

Global Analysis of Nuclear PDFs

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based on work in collaboration with

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Outline

- Parton distribution functions (PDFs)
- From protons to nuclei
- Global analysis of nCTEQ nuclear PDFs
- The nuclear gluon distribution
- The gluon from hard processes at the LHC/RHIC
- Nuclear corrections in neutrino DIS

Parton distribution functions (PDFs)

NUCLEAR PDFs (NPDF)

- Information on **hadronic structure**
- **Initial state** for hard processes in collisions involving hadrons
 - Deep inelastic scattering (DIS): $lA, \nu A$
 - Drell-Yan (DY): $A + B \rightarrow l^+ + l^-$
 - Jets, Photons, Hadrons at large p_T ; Heavy Quarks; ...
in $pA, AA, (\gamma A, eA)$ collisions
- Provide **nuclear corrections** for global analyses of **proton PDFs** in a **flexible way**

THEORETICAL BASIS: FACTORIZATION

- Factorization theorems
 - provide (field theoretical) **definitions of universal PDFs**
 - make the formalism **predictive**
 - make a statement about the **error**
- **PDFs** and predictions for **observables+uncertainties** **refer to this standard pQCD framework**
- There might be breaking of QCD factorization, deviations from **DGLAP** evolution — in particular in a nuclear environment

Still need solid understanding of standard framework to establish deviations!

In the nuclear case, consider factorization as a **working assumption** to be tested phenomenologically

Predictive Power

Universality: same PDFs/FFs enter different processes:

- DIS:
$$F_2^A(x, Q^2) = \sum_i [f_i^A \otimes C_{2,i}] (x, Q^2)$$
- DY:
$$\sigma_{A+B \rightarrow \ell^+ + \ell^- + X} = \sum_{i,j} f_i^A \otimes f_j^B \otimes \hat{\sigma}^{i+j \rightarrow \ell^+ + \ell^- + X}$$
- A+B \rightarrow H + X:
$$\sigma_{A+B \rightarrow H+X} = \sum_{i,j,k} f_i^A \otimes f_j^B \otimes \hat{\sigma}^{i+j \rightarrow k+X} \otimes D_k^H$$
- **Predictions** for unexplored kinematic regions and for your favorite **new physics** process

From protons to nuclei

FROM PROTONS TO NUCLEI

Starting point: (CTEQ) global analysis framework for free nucleons

Make sure it can be applied to the case of PDFs for nuclear targets (A, Z)

- Variable: $0 < x_N < A$
- Evolution equations
- Sum rules
- Observables

Apart from the validity of factorization which is (possibly up to precision effects) a working assumption and to be verified phenomenologically

DIS ON NUCLEAR TARGETS

Consider deep inelastic lepton–nucleon collisions: $l(k) + A(p_A) \rightarrow l'(k') + X$

Introduce the usual DIS variables: $q \equiv k - k'$, $Q^2 \equiv -q^2$, $x_A \equiv \frac{Q^2}{2p_A \cdot q}$

Hadronic tensor: $W_{\mu\nu}^A \propto \langle A(p_A) | J_\mu J_\nu^\dagger | A(p_A) \rangle = \sum_i a_{\mu\nu}^{(i)} \tilde{F}_i^A(x_A, Q^2)$,

where $a_{\mu\nu}^{(i)}$ are Lorentz-tensors composed out of the 4-vectors q and p_A and the metric $g_{\mu\nu}$

Express structure functions in the QCD improved parton model in terms of NPDFs

$$\tilde{\mathcal{F}}_k^A(x_A, Q^2) = \int_{x_A}^1 \frac{dy_A}{y_A} \tilde{f}_i^A(y_A, Q^2) C_{k,i}(x_A/y_A) + \tilde{\mathcal{F}}_k^{A, \tau \geq 4}(x_A, Q^2)$$

NPDFs: Fourier transforms of matrix elements of twist-two operators composed out of the quark and gluon fields:

$$\tilde{f}_i^A(x_A, Q^2) \propto \langle A(p_A) | O_i | A(p_A) \rangle$$

Definitions of $\tilde{F}_i^A(x_A, Q^2)$, $\tilde{f}_i^A(x_A, Q^2)$, and the variable $0 < x_A < 1$ carry over one-to-one from the well-known free nucleon case

EVOLUTION EQUATIONS AND SUM RULES

DGLAP as usual:

$$\begin{aligned}\frac{d\tilde{f}_i^A(x_A, Q^2)}{d\ln Q^2} &= \frac{\alpha_s(Q^2)}{2\pi} \int_{x_A}^1 \frac{dy_A}{y_A} P_{ij}(y_A) \tilde{f}_j^A(x_A/y_A, Q^2), \\ &= \frac{\alpha_s(Q^2)}{2\pi} \int_{x_A}^1 \frac{dy_A}{y_A} P_{ij}(x_A/y_A) \tilde{f}_j^A(y_A, Q^2),\end{aligned}$$

Sum rules:

$$\begin{aligned}\int_0^1 dx_A \tilde{u}_V^A(x_A, Q^2) &= 2Z + N, \\ \int_0^1 dx_A \tilde{d}_V^A(x_A, Q^2) &= Z + 2N,\end{aligned}$$

and the momentum sum rule

$$\int_0^1 dx_A x_A \left[\tilde{\Sigma}^A(x_A, Q^2) + \tilde{g}^A(x_A, Q^2) \right] = 1,$$

where $N = A - Z$ and $\tilde{\Sigma}^A(x_A) = \sum_i (\tilde{q}_i^A(x_A) + \tilde{\bar{q}}_i^A(x_A))$ is the quark singlet combination

RESCALED DEFINITIONS

Problem: average momentum fraction carried by a parton $\propto A^{-1}$
since there are 'A-times more partons' which have to share the momentum

- Different nuclei (A, Z) not directly comparable
- Functional form for x -shape would change drastically with A
- Need to rescale!

PDFs are number densities: $\tilde{f}_i^A(x_A) dx_A$ is the number of partons carrying a momentum fraction in the interval $[x_A, x_A + dx_A]$

Define rescaled NPDFs $f_i^A(x_N)$ with $0 < x_N := Ax_A < A$:

$$f_i^A(x_N) dx_N := \tilde{f}_i^A(x_A) dx_A$$

The variable x_N can be interpreted as parton momentum fraction w.r.t. the **average** nucleon momentum $\bar{p}_N := p_A/A$

RESCALED EVOLUTION EQUATIONS AND SUM RULES

Evolution:

$$\begin{aligned}\frac{df_i^A(x_N, Q^2)}{d \ln Q^2} &= \frac{\alpha_s(Q^2)}{2\pi} \int_{x_N/A}^1 \frac{dy_A}{y_A} P(y_A) f_i^A(x_N/y_A, Q^2), \\ &= \frac{\alpha_s(Q^2)}{2\pi} \int_{x_N}^A \frac{dy_N}{y_N} P(x_N/y_N) f_i^A(y_N, Q^2).\end{aligned}$$

Assume that $f_i^A(x_N) = 0$ for $x_N > 1$, then **original, symmetrical** form recovered:

$$\frac{df_i^A(x_N, Q^2)}{d \ln Q^2} = \begin{cases} \frac{\alpha_s(Q^2)}{2\pi} \int_{x_N}^1 \frac{dy_N}{y_N} P(y_N) f_i^A(x_N/y_N, Q^2) & : 0 < x_N \leq 1 \\ 0 & : 1 < x_N < A, \end{cases}$$

Sum rules for the rescaled PDFs:

$$\begin{aligned}\int_0^A dx_N u_v^A(x_N) &= 2Z + N, \\ \int_0^A dx_N d_v^A(x_N) &= Z + 2N,\end{aligned}$$

and

$$\int_0^A dx_N x_N \left[\Sigma^A(x_N) + g^A(x_N) \right] = A,$$

RESCALED STRUCTURE FUNCTIONS

The rescaled structure functions can be defined as

$$x_N \mathcal{F}_i^A(x_N) := x_A \tilde{\mathcal{F}}_i^A(x_A) ,$$

with $\mathcal{F}_{1,2,3}(x) = \{F_1(x), F_2(x)/x, F_3(x)\}$.

More explicitly:

$$\begin{aligned} F_2^A(x_N) &:= \tilde{F}_2^A(x_A) , \\ x_N F_1^A(x_N) &:= x_A \tilde{F}_1^A(x_A) , \\ x_N F_3^A(x_N) &:= x_A \tilde{F}_3^A(x_A) . \end{aligned}$$

This leads to consistent results in the parton model using the rescaled PDFs.

PDFS OF BOUND NUCLEONS

Further decompose the NPDFs $f_i^A(x_N)$ in terms of effective parton densities for **bound** protons, $f_i^{p/A}(x_N)$, and neutrons, $f_i^{n/A}(x_N)$, inside a nucleus A :

$$f_i^A(x_N, Q^2) = Z f_i^{p/A}(x_N, Q^2) + N f_i^{n/A}(x_N, Q^2)$$

- The bound proton PDFs have the **same** evolution equations and sum rules as the free proton PDFs **provided** we neglect any contributions from the region $x_N > 1$
- Neglecting the region $x_N > 1$, is consistent with the DGLAP evolution
- The region $x_N > 1$ is expected to have a minor influence on the sum rules of less than one or two percent (see also [[PRC73\(2006\)045206](#)])
- Isospin symmetry: $u^{n/A}(x_N) = d^{p/A}(x_N)$, $d^{n/A}(x_N) = u^{p/A}(x_N)$

An observable \mathcal{O}^A is then given by:

$$\mathcal{O}^A = Z \mathcal{O}^{p/A} + N \mathcal{O}^{n/A}$$

In conclusion: the free proton framework can be used to analyse nuclear data

Global analysis of nCTEQ nuclear PDF

Global Analysis: General Procedure

1.) Parameterize x -dependence of PDFs at **input scale** Q_0 :

$$f(x, Q_0) = A_0 x^{A_1} (1-x)^{A_2} P(x; A_3, \dots); f = u_v, d_v, g, \bar{u}, \bar{d}, s, \bar{s}$$

2.) **Evolve** from $Q_0 \rightarrow Q$ by solving the **DGLAP** evolution equations
 $\rightarrow f(x, Q)$

3.) Define suitable χ^2 function and **minimize** w.r.t. fit parameters

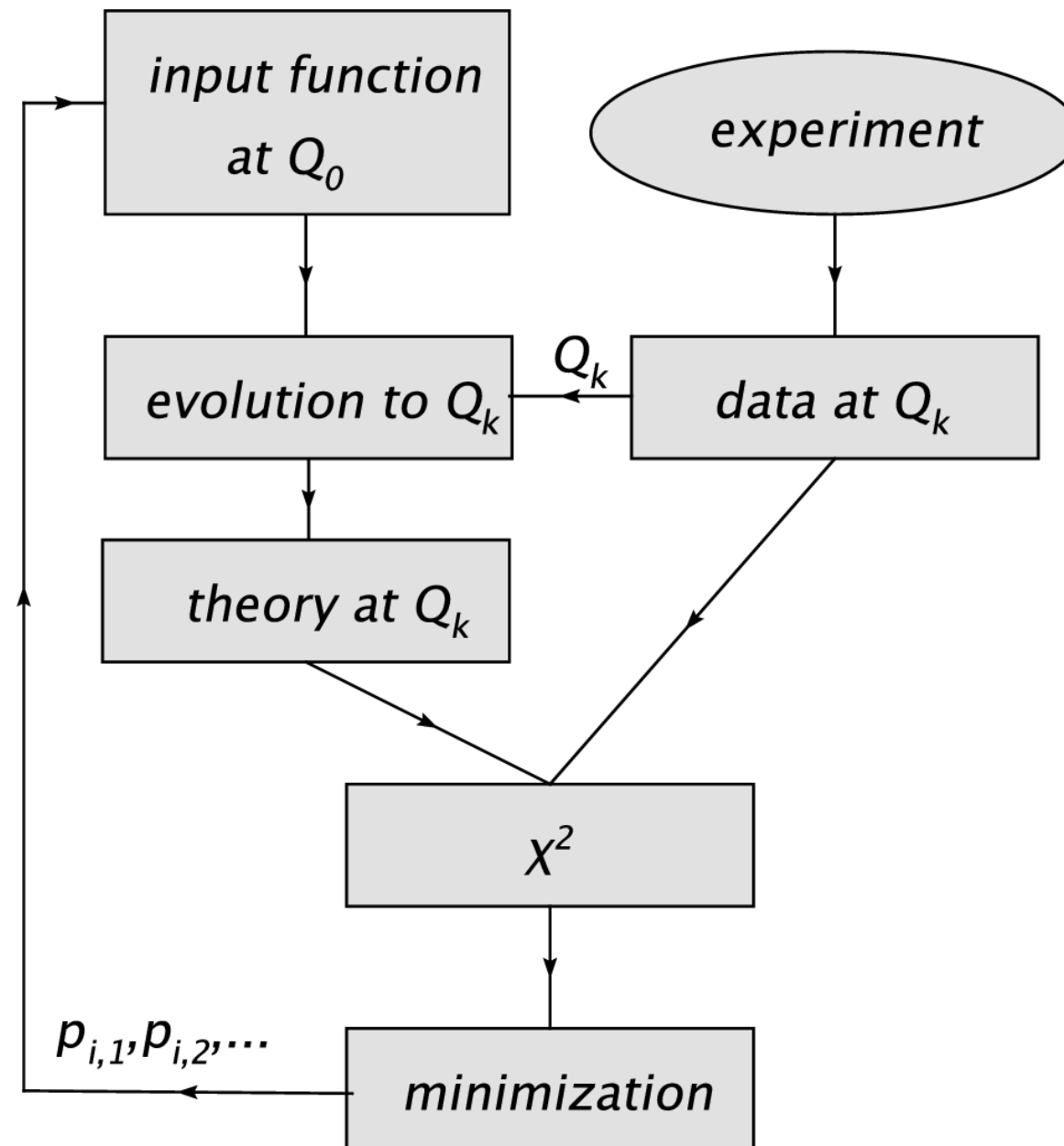
$$\chi^2_{global} [A_i] = \sum_n w_n \chi^2_n; \chi^2_n = \sum_I \left(\frac{D_{nI} - T_{nI}}{\sigma_{nI}} \right)^2$$

Sum over experiments

Sum over data points

weights: default=1, allows to emphasize certain data sets

Flowchart



NPDFS FROM ℓA DIS AND DY DATA

Global analyses of NPPDF by four groups:

- **HKN'07** [[PRC76\(2007\)065207](#)]
LO, NLO, error PDFs, $\chi^2/dof = 1.2$
- **EPS'09** [[JHEP0904\(2009\)065](#)]
LO, NLO, error PDFs, $\chi^2/dof = 0.8$
Use also inclusive π^0 data at midrap. from $d + Au$ and $p + p$ coll. at RHIC \rightarrow gluon
- **DS'04** [[PRD69\(2004\)074028](#)]
first NLO analysis, 'semi-global', no error PDFs, $\chi^2/dof = 0.76$
- **nCTEQ** [[PRD80\(2009\)094004](#)]
NLO, same data as HKN'07 (up to cuts), no error PDFs (so far), $\chi^2/dof = 0.95$, official release soon

Table from [Hirai et al., arXiv:0909.2329](#)

	R	Nucleus	Experiment	EPS09	HKN07	DS04	
DIS	A/D	D/p	NMC		0		
		4He	SLAC E139	0	0	0	
			NMC95	0 (5)	0	0	
		Li	NMC95	0	0		
		Be	SLAC E139	0	0	0	
		C	EMC-88, 90			0	
			NMC 95	0	0	0	
			SLAC E139	0	0	0	
		N	FNAL-E665			0	
			BCDMS 85			0	
		Al	HERMES 03			0	
			SLAC E49			0	
		Ca	SLAC E139	0	0	0	0
			EMC 90			0	
			NMC 95	0	0	0	0
			SLAC E139	0	0	0	0
		Fe	FNAL-E665			0	
			SLAC E87			0	
			SLAC E139	0 (15)	0	0	0
			SLAC E140			0	
		Cu	BCDMS 87			0	
			EMC 93	0	0		
		Kr	HERMES 03			0	
Ag	SLAC E139	0	0	0	0		
Sn	EMC 88			0			
Au	SLAC E139	0	0	0	0		
	SLAC E140			0			
Pb	FNAL-E665			0			
A/C	Be	NMC 96	0	0	0		
	Al	NMC 96	0	0	0		
	Ca	NMC 95			0		
		NMC 96	0	0	0	0	
	Fe	NMC 96	0	0	0	0	
	Sn	NMC 96	0 (10)	0	0	0	
A/Li	Pb	NMC 96	0	0	0		
	C	NMC 95	0	0			
DY	A/D	Ca	FNAL-E772	0	0	0	
		Fe		0 (15)	0	0	
		W		0 (15)	0	0	
				0 (10)	0	0	
A/Be	Fe	FNAL E866	0	0			
	W		0	0			
π pro	dA/pp	Au	RHIC-PHENIX	0 (20)			

WHAT ARE THE DIFFERENCES?

Main differences:

- **Choice of data sets** (see previous table)
- **Parametrization of input distributions**
- **Assumptions on PDFs**
 - Data less constraining than in proton case → need to make more assumptions (otherwise flat directions in χ^2 function and fits don't converge)
 - Assumptions replace uncertainty! → error bands (of a single fit) underestimate true uncertainties

Consequences?

- **Use different sets of NPDFs to scan over assumptions**
- Include more data sets → allows to relax assumptions
- New ideas to handle flat directions?
- Neural Network NPDFs?

WHAT ARE THE DIFFERENCES?

Further differences:

- **Heavy flavor schemes**

- **DS'04**: 3-Fixed Flavor Number Scheme (3-FFNS) → **no charm PDF**
- **HKN'07, EPS'09, nCTEQ**: Variable Flavor Number Schemes (VFNS)

→ Beware of comparing 'apples with oranges'!

