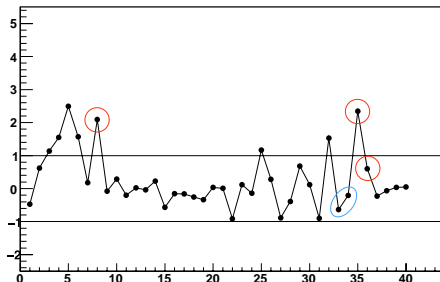


- Y-axis: C_{90} ; X-axis: Number of the data set ($n = 1, \dots, 40$)
- Important data sets:
 - $n = 8$ (red circle): Fe/D charged lepton data
 - blue ellipse: CHORUS $\nu Pb, \bar{\nu} Pb$ cross section data
 - $n = 35, 36$ (red ellipse): NuTeV $\nu Fe, \bar{\nu} Fe$ cross section data

TOLERANCE CRITERION $C_{90} < 1$:

INDIVIDUAL DATA SETS: $n = 1, \dots, 32$ VS DECUT3; $n = 33, \dots, 40$ VS NUANUA1

glofac1b ($w = 1/2$)

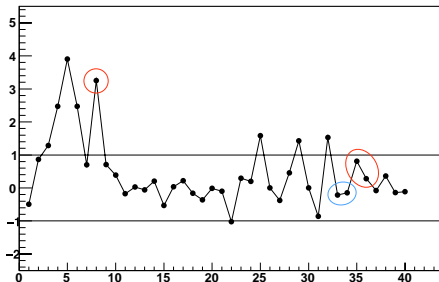


- Y-axis: C_{90} ; X-axis: Number of the data set ($n = 1, \dots, 40$)
- Important data sets:
 - $n = 8$ (red circle): Fe/D charged lepton data
 - blue ellipse: CHORUS $\nu Pb, \bar{\nu} Pb$ cross section data
 - $n = 35, 36$ (red ellipse): NuTeV $\nu Fe, \bar{\nu} Fe$ cross section data

TOLERANCE CRITERION $C_{90} < 1$:

INDIVIDUAL DATA SETS: $n = 1, \dots, 32$ VS DECUT3; $n = 33, \dots, 40$ VS NUANUA1

global2b ($w = 1$)



- Y-axis: C_{90} ; X-axis: Number of the data set ($n = 1, \dots, 40$)
- Important data sets:
 - $n = 8$ (red circle): Fe/D charged lepton data
 - blue ellipse: CHORUS $\nu Pb, \bar{\nu} Pb$ cross section data
 - $n = 35, 36$ (red ellipse): NuTeV $\nu Fe, \bar{\nu} Fe$ cross section data

TOLERANCE CRITERION $C_{90} < 1$:

INDIVIDUAL DATA SETS

Observations:

- $w = 1/7$: $C_{90} > 5$ for NuTeV νFe ; $C_{90} \simeq 1.8$ for NuTeV $\bar{\nu} Fe$
- CHORUS data (blue ellipse) always compatible; little dependence on weight w
- increasing weight: NuTeV cross section data improve; charged lepton Fe/D data get worse
- our best candidate ($w = 1/2$)
 - Fe/D ($n = 8$): $C_{90} \simeq 2$
 - NuTeV νFe ($n = 35$): $C_{90} \simeq 2.2$
 - NuTeV $\bar{\nu} Fe$ ($n = 36$): $C_{90} < 1$
 - some other data sets $n = 3, 4, 5, 6, 32$ with $C_{90} > 1$
- $w = 1$: Fe/D ($n = 8$): $C_{90} > 3$
- Confirms and quantifies observations based on R plots

CONCLUSIONS

Based on nuclear corrections factors R and the tolerance criterion $C_{90} < 1$:

- There is no good compromise fit to the $\ell A \text{ DIS} + \text{DY} + \nu A \text{ DIS}$ data.
- Most problematic: tension between NuTeV νFe cross section data and Fe/D data in charged lepton DIS.
- The NuTeV $\bar{\nu} Fe$ data are less problematic. They have larger errors.
- The CHORUS νPb and $\bar{\nu} Pb$ data are compatible with both, the $\ell A\text{-DIS}+\text{DY}$ and the NuTeV νFe and $\bar{\nu} Fe$ data, as is well known. They also have larger errors.

- Relaxing the tolerance criterion to $C_{90} \lesssim 2$ the fit with weight $w = 1/2$ would be *marginally* acceptable.
- This can also (qualitatively) be verified with the R -plots.
- A larger Q^2 -cut, say $Q^2 > 5 \text{ GeV}^2$, could also help to reduce the tension. (In particular, this would remove some of the rather precise NuTeV cross section data at small x .)