- **Parameters and other**

- Input scale Q_0 , $\alpha_s(M_Z)$, m_c , m_b
- Evolution in n -space (**DS**) and x -space (**HKN, EPS, nCTEQ**)
- Target Mass Corrections (TMC)
see, e.g., [**IS et al., JPG35(2008)053101**; **Qiu, Accardi, JHEP0807(2008)090**]

NUCLEAR PDFS

Review of existing global analyses of nuclear PDF

1. Multiplicative nuclear correction factor

$$f_i^A(x_N, Q_0^2) = R_i(x_N, Q_0, A, Z) f_i(x_N, Q_0^2)$$

↑
free parton density

Hirai, Kumano, Nagai [[PRC76\(2007\)065207](#)] arXiv: 0709.0338

Eskola, Paukkunen, Salgado [[JHEP0904\(2009\)065](#)] arXiv: 0902.4154



de Florian, Sassot, Stratmann, Zurita arXiv: 1112.6324

2. Convolution relation

$$f_i^A(x_N, Q_0^2) = \int_{x_N}^A \frac{dy}{y} W_i(y, A, Z) f_i(x_N/y, Q_0^2)$$

↑
nucleon density in nucleus with y/A mom. fraction

de Florian, Sassot [[PRD69\(2004\)074028](#)] hep-ph/0311227

3. Native nuclear PDF

$$f_i^A(x_N, Q_0^2) = f_i(x_N, A, Q_0^2) \qquad f_i(x_N, Q_0^2) = f_i(x_N, A = 1, Q_0^2)$$

↑
bound parton density

↑
free parton density

nCTEQ [[PRD80\(2009\)094004](#)] arXiv: 0907.2357

Connected to GRV'98 proton PDFs $f_i^p(x, Q)$:

- $Q_0^2 = 0.4 \text{ GeV}^2$ (NLO), $Q_0^2 = 0.26 \text{ GeV}^2$ (LO), m_c, m_b, α_s as in GRV'98
- 3-Fixed flavor scheme (no charm PDF)
- strange PDF dynamically generated, i.e., $s^p(x, Q_0^2) = 0$

Parametrization of input distributions:

- PDFs for bound protons inside nucleus A : $f_i^{p/A}(x, Q)$

- Convolution relation: $f_i^{p/A}(x_N, Q_0^2) = \int_{x_N}^A \frac{dy}{y} W_i(y, A, Z) f_i^p(x_N/y, Q_0^2)$

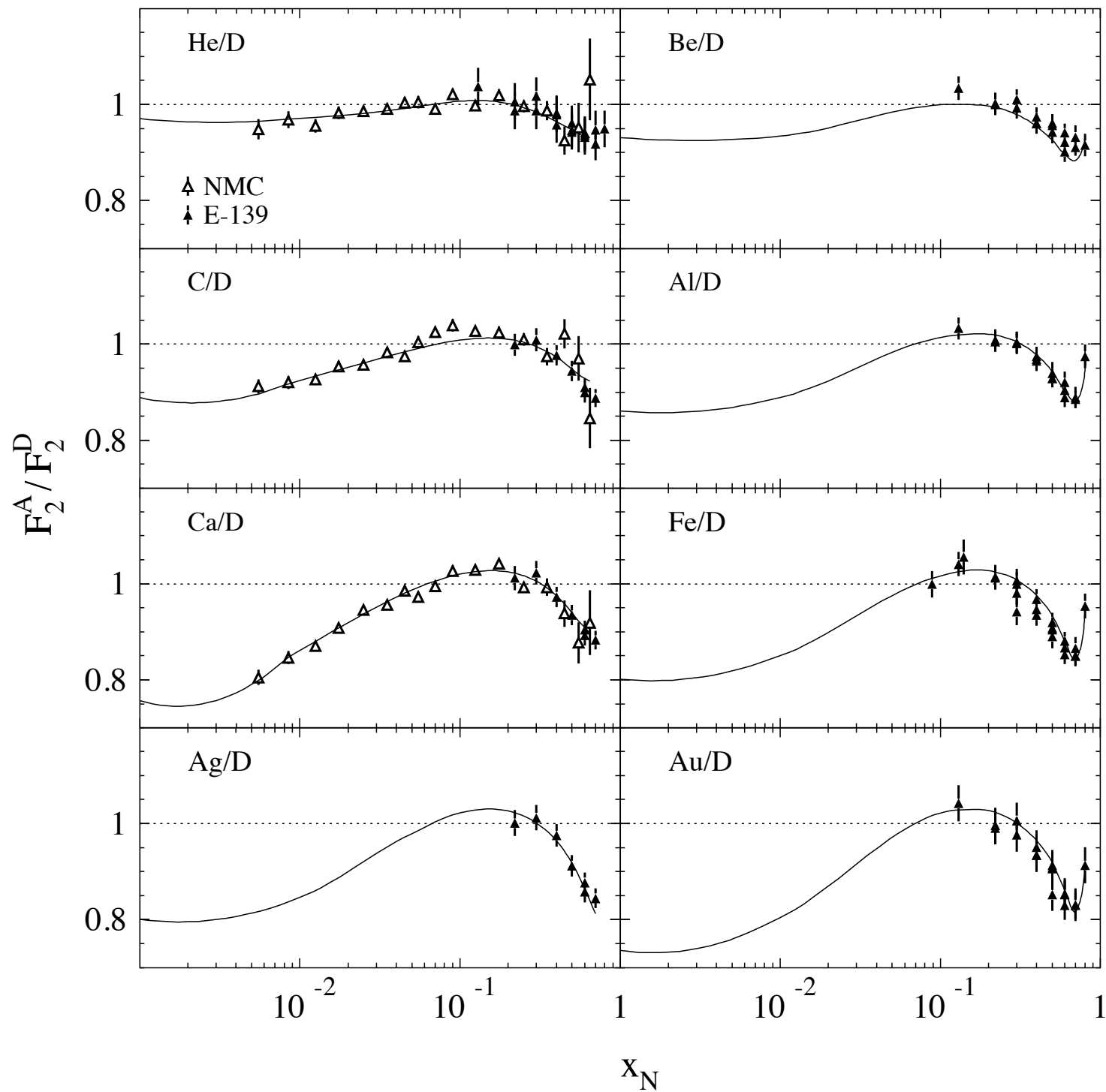
- Weight functions W_v (valence), W_s (sea), W_g (gluon). For example:

$$W_v(y, A, Z) = A[a_v \delta(1 - \epsilon_v - y) + (1 - a_v) \delta(1 - \epsilon_{v'} - y)] + n_v (y/A)^{\alpha_v} (1 - y/A)^{\beta_v} + n_s (y/A)^{\alpha_s} (1 - y/A)^{\beta_s}$$

- Note:

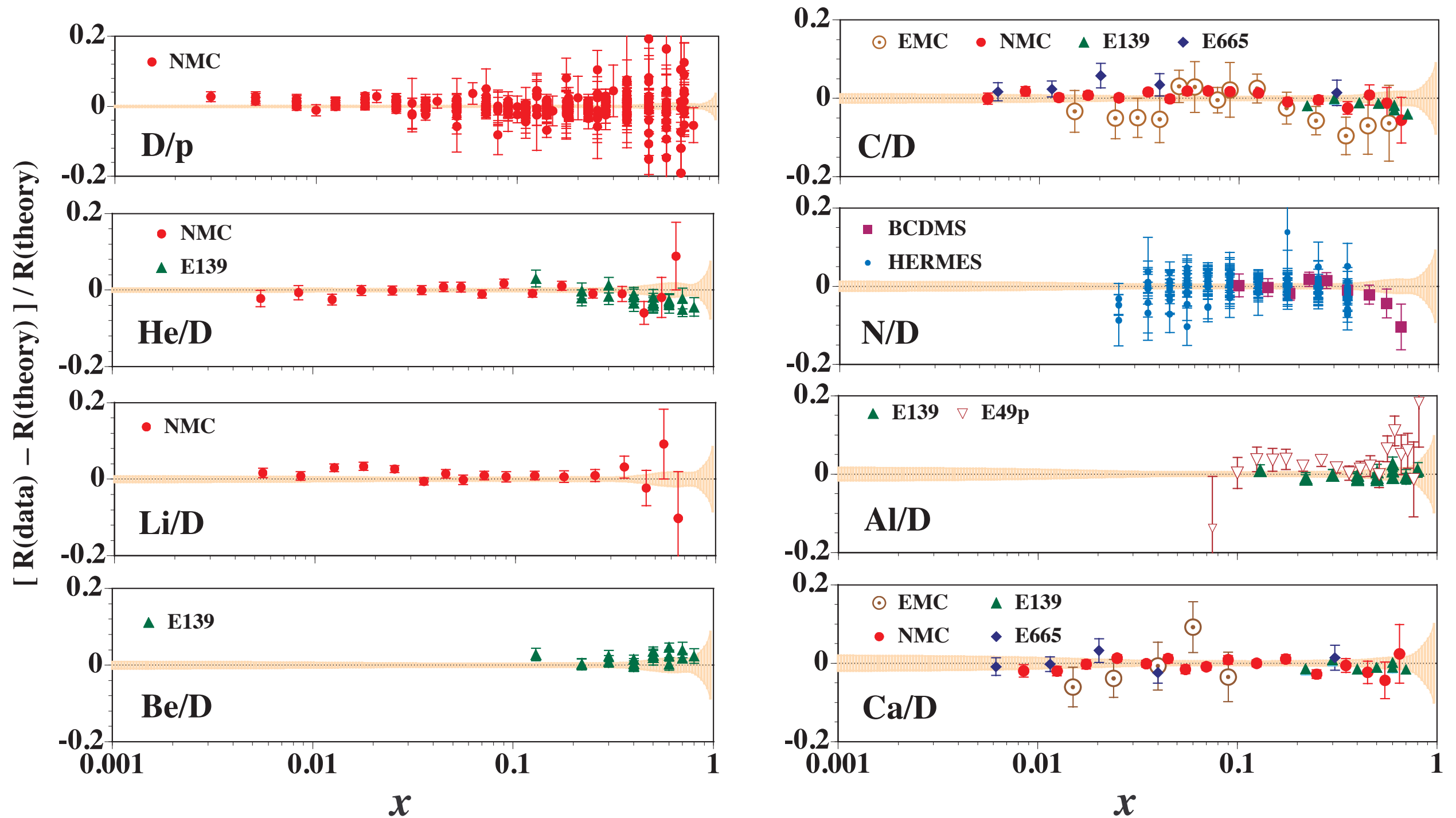
- Convolution simple product in Mellin moment space: very elegant
- Ansatz valid for $0 < x_N < A$!
- The x -space approaches (HKN, EPS, nCTEQ) are restricted to $0 < x_N < 1$
- However, the DS'04 PDF grids apparently are restricted to $0 < x_N < 1$ (and the momentum sum rule integrates to unity in this range)

Excellent fit to a **restricted** data set (420 points): $\chi^2/dof = 0.75$



- LO, NLO, error PDFs
- Related to MRST'98 proton PDF: $Q_0^2 = 1 \text{ GeV}^2$
- Uses multiplicative ansatz: $f_i^{p/A}(x_N, Q_0^2) = R_i(x_N, Q_0, A, Z) f_i^p(x_N, Q_0^2)$
- Weight factor: $R_i(x, A, Z) = 1 + \left(1 - \frac{1}{A^\alpha}\right) \frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1-x)^{\beta_i}} \quad (i = u_v, d_v, \bar{q}, g)$
- neglects region $x_N > 1$
- includes all current DIS & DY data sets, in particular deuterium data
- uses Hessian method to produce error PDFs

- Reasonable fits: $\chi^2/dof = 1.2$



- LO, NLO, error PDFs
- Related to CTEQ6.1M proton PDF: $Q_0 = 1.3 \text{ GeV}$

- Uses multiplicative ansatz: $f_i^{p/A}(x_N, Q_0^2) = R_i(x_N, Q_0, A, Z) f_i^p(x_N, Q_0^2)$

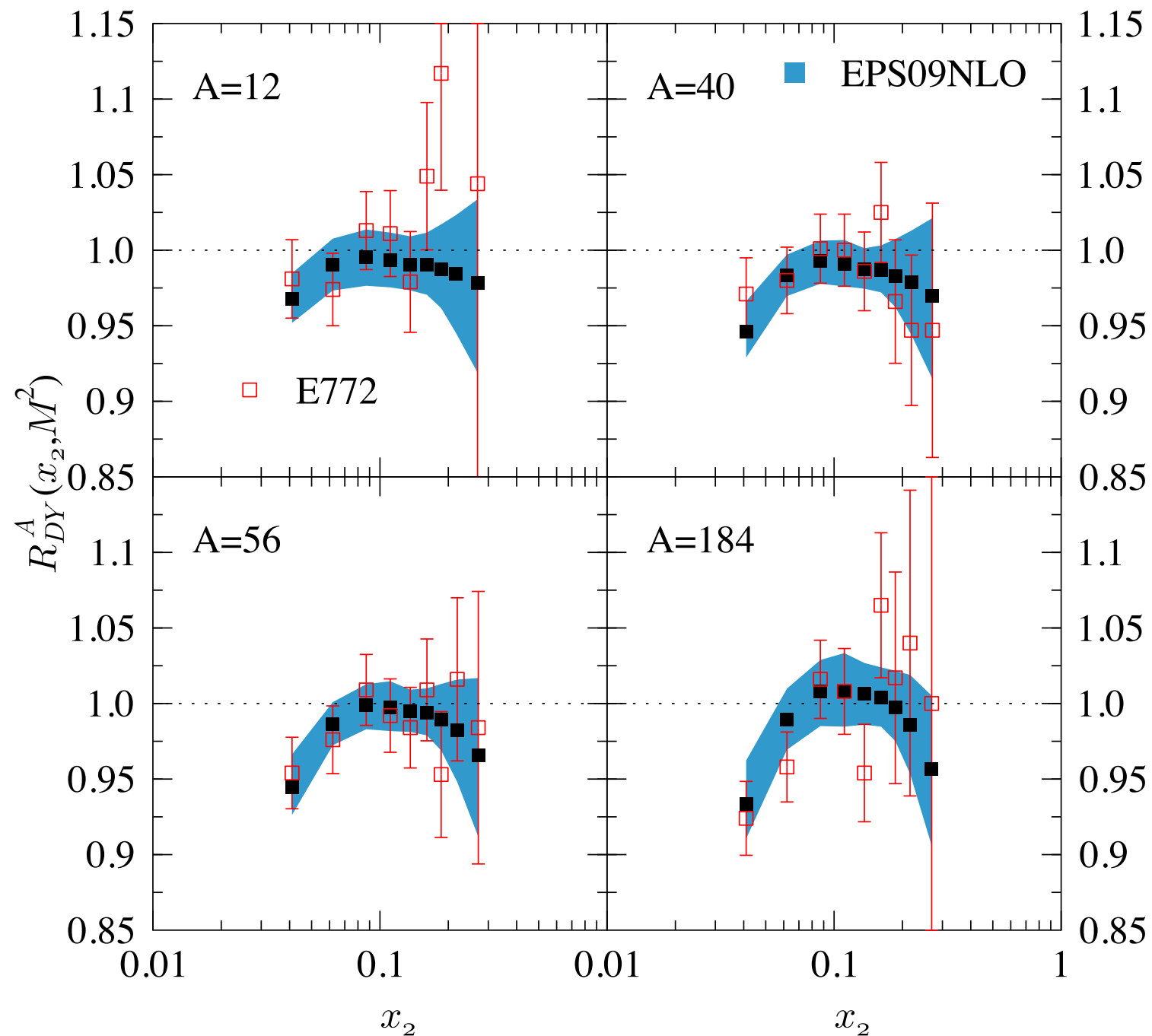
- Weight factor is a piecewise defined function:

$$R_i(x, A, Z) = \begin{cases} a_0 + (a_1 + a_2 x)(e^{-x} - e^{-x_a}) & x \leq x_a \\ b_0 + b_1 x + b_2 x^2 + b_3 x^3 & x_a \leq x \leq x_e \\ c_0 + (c_1 - c_2 x)(1 - x)^{-\beta} & x_e \leq x \leq 1 \end{cases}$$

where the parameters $a_i, b_i, c_i, \beta, x_a, x_e$ are A -dependent

- neglects region $x_N > 1$
- includes π^0 RHIC data with a weight 20 to constrain gluon
- uses Hessian method to produce error PDFs

- Excellent fit: $\chi^2/dof = 0.8$
- Show here, as an example, comparison with DY data



Work in collaboration with:

- People from LPSC Grenoble: K. Kovarik, J. Y. Yu, T. Stavreva, IS
- CTEQ-members: F. Olness (SMU), J. Owens (FSU), J. Morfin (FNAL), C. Keppel (JLAB)

-
- The results shown in the following are from
IS, Yu, Kovarik, Keppel, Morfin, Olness, Owens, PRD80(2009)094004

nCTEQ PDFs available at: <http://projects.hepforge.org/ncteq>

NUCLEAR CTEQ

Framework as in CTEQ6M proton fit:

- **Same functional form** for **bound proton PDFs** inside a nucleus A as for free proton PDFs (restrict x to $0 < x < 1$):

$$x f_k^{p/A}(x, Q_0) = c_0 x^{c_1} (1-x)^{c_2} e^{c_3 x} (1 + e^{c_4 x})^{c_5}, \quad k = u_v, d_v, g, \bar{u} + \bar{d}, s, \bar{s},$$
$$\bar{d}(x, Q_0)/\bar{u}(x, Q_0) = c_0 x^{c_1} (1-x)^{c_2} + (1 + c_3 x)(1-x)^{c_4}$$

(bound neutron PDFs $f_k^{n/A}$ by isospin symmetry)

- **A -dependent fit parameters:** (reduces to free proton parameters $c_{k,0}$ for $A = 1$)

$$c_k \rightarrow c_k(A) \equiv c_{k,0} + c_{k,1}(1 - A^{-c_{k,2}}), \quad k = 1, \dots, 5$$

- **PDFs for a nucleus (A, Z) :** $f_i^{(A,Z)}(x, Q) = \frac{Z}{A} f_i^{p/A}(x, Q) + \frac{A-Z}{A} f_i^{n/A}(x, Q)$
- **Input parameters:** $Q_0 = m_c = 1.3 \text{ GeV}$, $m_b = 4.5 \text{ GeV}$, $\alpha_s^{NLO, \overline{\text{MS}}}(M_Z) = 0.118$
- **Heavy quark treatment:** ACOT scheme
- **Standard DIS-cuts:** $Q > 2 \text{ GeV}$, $W > 3.5 \text{ GeV}$

EXPERIMENTAL INPUT



Use same data as HKN'07 (up to cuts)

- DIS F_2^A / F_2^D data sets: 862 points (before cuts)
- DIS $F_2^A / F_2^{A'}$ data sets: 297 points (before cuts)
- DY data sets $\sigma_{DY}^{pA} / \sigma_{DY}^{pA'}$: 92 points (before cuts)

Table from [Hirai et al., arXiv:0909.2329](https://arxiv.org/abs/0909.2329)

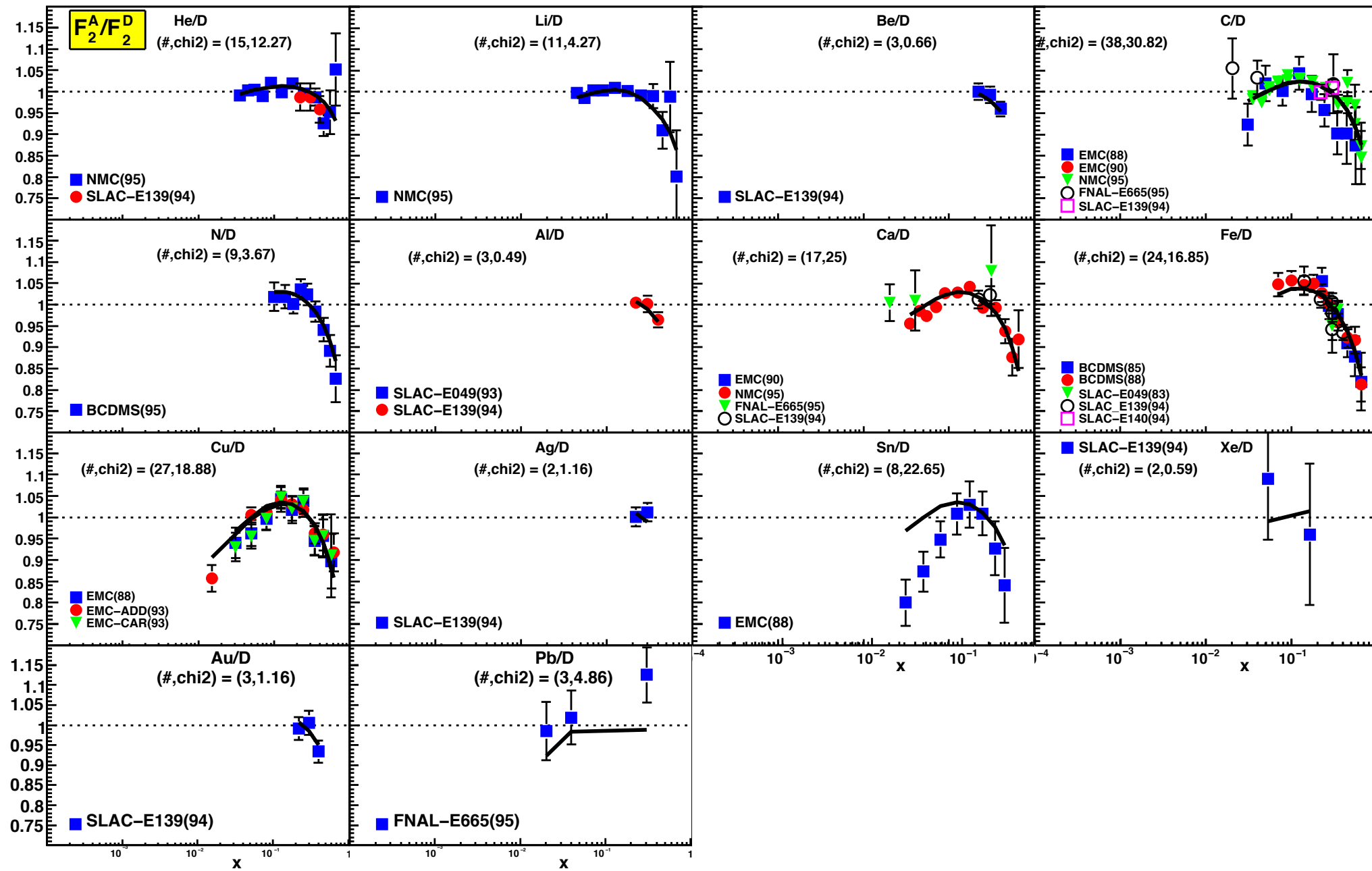
	R	Nucleus	Experiment	EPS09	HKN07	DS04	
DIS	A/D	D/p	NMC		0		
		4He	SLAC E139	0	0	0	
			NMC95	0 (5)	0	0	
		Li	NMC95	0	0		
		Be	SLAC E139	0	0	0	
		C	EMC-88, 90			0	
			NMC 95	0	0	0	
			SLAC E139	0	0	0	
		N	FNAL-E665			0	
			BCDMS 85			0	
		Al	HERMES 03			0	
			SLAC E49			0	
		Ca	SLAC E139	0	0	0	
			EMC 90			0	
			NMC 95	0	0	0	
			SLAC E139	0	0	0	
		Fe	FNAL-E665			0	
			SLAC E87			0	
			SLAC E139	0 (15)	0	0	
			SLAC E140			0	
		Cu	BCDMS 87			0	
		Kr	EMC 93	0	0		
		Ag	HERMES 03			0	
		Sn	SLAC E139	0	0	0	
		Au	EMC 88			0	
			SLAC E139	0	0	0	
		A/C	SLAC E140			0	
Pb	FNAL-E665			0			
Be	NMC 96		0	0	0		
Al	NMC 96		0	0	0		
Ca	NMC 95				0		
	NMC 96		0	0	0		
Fe	NMC 96		0	0	0		
Sn	NMC 96	0 (10)	0	0			
Pb	NMC 96	0	0	0			
A/Li	C	NMC 95	0	0			
	Ca	NMC 95	0	0			
DY	A/D	C	FNAL-E772	0	0	0	
		Ca		0 (15)	0	0	
		Fe		0 (15)	0	0	
		W		0 (10)	0	0	
A/Be	Fe	FNAL E866	0	0			
	W		0	0			
π pro	dA/pp	Au	RHIC-PHENIX	0 (20)			

RESULTS: DECUT3 FIT

- 708 (1233) data points after (before) cuts
- 32 free parameters; 675 d.o.f.
- Overall $\chi^2/\text{d.o.f.} = 0.95$
- individually:
 - for F_2^A/F_2^D : $\chi^2/\text{pt} = 0.92$
 - for $F_2^A/F_2^{A'}$: $\chi^2/\text{pt} = 0.69$
 - for DY: $\chi^2/\text{pt} = 1.08$

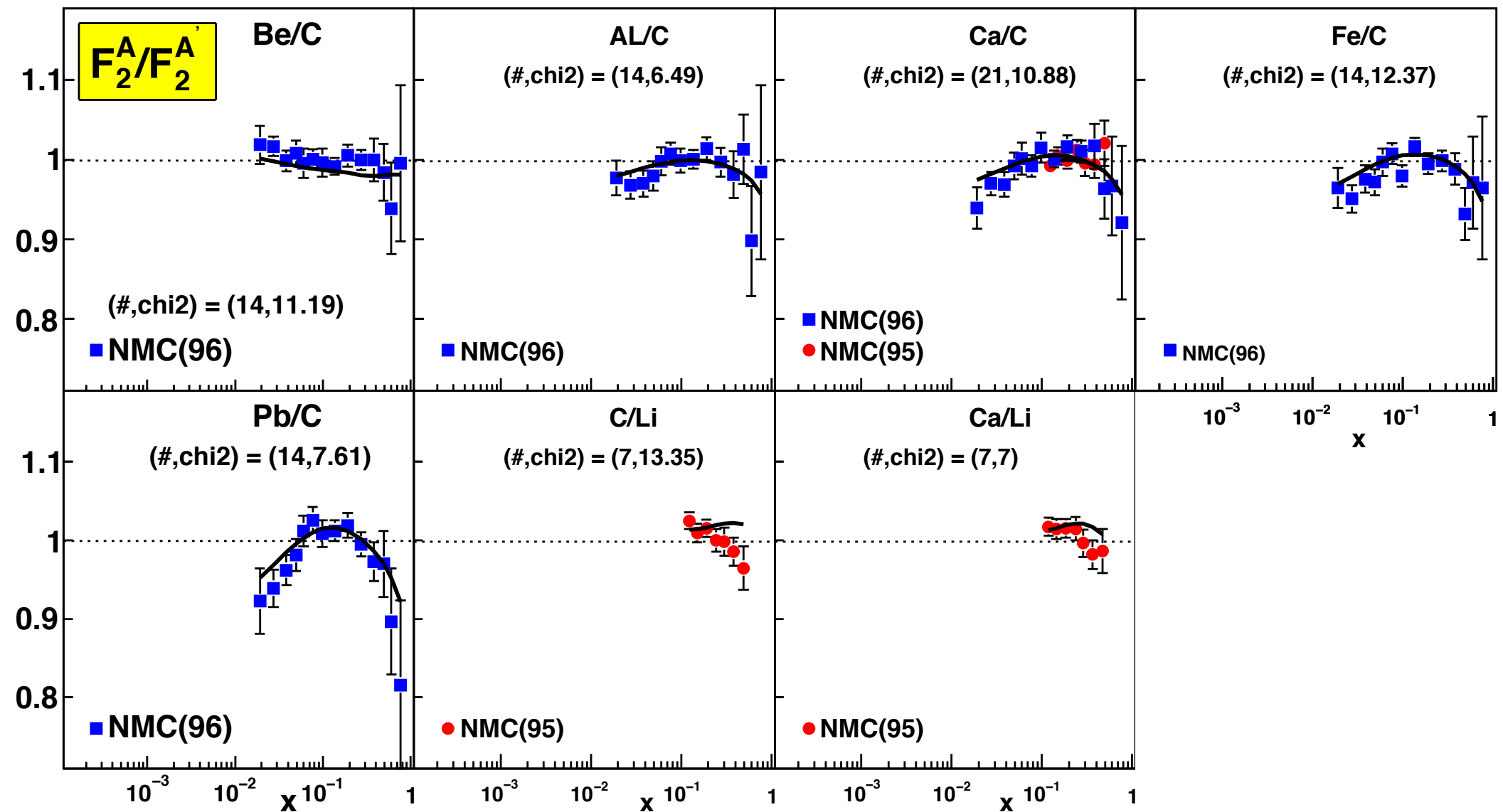
RESULTS: DECUT3 FIT

DIS DATA VS x



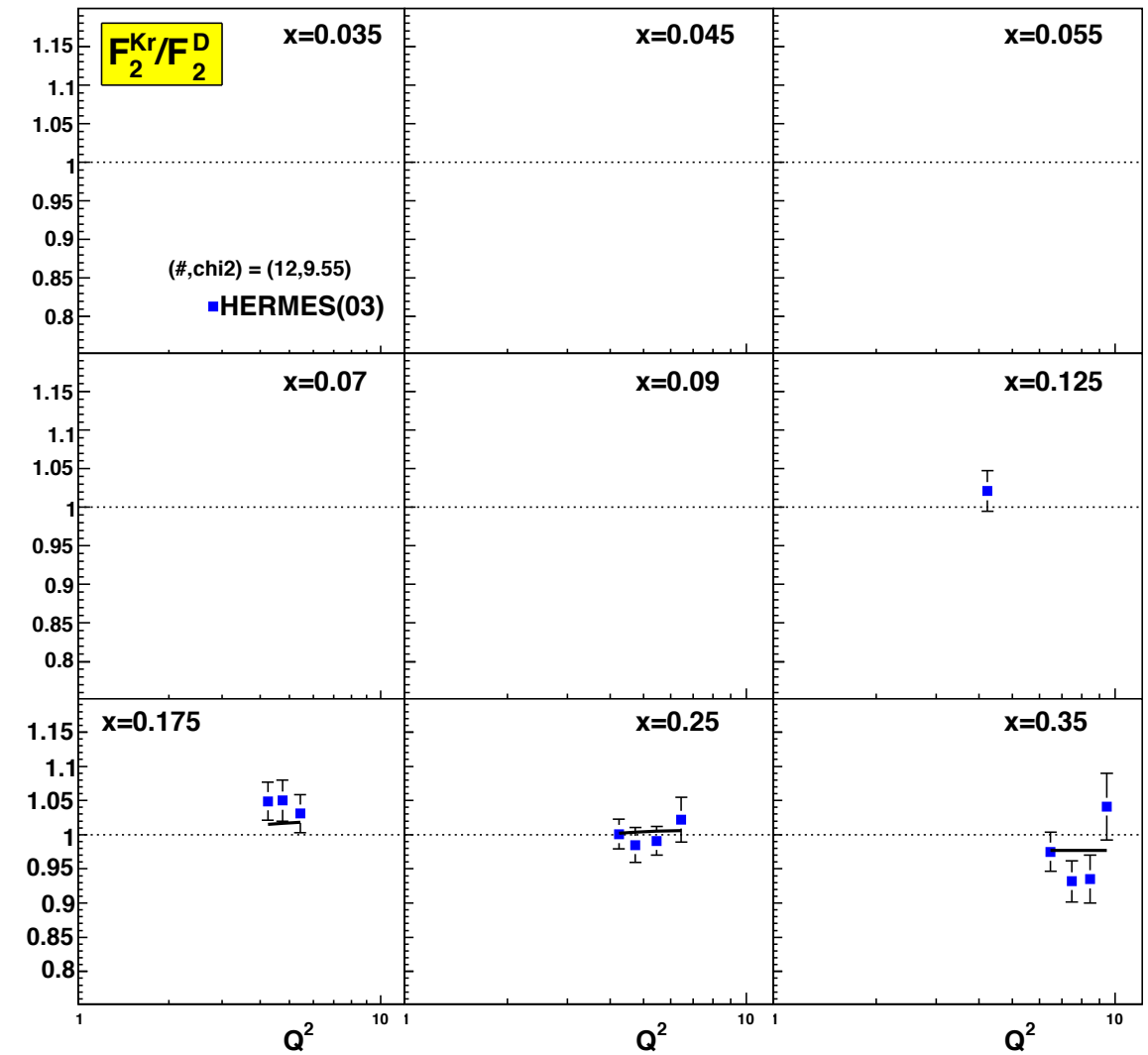
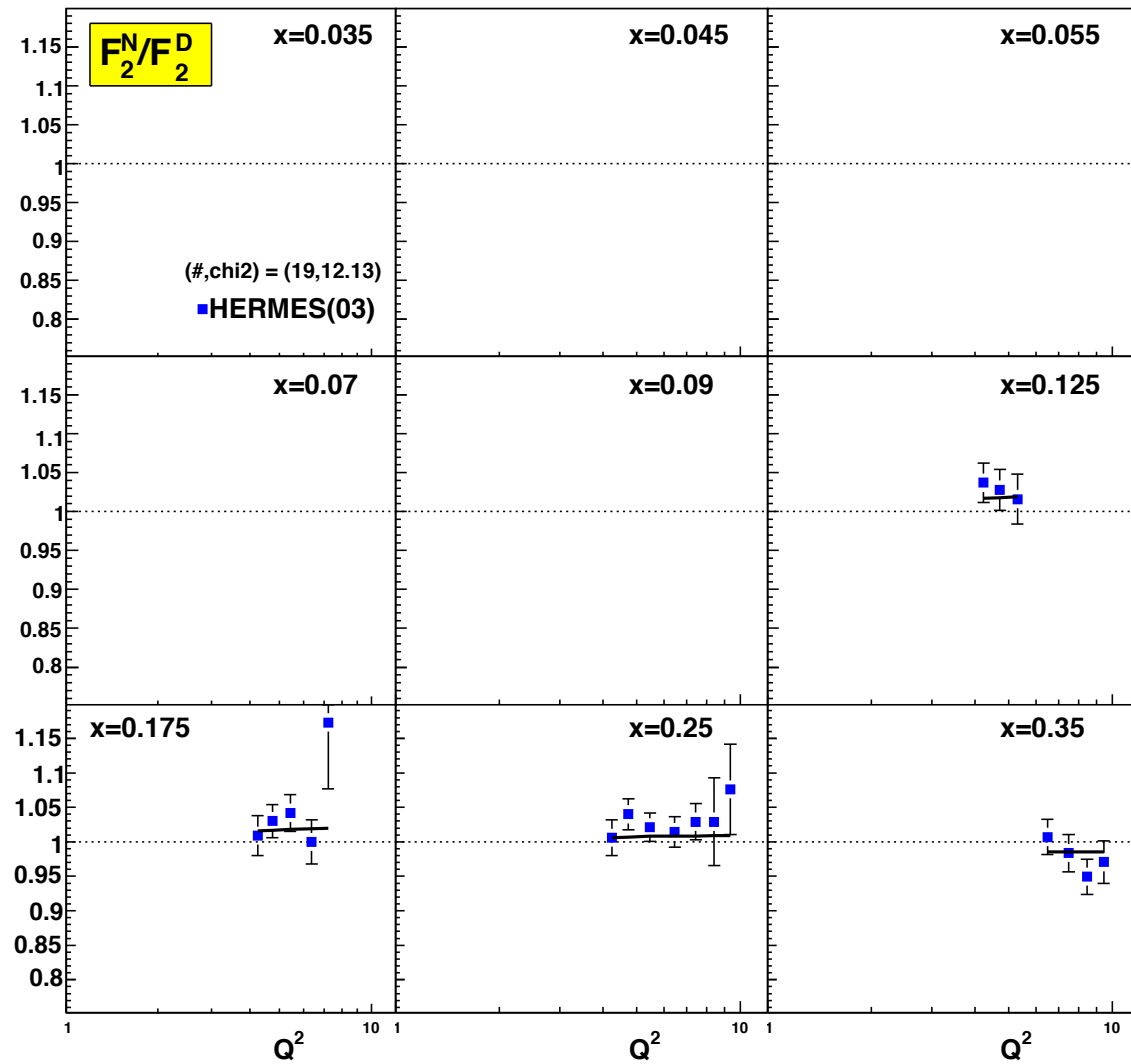
RESULTS: DEECUT3 FIT

DIS DATA VS X



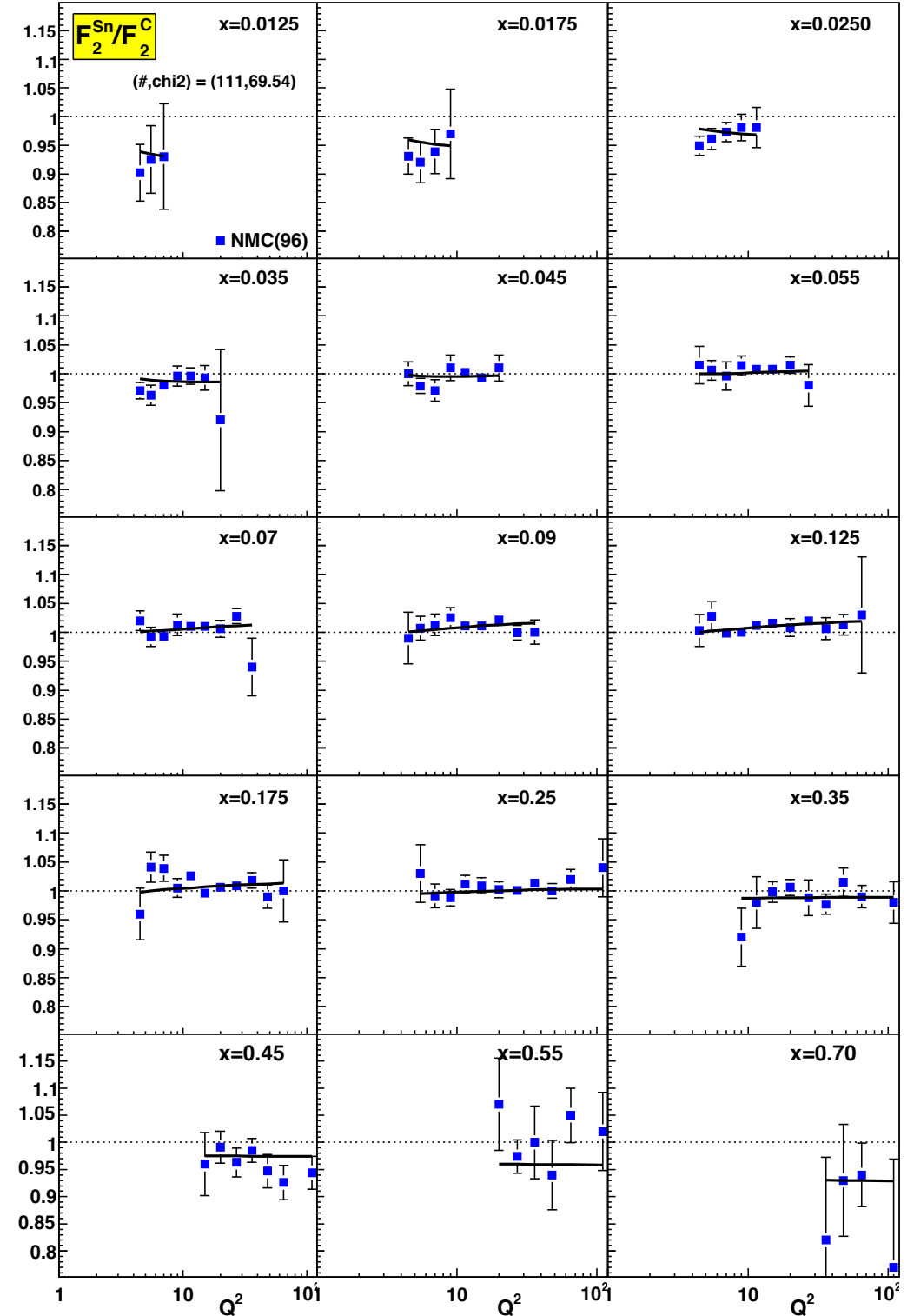
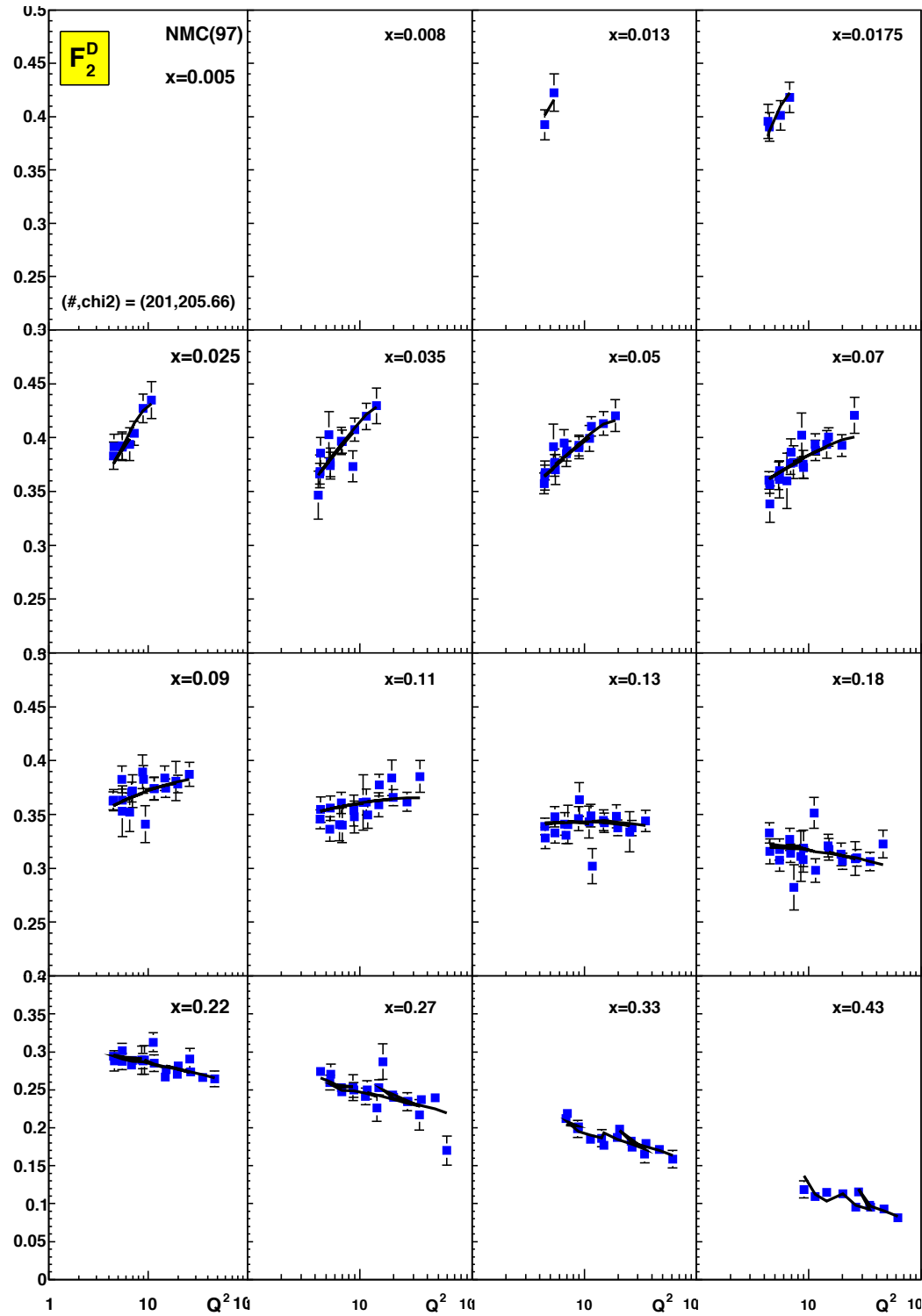
RESULTS: DECUT3 FIT

HERMES DATA VS Q^2



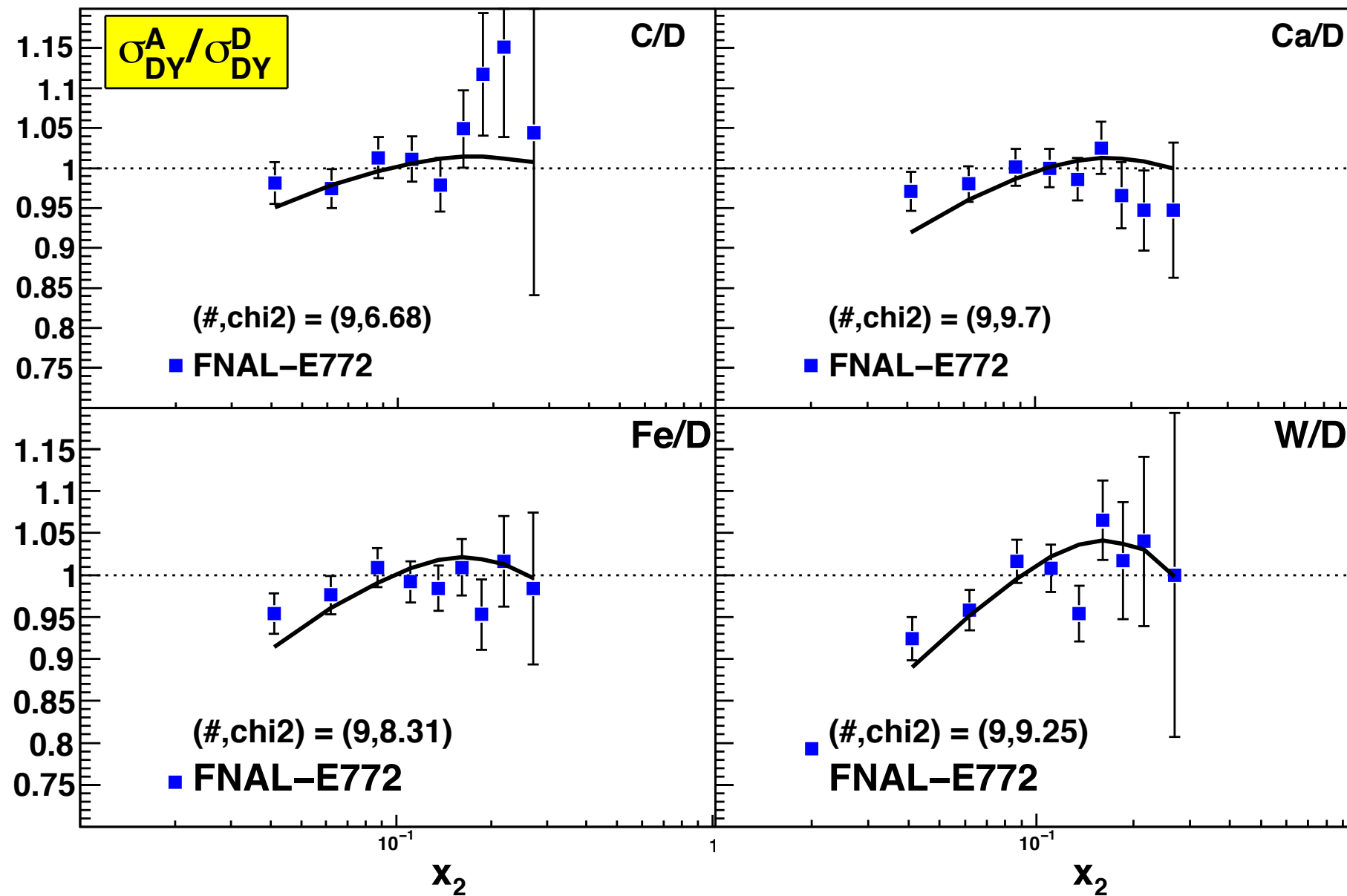
RESULTS: DECUT3 FIT

NMC DATA FOR D AND S_n/C vs Q^2



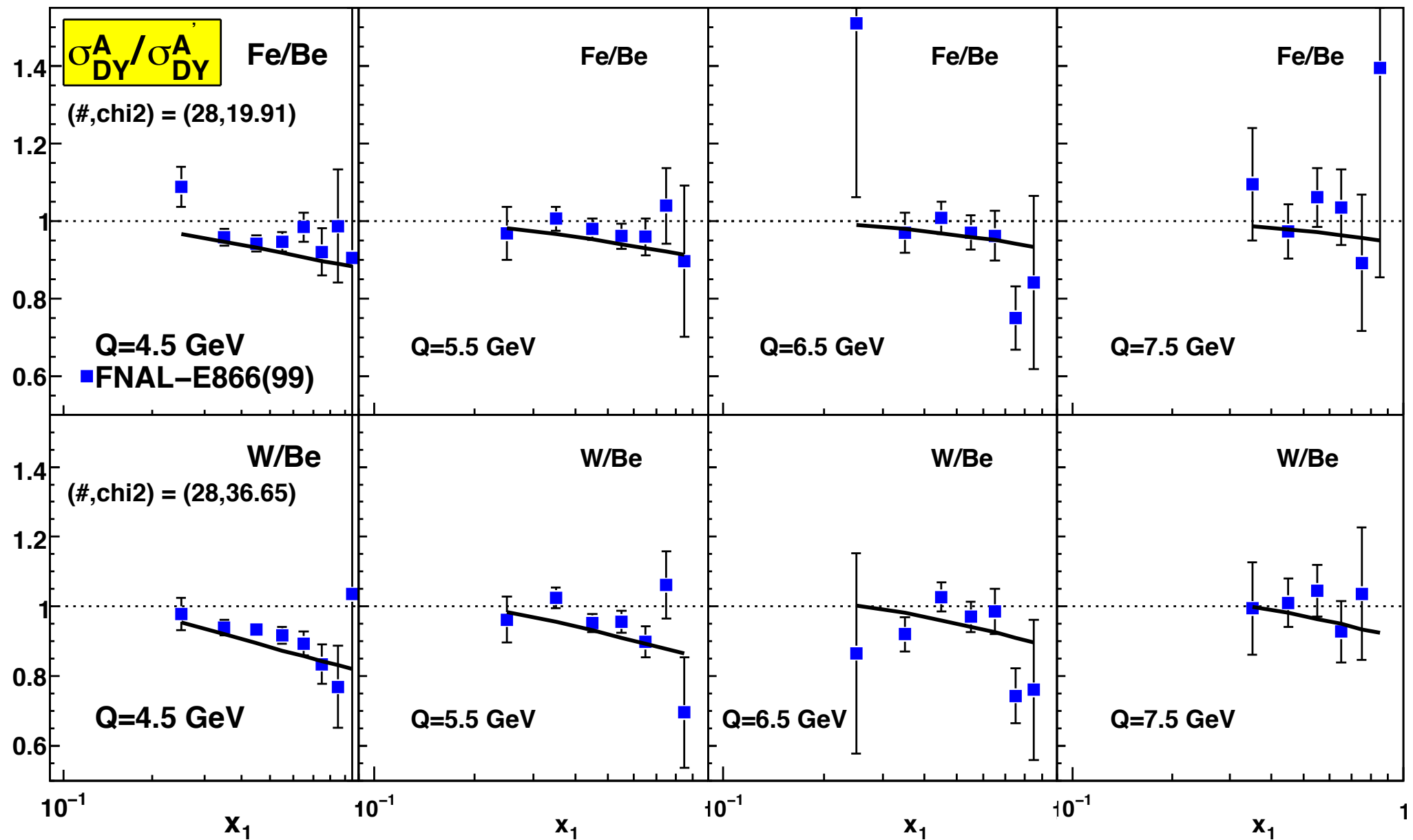
RESULTS: DEECUT3 FIT

DRELL-YAN DATA

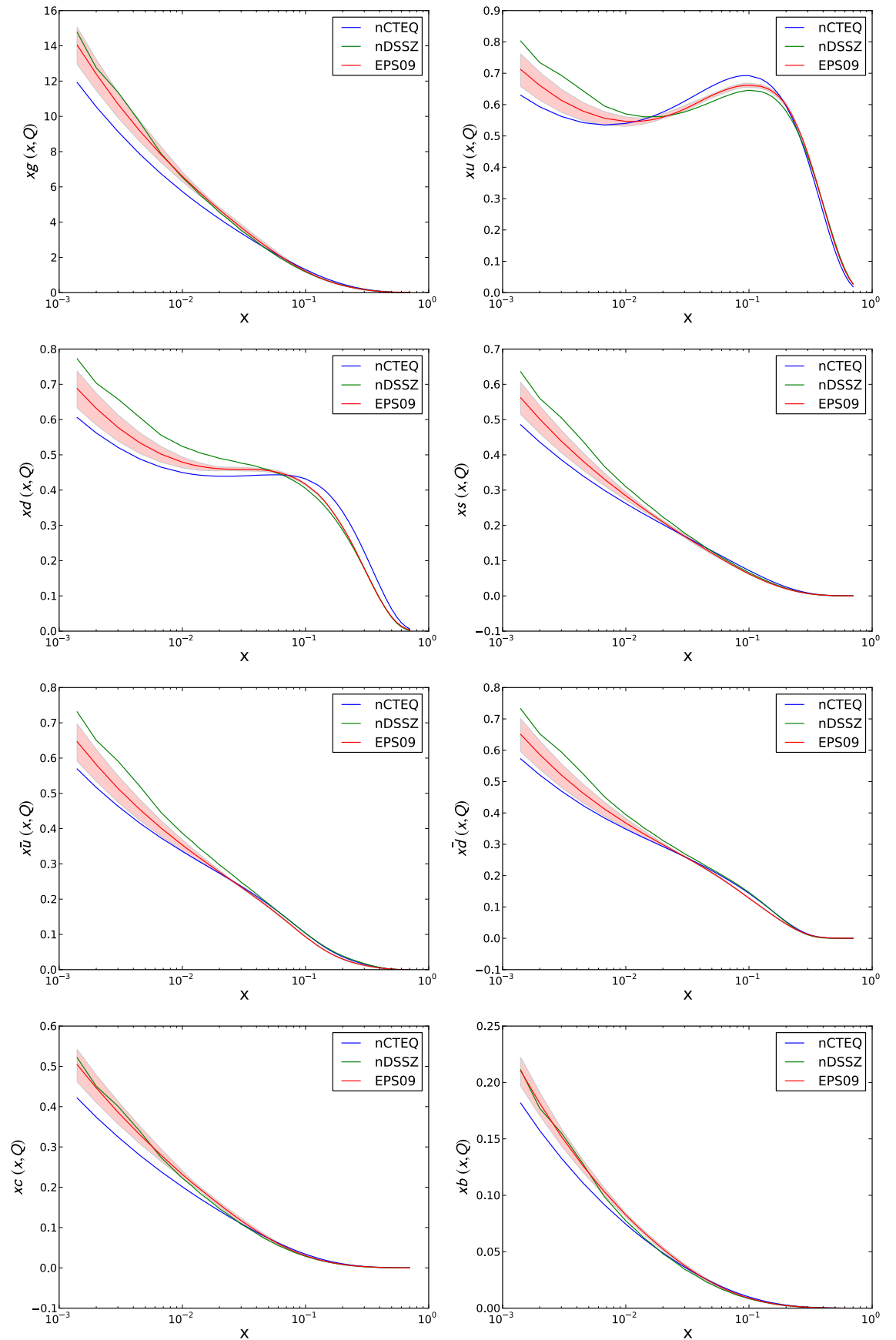
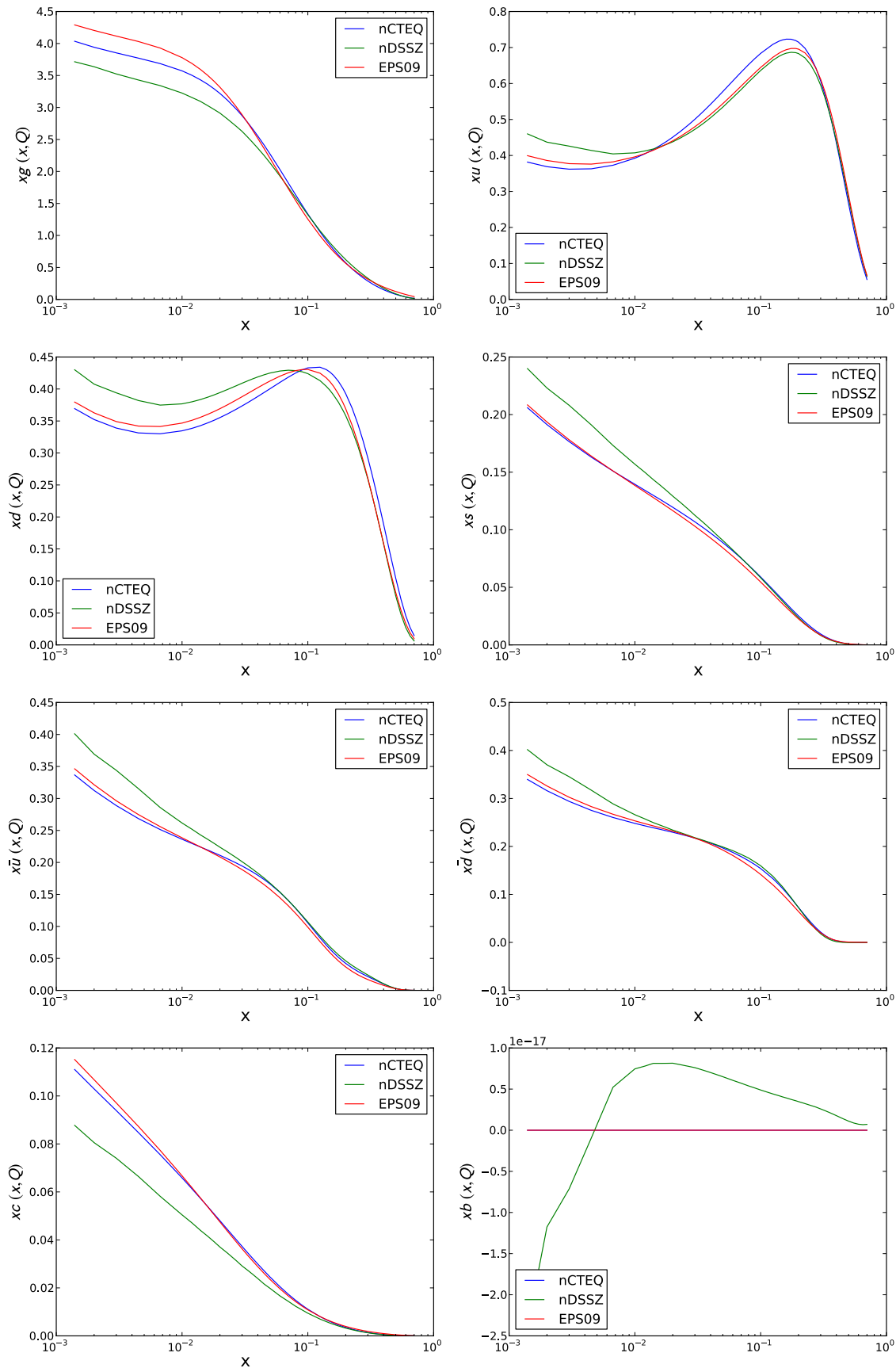


RESULTS: DECUT3 FIT

DRELL-YAN DATA



Talk by K. Kovarik at DIS12



Conclusion I

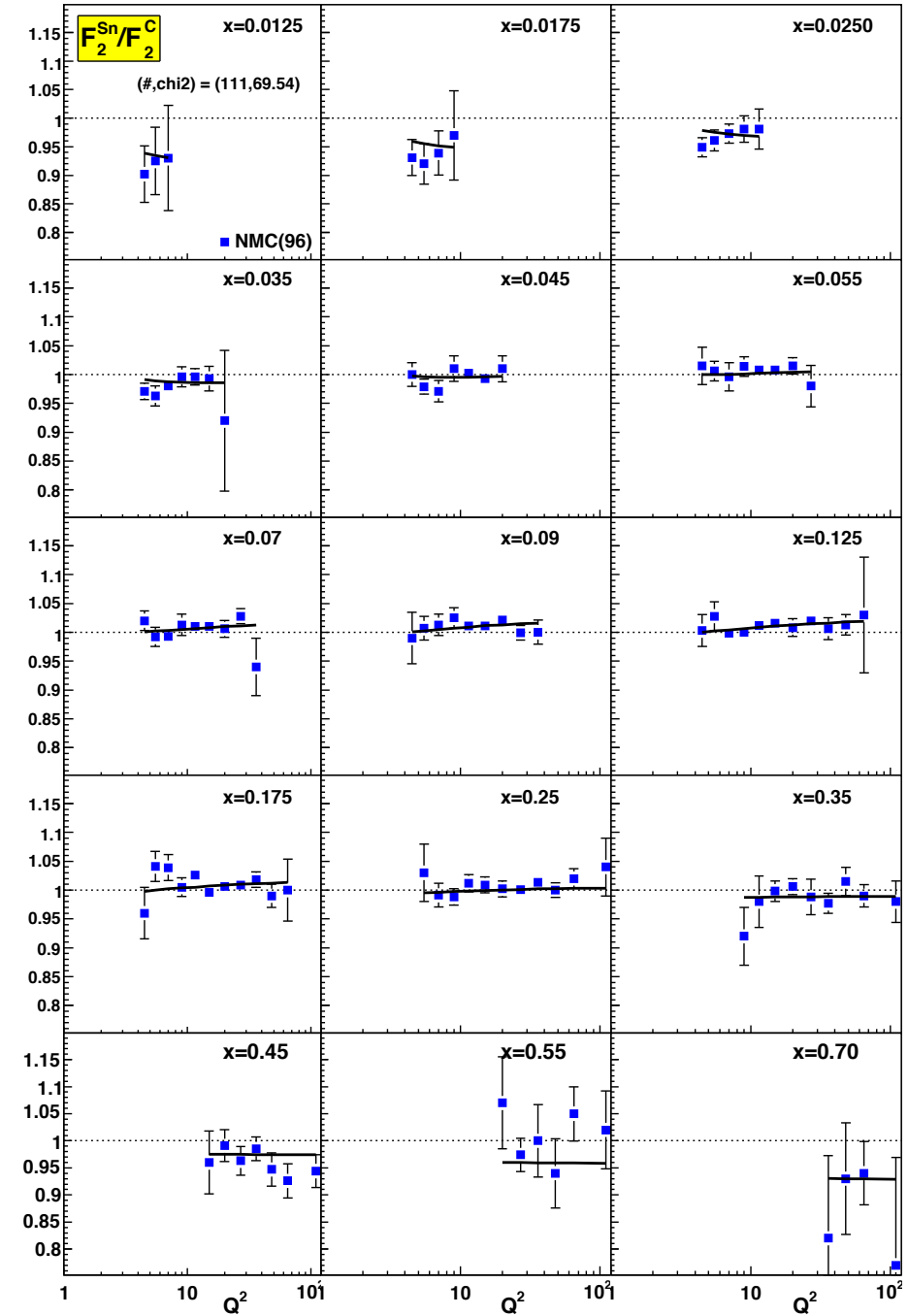
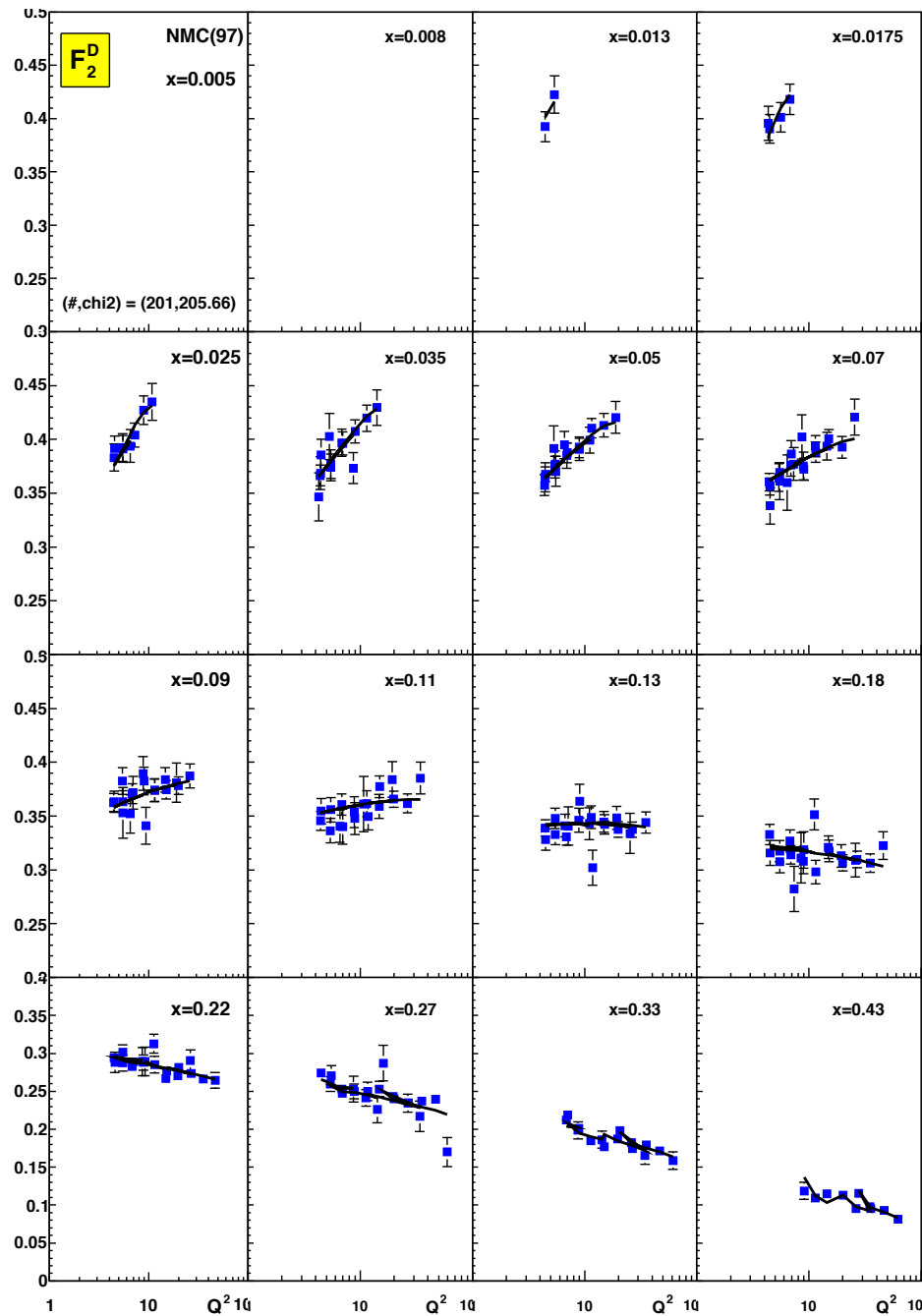
Excellent agreement between NLO pQCD and the IA DIS and DY data in the kinematical range $0.02 < x < 1$, $m_c^2 < Q^2 < 150 \text{ GeV}^2$ is found.

Factorization theorem in hard nuclear processes seems to work well.

nCTEQ PDFs available upon request

The nuclear gluon distribution

$g^A(x, Q^2)$ WEAKLY CONSTRAINED BY Q^2 -DEPENDENCE OF NMC DATA



- $x \sim 0.01 \dots 0.4, Q^2 \sim 10 \dots 100 \text{ GeV}^2$

gluon nCTEQ decut3gx fits

$$c_1 = c_{1,0} + c_{1,1}(1 - A^{-c_{1,2}})$$

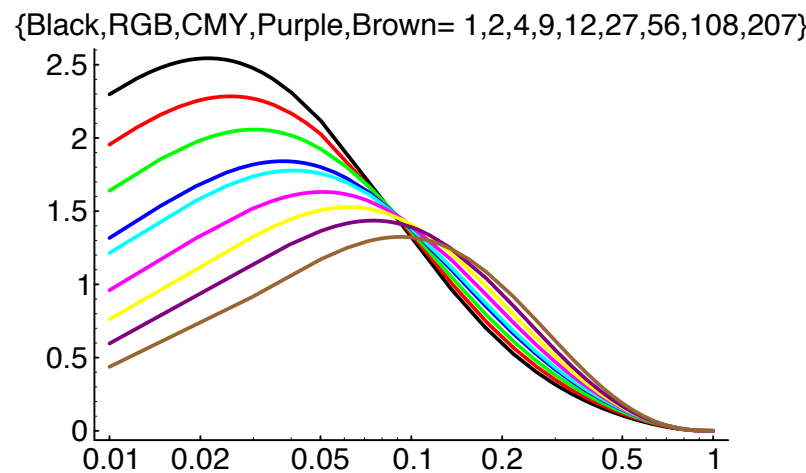
Name	(initial) fit parameter	$c_{1,1}$	$c_{1,2}$
decut3	free	-0.29	-0.09
decut3g1	fixed	0.2	50.0
decut3g2	fixed	-0.1	-0.15
decut3g3	fixed	0.2	-0.15
decut3g4	free	0.2	-0.15
decut3g5	fixed	0.2	-0.25
decut3g7	fixed	0.2	-0.23
decut3g8	fixed	0.35	-0.15
decut3g9	fixed – free proton	0.0	—

- Vary c_1 influencing small-x behaviour of gluon nPDF
- **Each fit equally acceptable** with excellent $\chi^2/\text{dof} \sim 0.9$

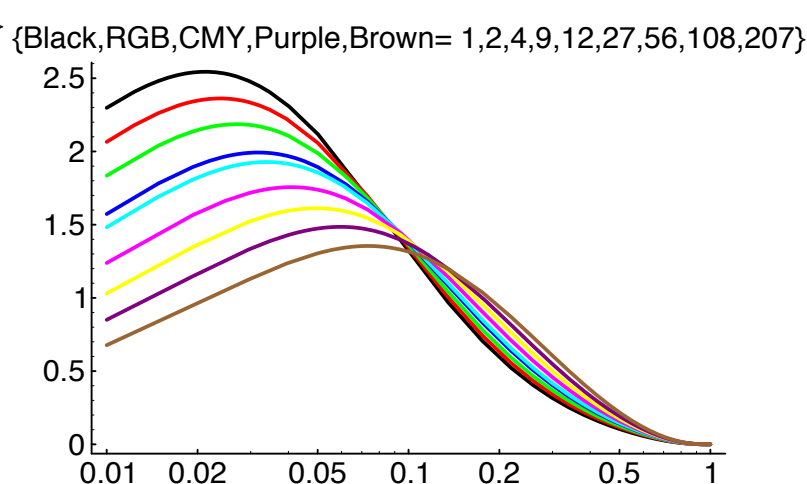
THE NUCLEAR GLUON DISTRIBUTION

A series of **equally good fits** ($\chi^2/pt \simeq 0.9$) to $\ell A+DY$ data with different gluons

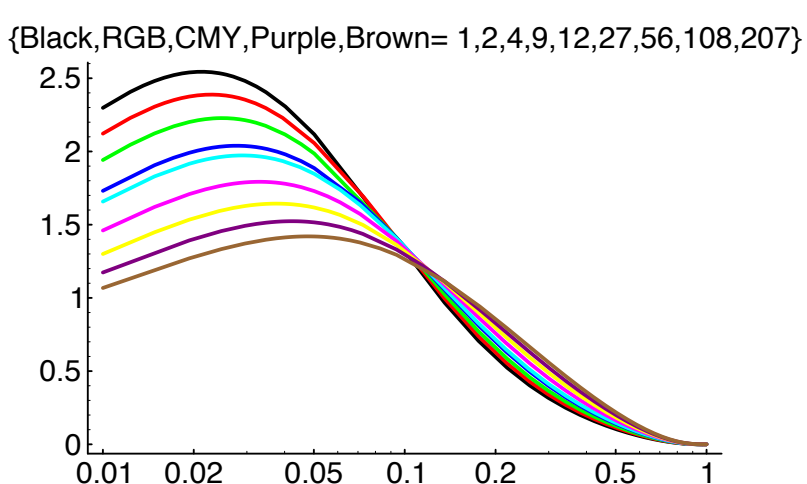
decut3



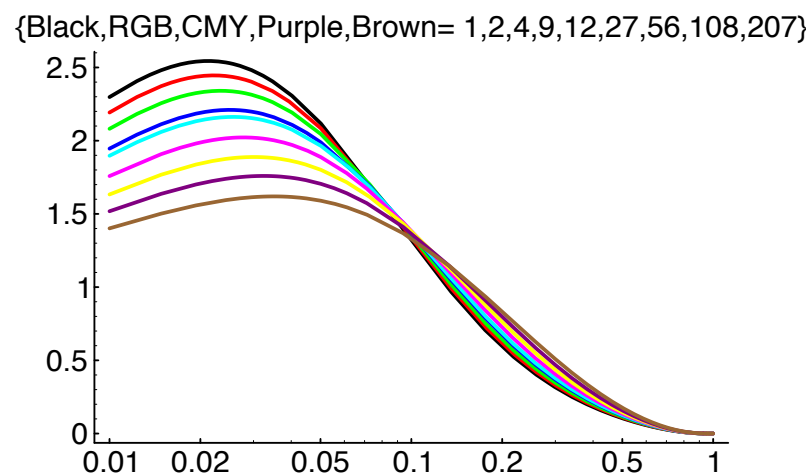
decut3g2



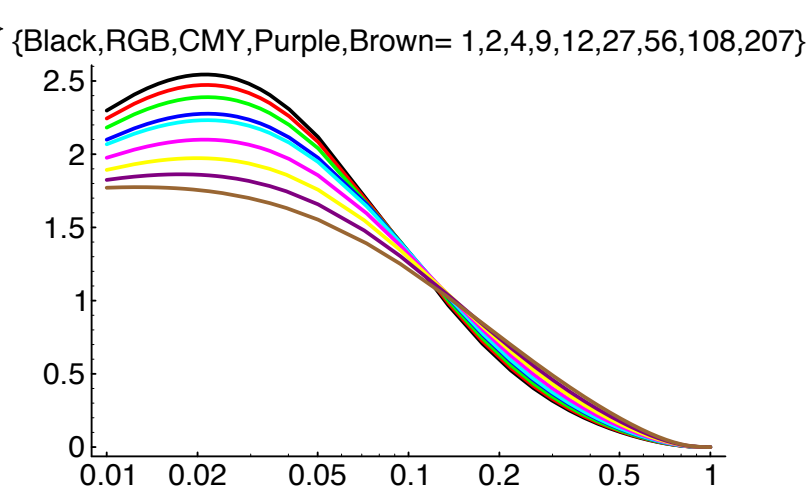
decut3g4



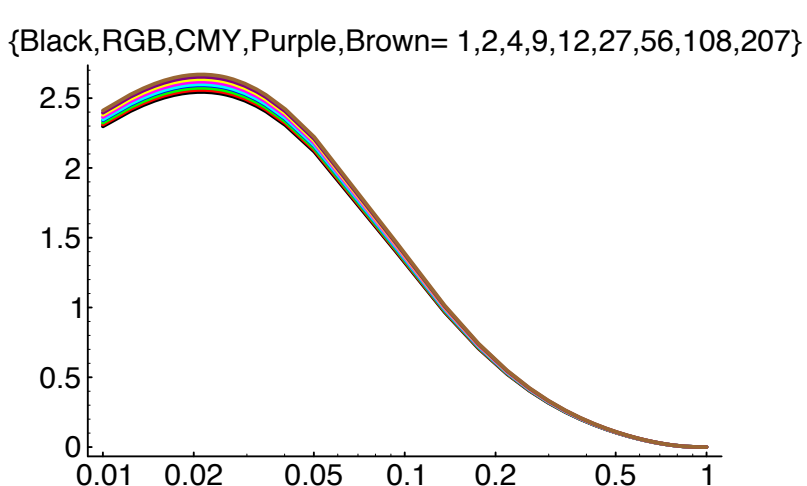
decut3g3



decut3g8

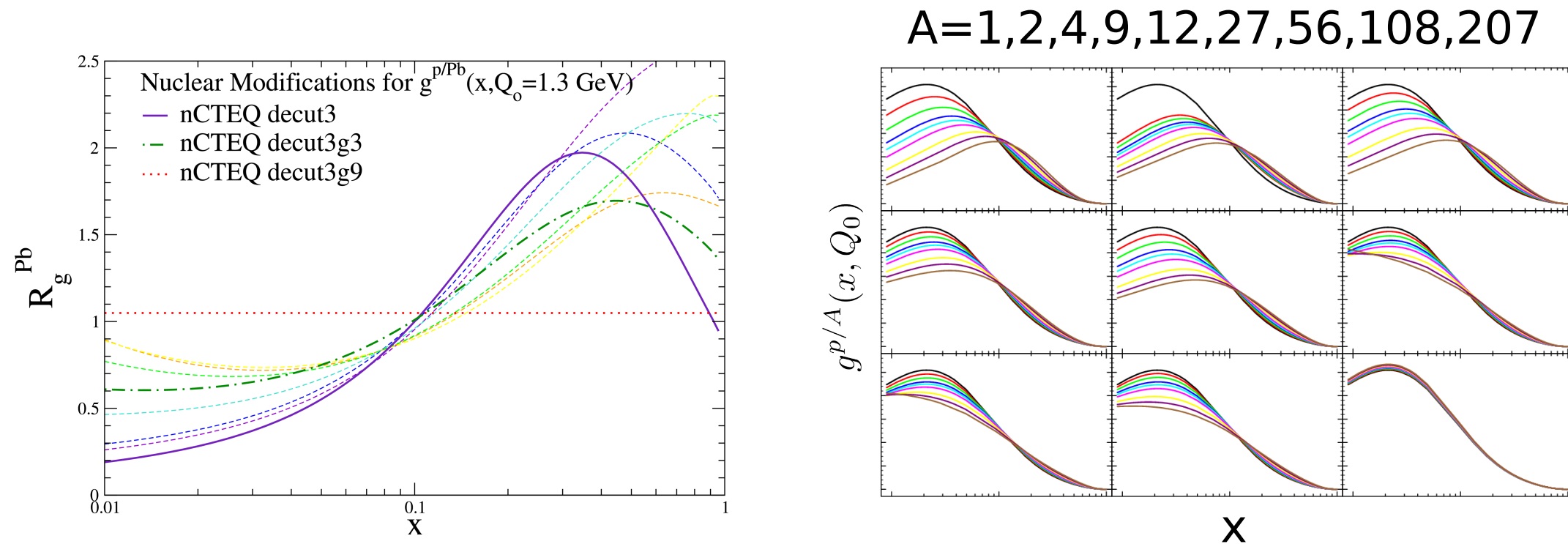


decut3g9



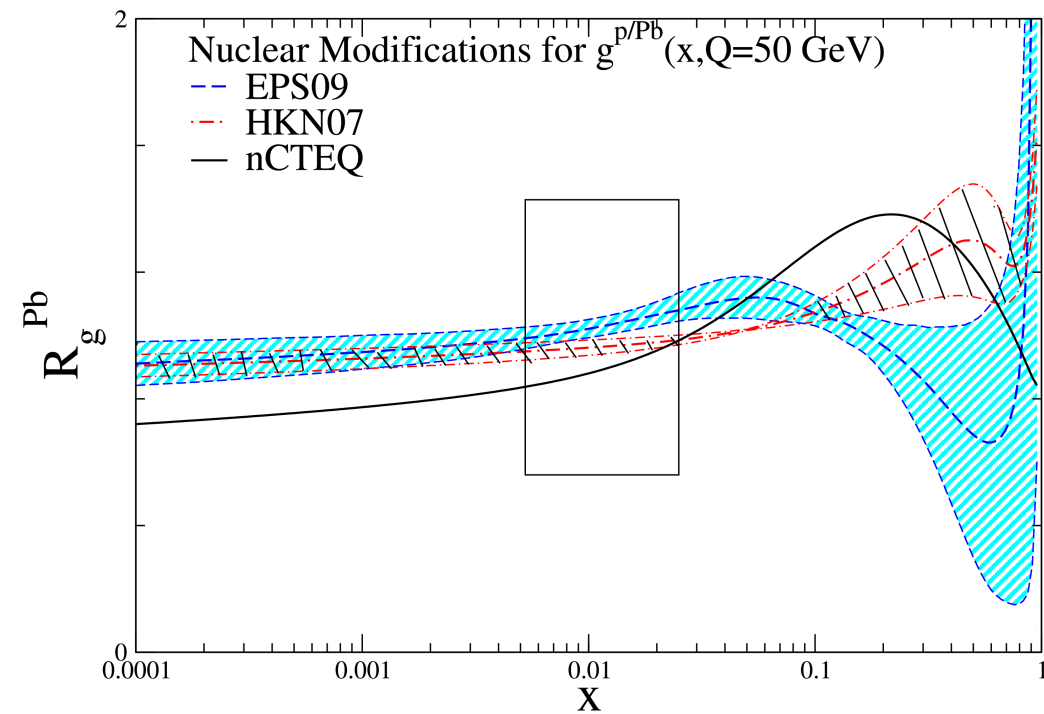
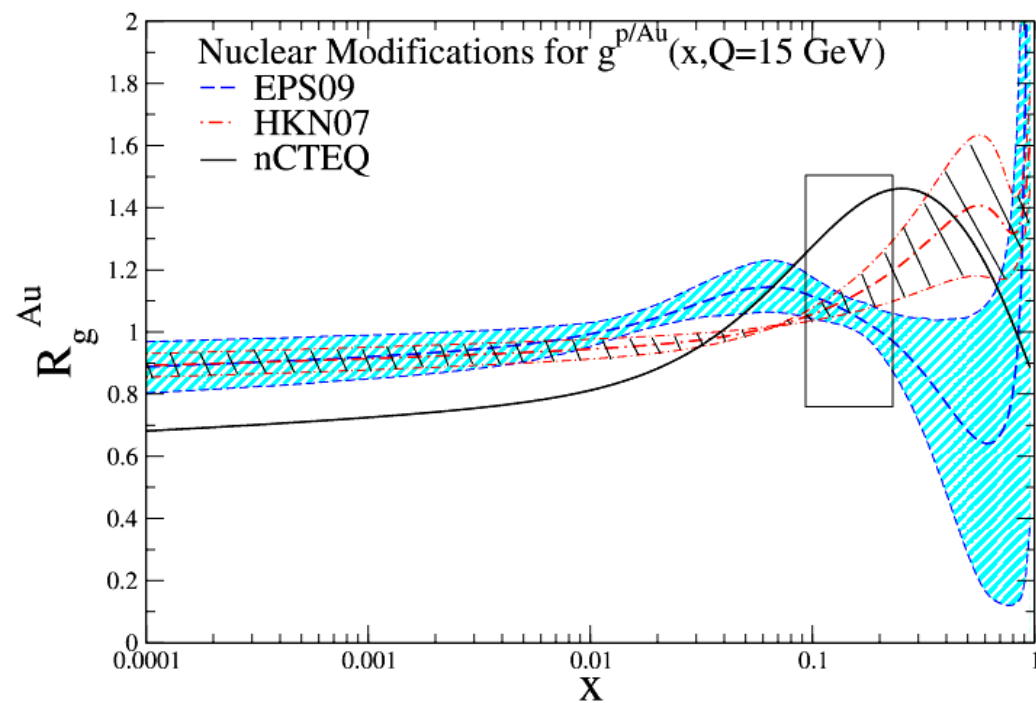
Shown are the gluon distributions at the scale $Q_0 = 1.3 \text{ GeV}$ for different A vs x

gluon nCTEQ decut3gx fits



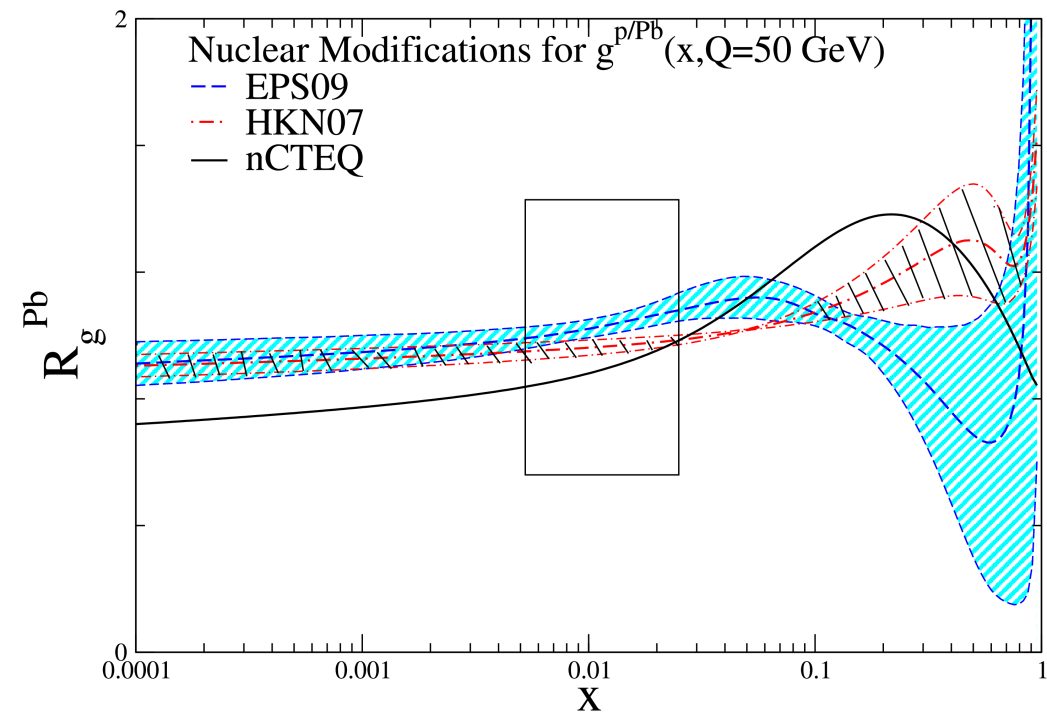
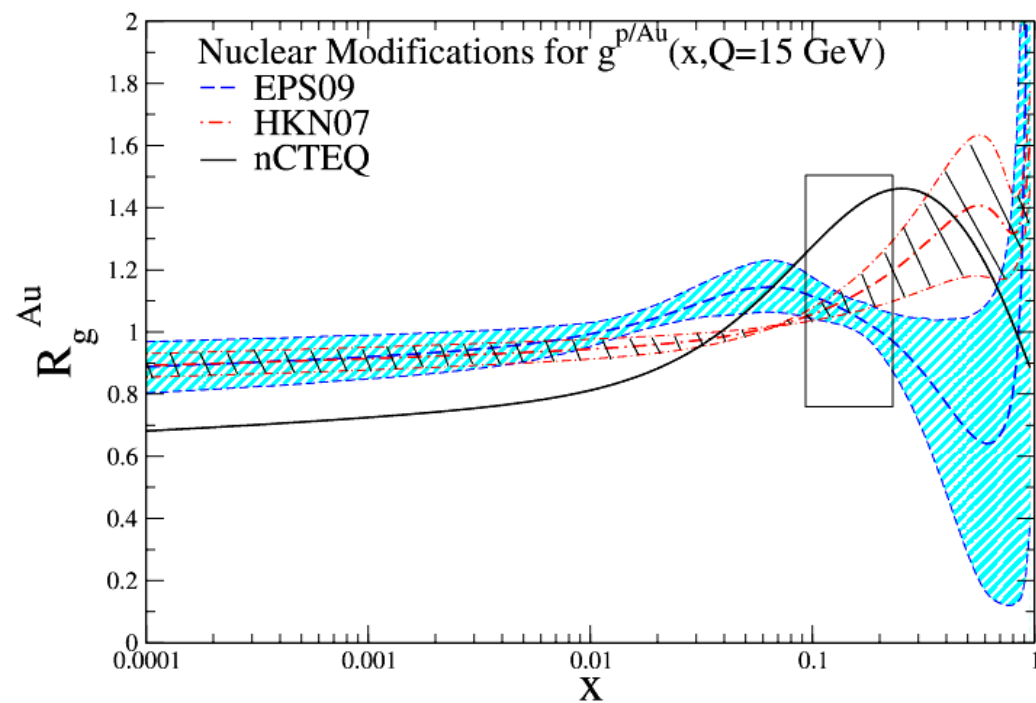
- This still **underestimates** the true uncertainty
- Some **curves lie outside the error bands** of EPS'09 and/or HKN'07!

At higher scales



- At larger Q error still large
- nPDFs quite different – individual error bands underestimate uncertainty
- Need more experimental constraints!

At higher scales

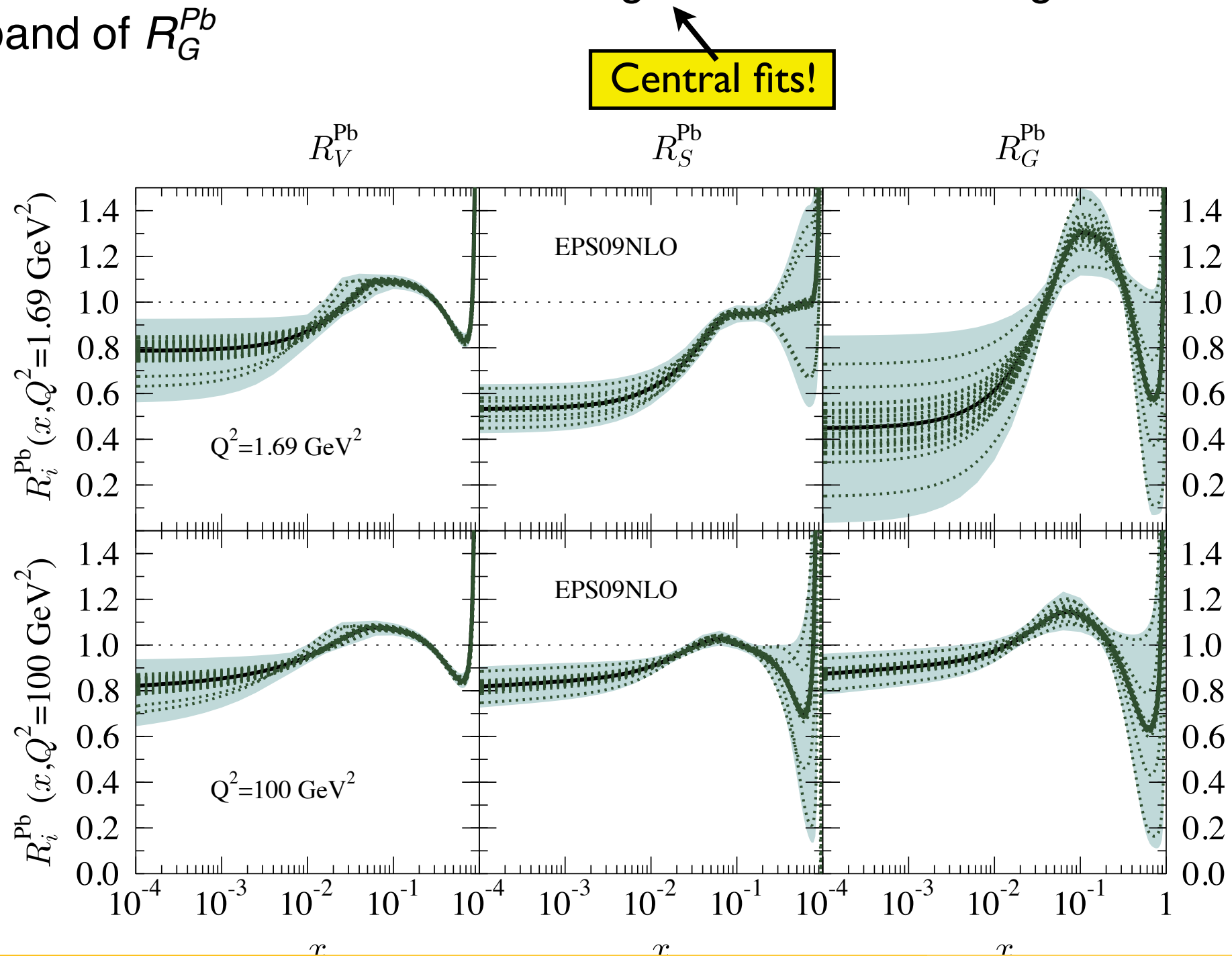


- At larger Q error still large
- nPDFs quite different – individual error bands underestimate uncertainty
- Need more experimental constraints!

Essential for ion-ion physics at LHC

GLUON UNCERTAINTY IN EPS'09

- EPS'09 also uses RHIC data for inclusive pion production to constrain the gluon
- This involves fragmentation functions $D_i^\pi(z, \mu^2)$ into pions
- Large uncertainties! Still some of the gluons of the decut3g series lie outside the error band of R_G^{Pb}



The gluon from hard processes at the LHC and RHIC

NEED HARD PROBES IN pA TO CONSTRAIN NPDFS

Hard probes in pp , $p\bar{p}$ to constrain proton PDFs:

- Tevatron inclusive jet data \rightarrow gluon
- Lepton pair production \rightarrow sea quarks
- Vector boson production \rightarrow sea quarks

(need high precision due to high scale $Q \sim M_W$)

The gluon from hard processes

- Inclusive jet production
- Inclusive hadron production
- Heavy quark production
- Heavy quarkonium production?
- Direct photon production
- Direct photon + jet
- **Direct photon + heavy quark jet**

Tevatron inclusive jet data
used in proton case

Cacciari et al,
Kniehl, Kramer, IS, Spiesberger

Arleo, d'Enterria

Stavreva et al

Photon + Q production

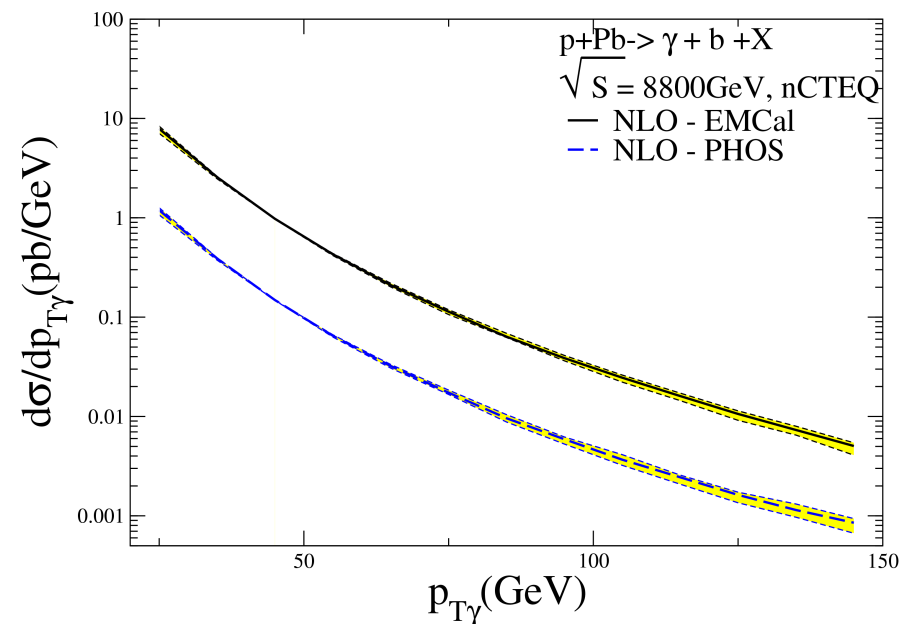
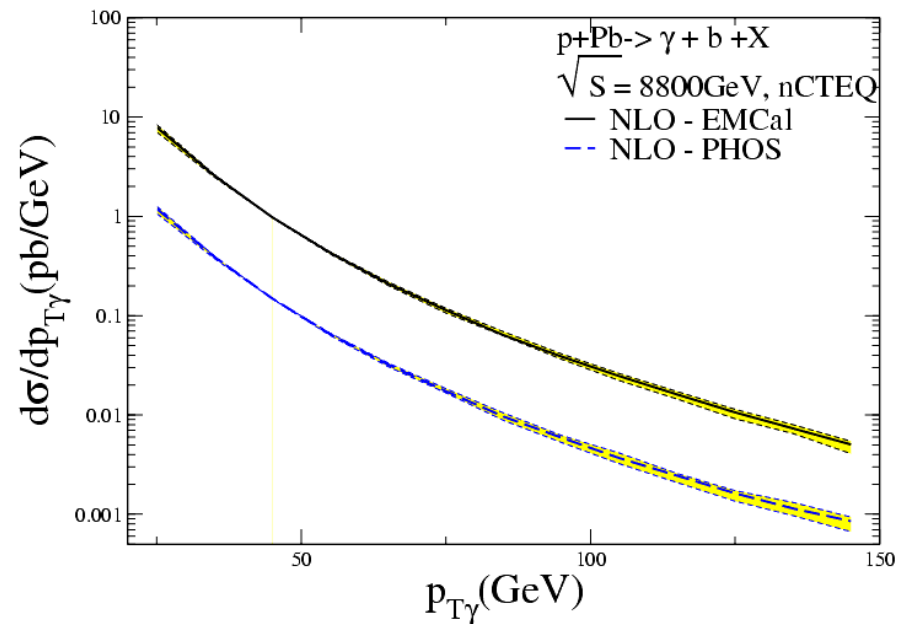
- Dominated by **Compton subprocess**:

$$g + Q \rightarrow \gamma + Q$$

- Heavy quark PDF depends entirely on gluon PDF (disregarding possible intrinsic charm)
- Use **nuclear correction ratio** to determine gluon:

$$R_{pA}^{\gamma Q} = \frac{\sigma(pA \rightarrow \gamma Q X)}{A\sigma(pp \rightarrow \gamma Q X)}$$

Photon + Q at the LHC

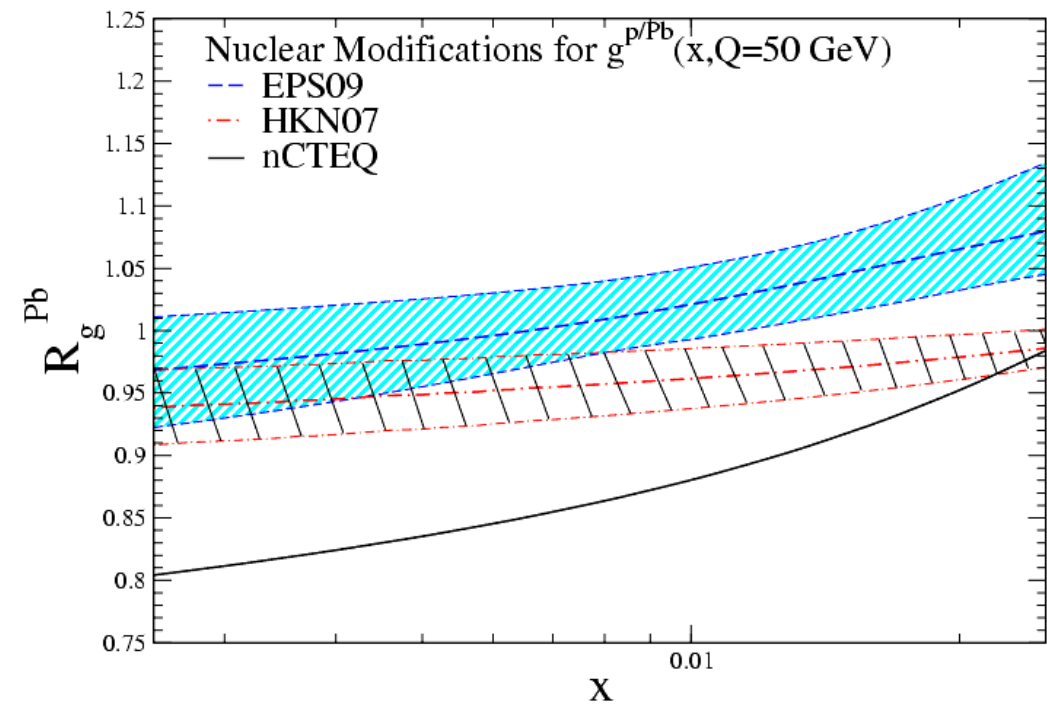
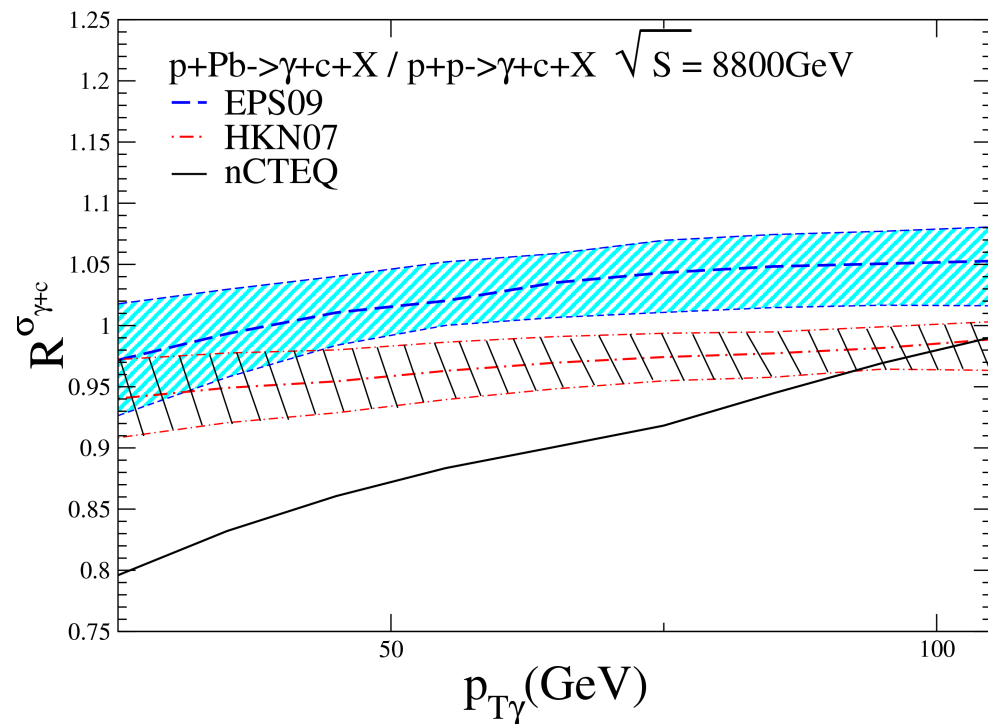


	σ^{tot}	N_{event}
$\gamma + c$ PHOS	131 pb	2700
$\gamma + b$ PHOS	20 pb	400
$\gamma + c$ EMCAL	684 pb	14200
$\gamma + b$ EMCAL	131 pb	2700

ALICE

- σ sufficiently large to measure $\gamma + c$ and $\gamma + b$

Constraining the gluon nPDF



- The nuclear ratio follows closely the gluon ratio
- Measurement with sufficiently small errors will allow to constrain the gluon distribution
- Similar at RHIC, but higher x probed

Conclusions II

- Nuclear gluon poorly known!
- Problem for heavy ion physics Disentangle:
 - initial state
 - cold nuclear matter effects
 - hot nuclear matter effects (QGP)
- Photon + Q useful to constrain gluon
- decut3gx series of nCTEQ fits available

Nuclear corrections in neutrino DIS

Why neutrino DIS?

- **Data interesting for global analyses of proton PDF and nuclear PDF (nPDF)**
 - **Flavor separation:**
Neutrino structure functions depend on different combinations of PDFs
- **Dimuon production:**
 - Main source of information on the strange sea
 - Large uncertainty on $s(x, Q^2)$ has significant influence on the **W and Z benchmark processes at LHC**
- **For proton PDF: need nuclear corrections!**

Why neutrino DIS?

- **LBL precision neutrino experiments:**

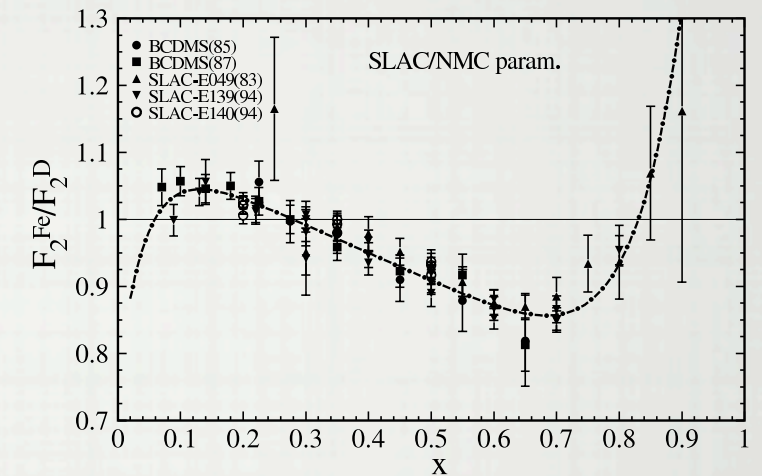
Need good understanding of ν -A cross sections
(A=Oxygen, Carbon)

- **EW precision measurements:**

Paschos-Wolfenstein analysis: extraction of $\sin^2 \theta_w$

NUCLEAR PDFS

- What are nuclear parton density functions (nPDF) ?
 - parton densities for partons in bound proton & neutron

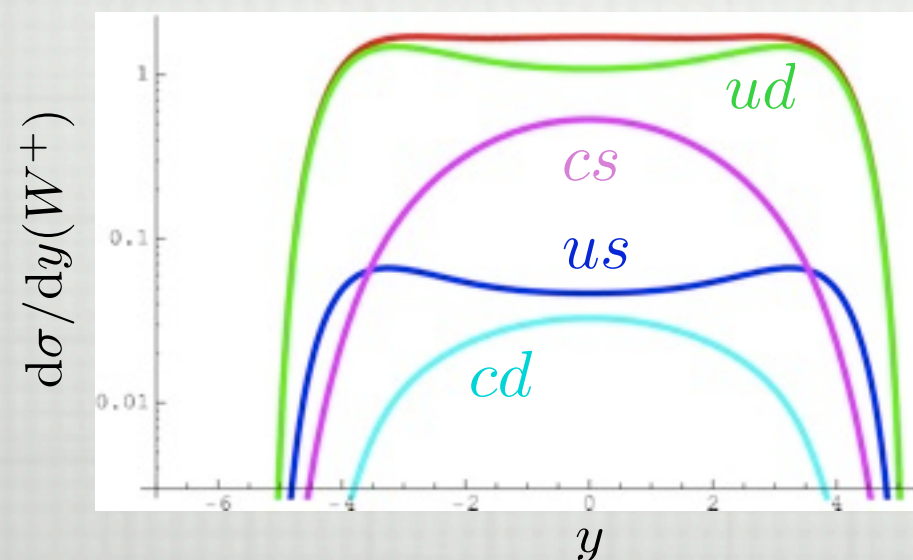


- Where are nuclear parton density functions useful ?

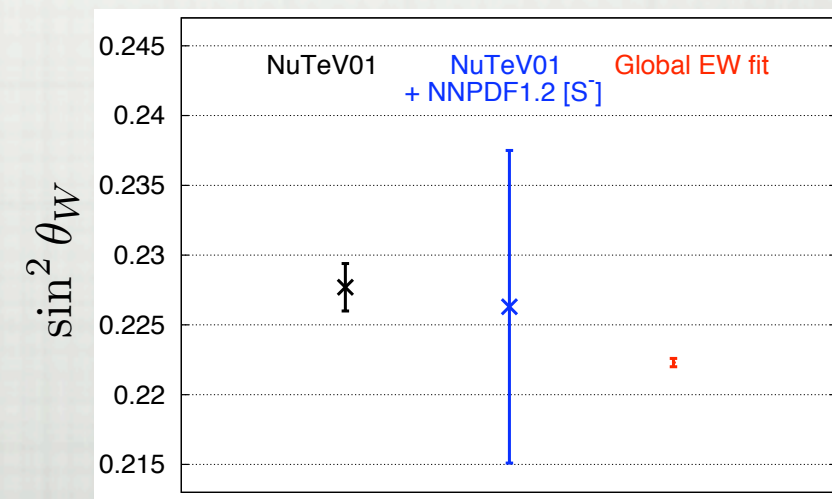
I. Strange quark content of the proton

(anti-)strange PDF from (anti-)neutrino DIS with heavy nuclei - nuclear effects important

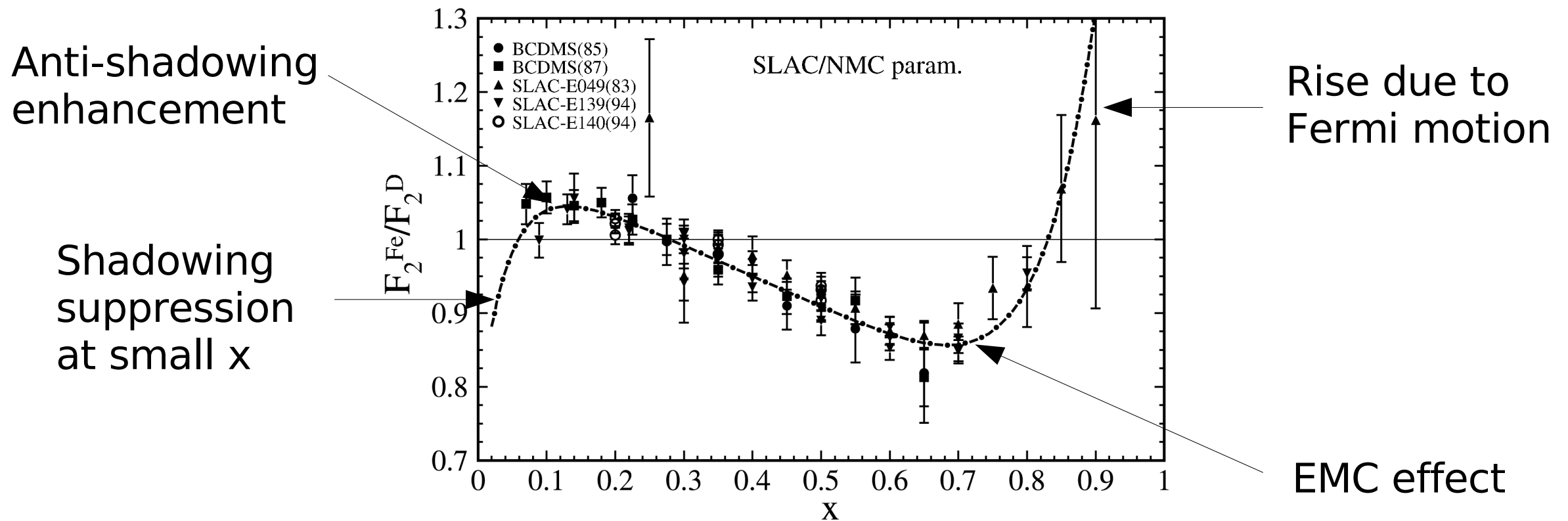
crucial for: *W*-boson production at the LHC
(standard candle process)



crucial for: determining weak mixing angle from
NuTeV experiment



Nuclear corrections: Historically



- Historically, nuclear corrections **from charged-lepton DIS data** are applied to neutrino DIS data
- Same correction for **all scales** Q^2
- Same correction for **all observables** (F_2 , F_3 , cross section, dimuon production)
- **Idea:** study nuclear corrections in the parton model (PM) using nuclear PDF

Nuclear correction factors in the PM

- Be O an **observable** calculable in the parton model

Define **nuclear correction factor**:

$$R[\mathcal{O}] = \frac{O[\text{nuc.PDF}]}{O[\text{freePDF}]}$$

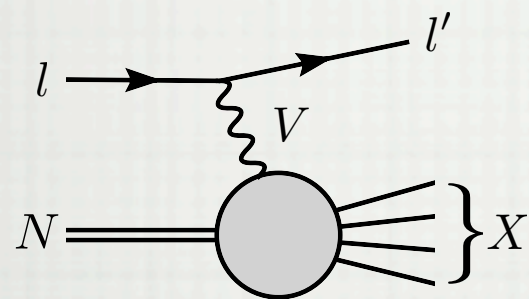
- Compare below: $R[F_2^{IA}]$ (IA DIS) with $R[F_2^{\nu A}]$ (νA DIS)
- Advantage:
 - very flexible (applicable to other observables: F_3 , $d\sigma$, ...)
 - scale dependent

NEUTRINO DIS

- Experiments included in the analysis:

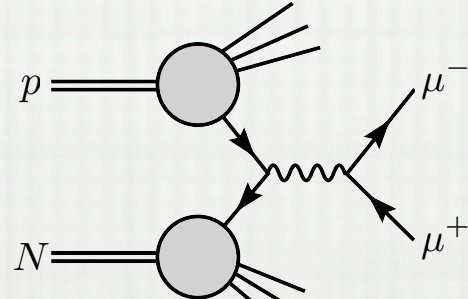
Charged lepton

Deep Inelastic Scattering



$$l + N \rightarrow l' + X$$

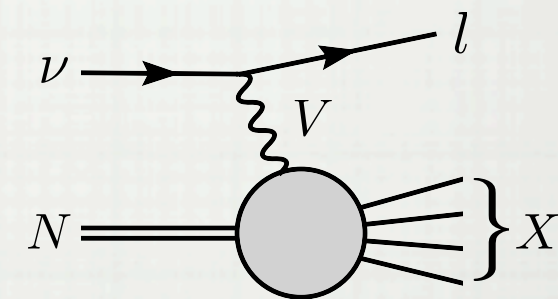
Drell-Yan process



$$p + N \rightarrow \mu^+ \mu^- + X$$

Neutrino

Deep Inelastic Scattering



$$\nu(\bar{\nu}) + N \rightarrow l + X$$

CERN BCDMS & EMC & NMC

$N = (\text{D, Al, Be, C, Ca, Cu, Fe, Li, Pb, Sn, W})$

FNAL E-665

$N = (\text{D, C, Ca, Pb, Xe})$

DESY Hermes

$N = (\text{D, He, N, Kr})$

SLAC E-139 & E-049

$N = (\text{D, Ag, Al, Au, Be, C, Ca, Fe, He})$

FNAL E-772 & E-886

$N = (\text{D, C, Ca, Fe, W})$

CHORUS

$N = \text{Pb}$

CCFR & NuTeV

$N = \text{Fe}$

1233 data points (708 after cuts)

3832 data points (3134 after cuts)

Neutrino data

- Correlated errors
- Radiative correct.
- with and w/o iso-scalar corrections

ID	$d\sigma^{\nu A}/dx dy :$ Observable	Experiment	# data
33	Pb	CHORUS ν	607 (412)
34	Pb	CHORUS $\bar{\nu}$	607 (412)
35	Fe	NuTeV ν	1423 (1170)
36	Fe	NuTeV $\bar{\nu}$	1195 (966)
37	Fe	CCFR ν di-muon	44 (44)
38	Fe	NuTeV ν di-muon	44 (44)
39	Fe	CCFR $\bar{\nu}$ di-muon	44 (44)
40	Fe	NuTeV $\bar{\nu}$ di-muon	42 (42)
	Total:		4006 (3134)

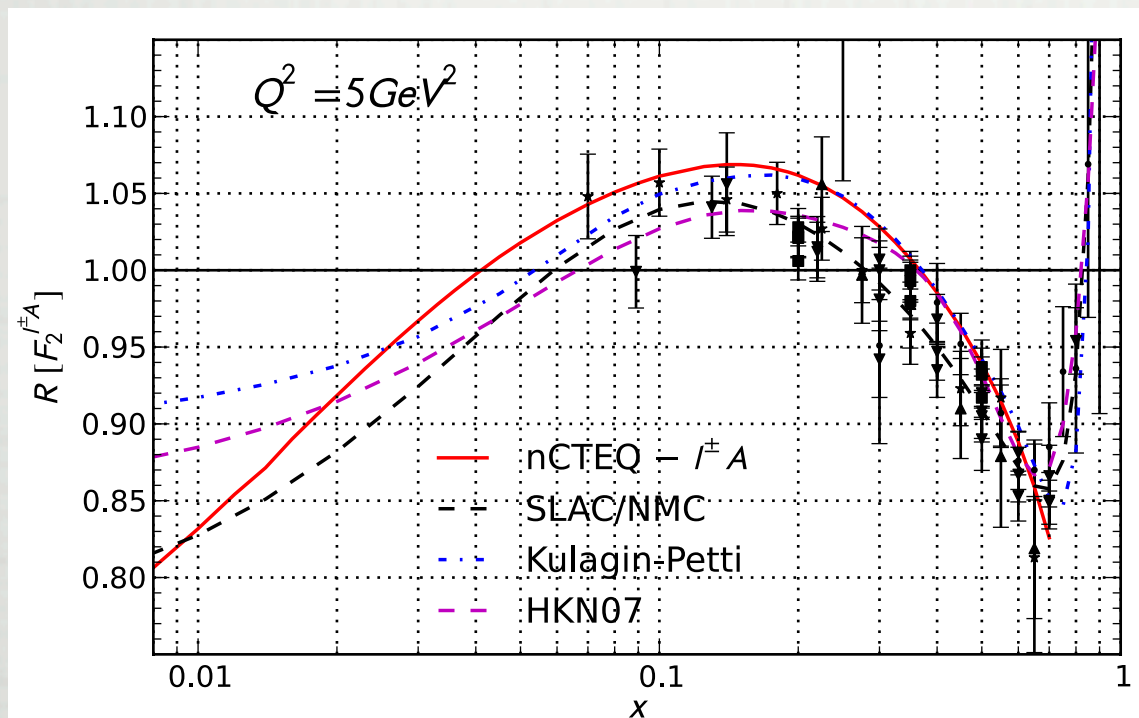
NEUTRINO DIS

- Comparison of charged lepton and neutrino fits

Fit to charged lepton data

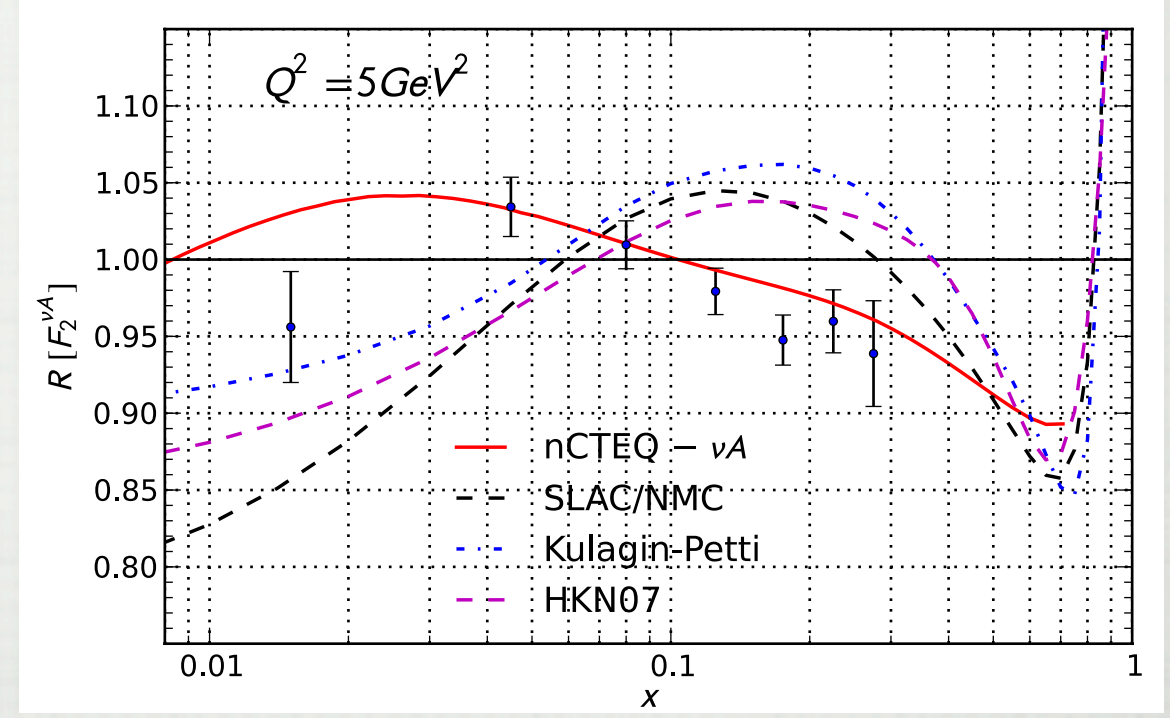
DIS & DY

$$\chi^2/\text{d.o.f} = 0.89$$



Fit to only neutrino DIS

$$\chi^2/\text{d.o.f} = 1.33$$



- can we explain the difference and fit all data together in a global fit ?

Fits to IA, DY and ν A data

- Many neutrino data points
- Use a weight parameter w to combine data sets
- $w=0$: only IA+DY data
- $w=\infty$: only ν A data

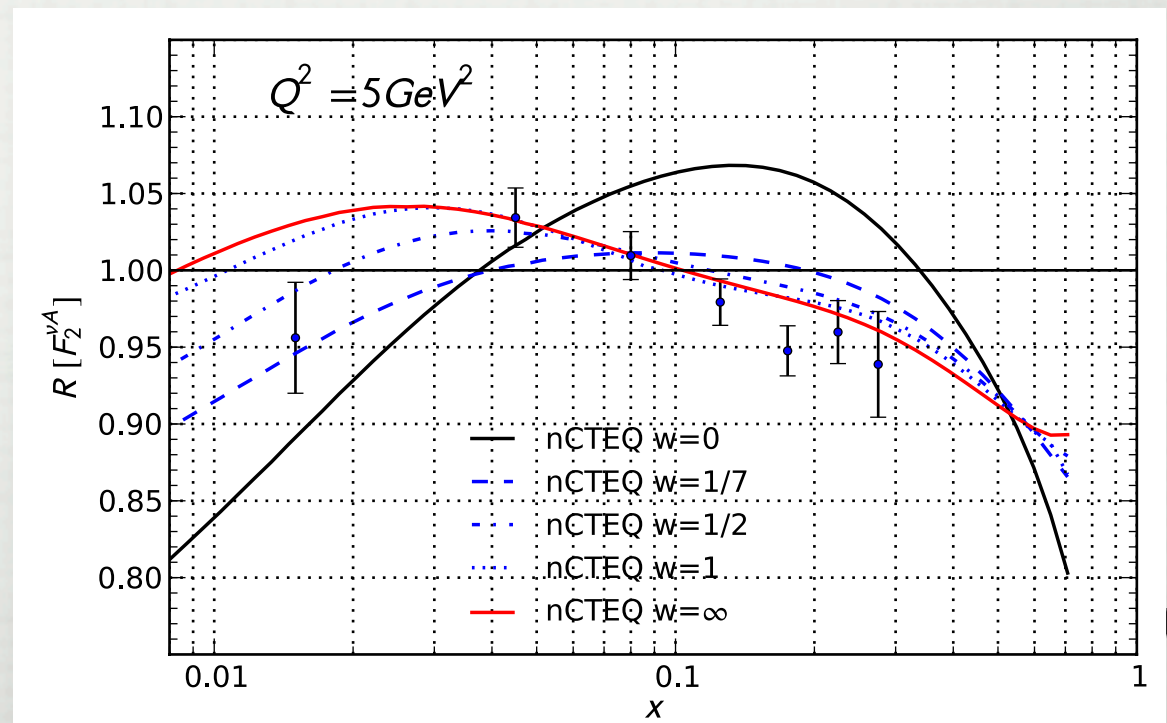
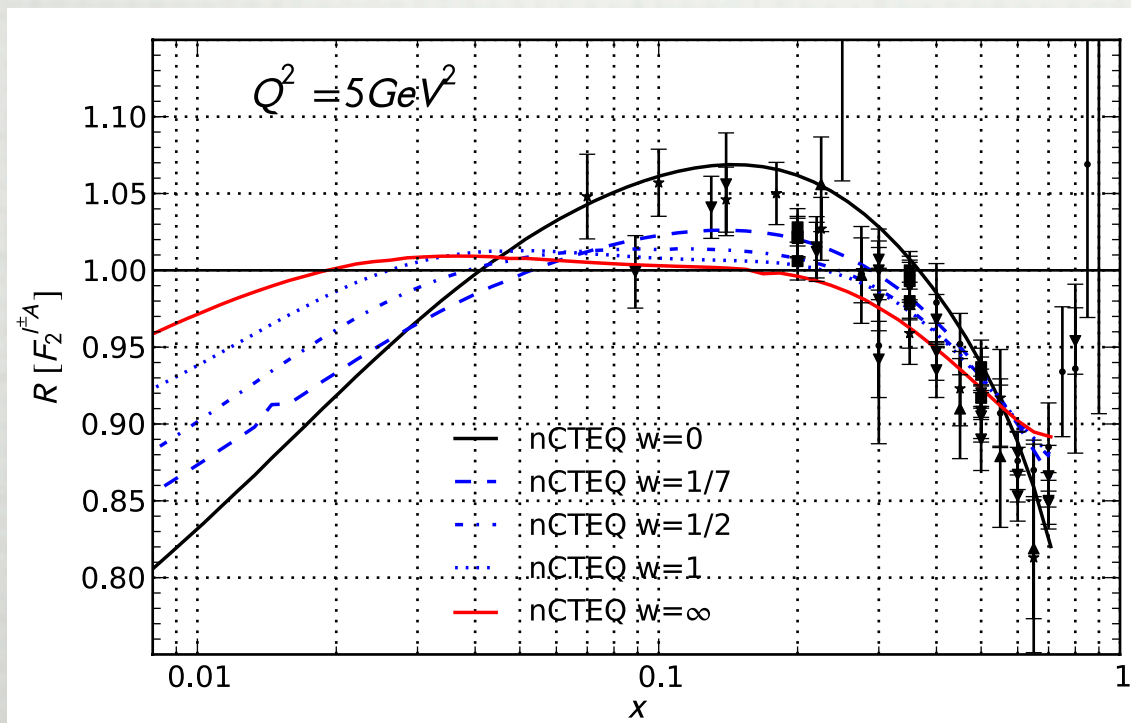
Weight	ℓ data	χ^2 (/pt)	ν data	χ^2 (/pt)	total χ^2 (/pt)
$w = 0$	708	639 (0.90)	-	-	639 (0.90)
$w = 1/7$	708	645 (0.91)	3134	4710 (1.50)	5355 (1.39)
$w = 1/4$	708	654 (0.92)	3134	4501 (1.43)	5155 (1.34)
$w = 1/2$	708	680 (0.96)	3134	4405 (1.40)	5085 (1.32)
$w = 1$	708	736 (1.04)	3134	4277 (1.36)	5014 (1.30)
$w = \infty$	-	-	3134	4192 (1.33)	4192 (1.33)

NEUTRINO DIS

Analysis of fits with different weights of neutrino DIS (correlated errors)

w	$l^\pm A$	χ^2 (/pt)	νA	χ^2 (/pt)	total χ^2 (/pt)
0	708	630 (0.89)	-	-	630 ± 58
1/7	708	645 (0.91)	3134	4681 (1.50)	5326 ± 203
1/2	708	680 (0.96)	3134	4375 (1.40)	5055 ± 192
1	708	736 (1.04)	3134	4246 (1.36)	4983 ± 190
∞	-	-	3134	4167 (1.33)	4167 ± 176

$$P(\chi^2, N) = \frac{(\chi^2)^{N/2-1} e^{-\chi^2/2}}{2^{N/2} \Gamma(N/2)}$$



NEUTRINO DIS

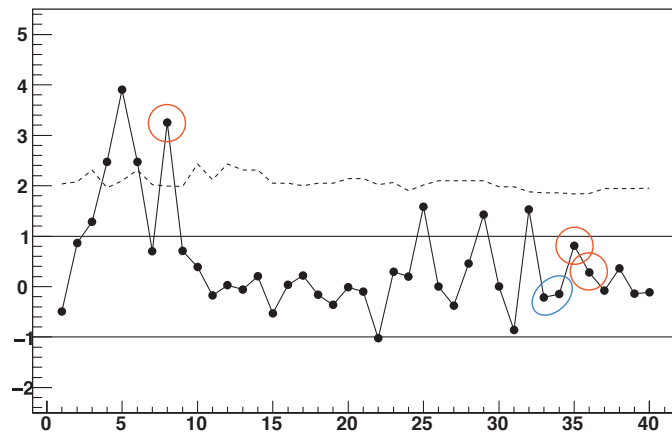
Analysis of fits with different weights of neutrino DIS (corr. errors)

- χ^2 -distribution criterion
$$P(\chi^2, N) = \frac{(\chi^2)^{N/2-1} e^{-\chi^2/2}}{2^{N/2} \Gamma(N/2)}$$

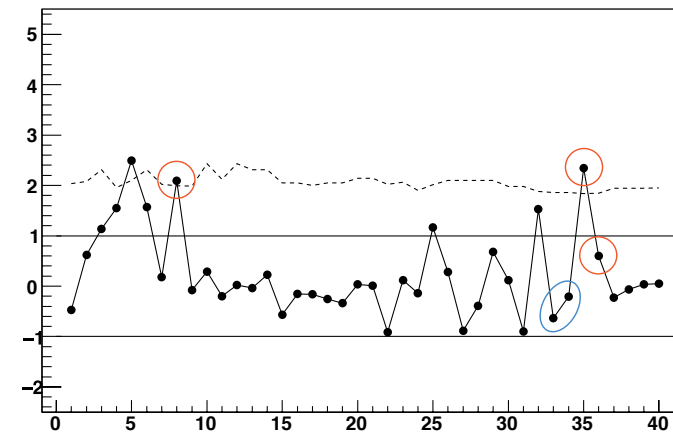
90% percentile (solid line at 1)
$$\int_0^{\xi_{90}} P(\chi^2, N) d\chi^2 = 0.90$$

99% percentile (dotted line)
$$\int_0^{\xi_{99}} P(\chi^2, N) d\chi^2 = 0.99$$

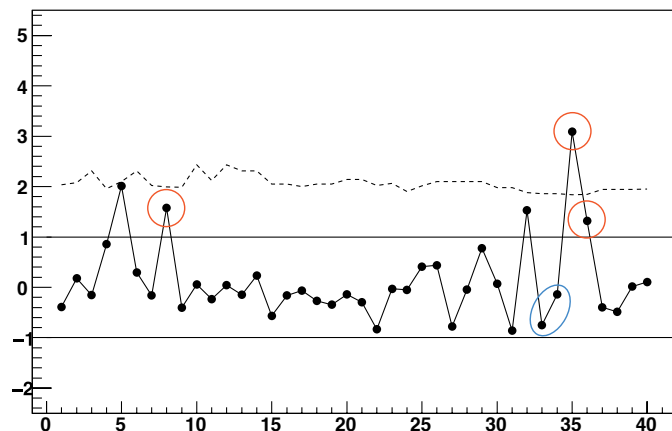
(w=1)



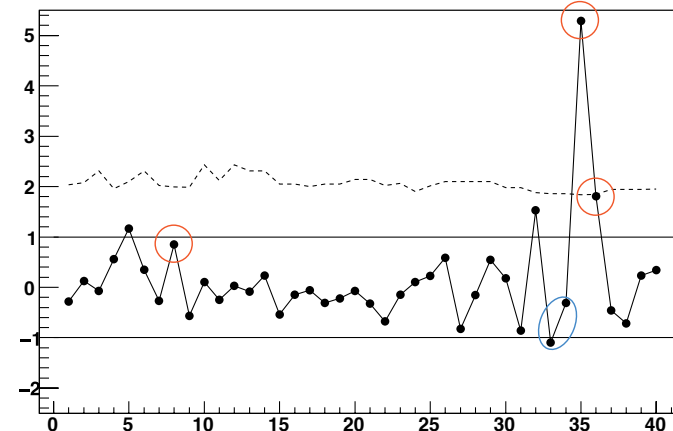
(w=1/2)



(w=1/4)



(w=1/7)

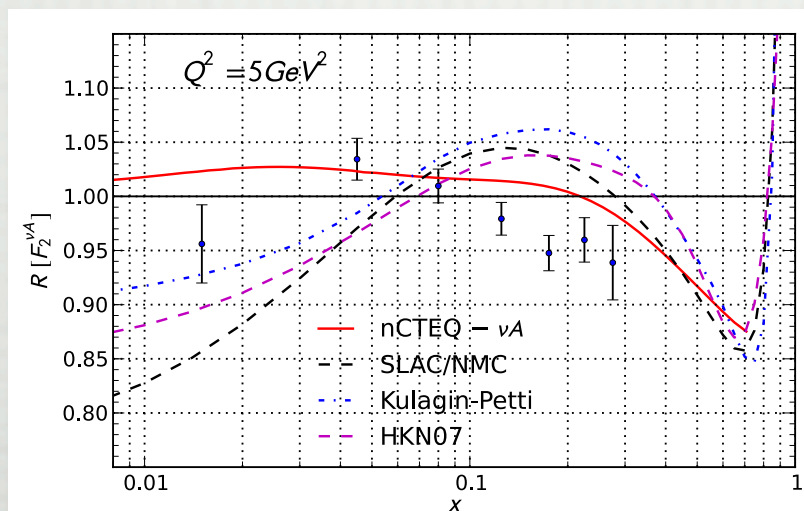
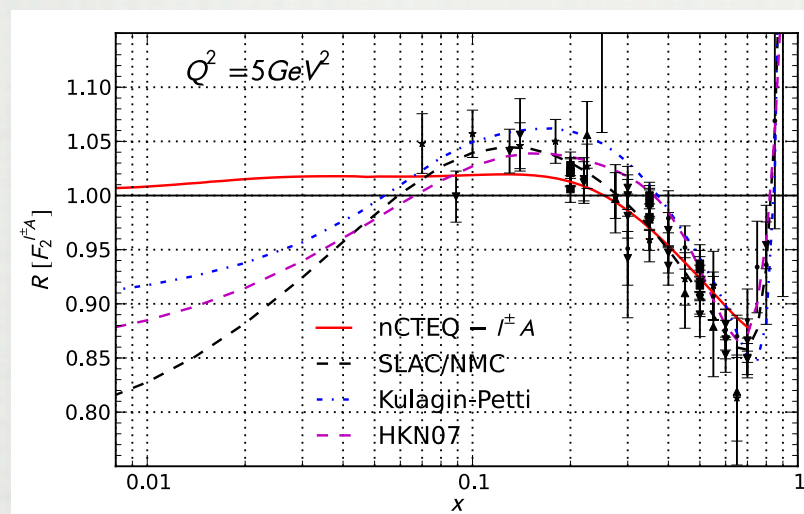


NEUTRINO DIS

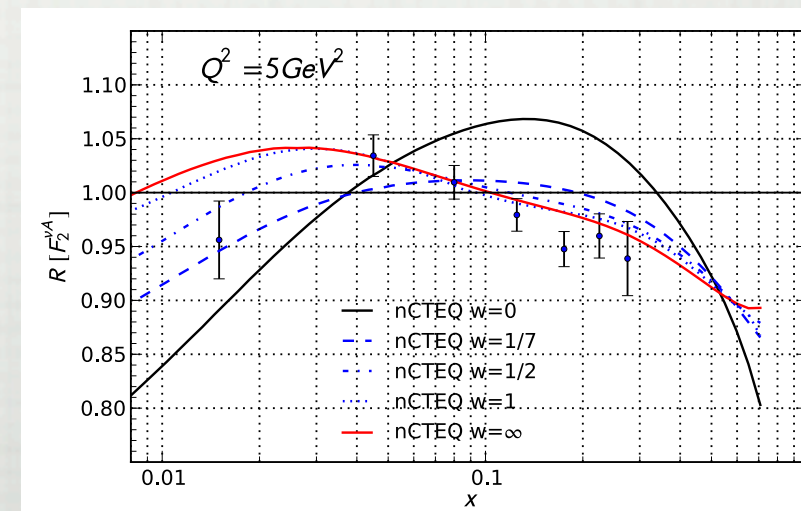
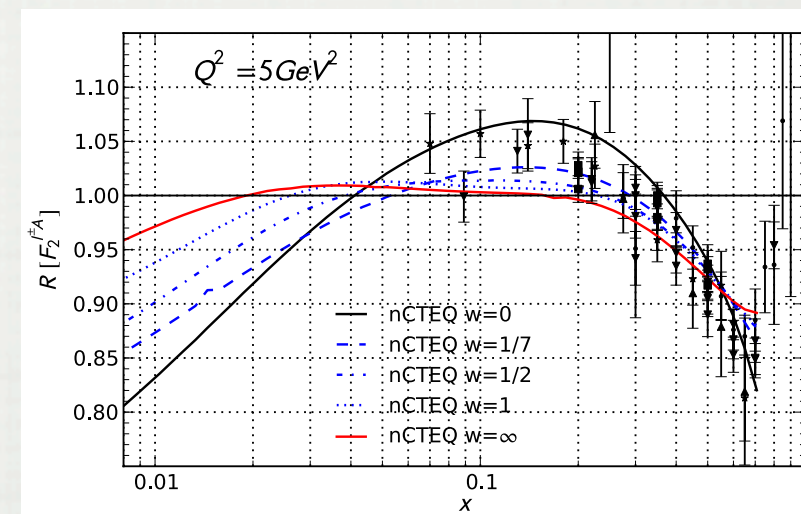
Analysis of fits with neutrino DIS (uncorrelated errors)

w	$l^\pm A$	χ^2 (/pt)	νA	χ^2 (/pt)	total χ^2 (/pt)
1-corr	708	736 (1.04)	3134	4246 (1.36)	4983 (1.30)
1-uncorr	708	809 (1.14)	3110	3115 (1.00)	3924 (1.02)

uncorrelated errors



correlated errors



Conclusions III

- **Incompatibility of neutrino DIS with charged lepton DIS?**
 - Conclusions heavily rely on NuTeV data (most precise)
 - Incompatibility a “precision effect” - the result changes e.g. when using uncorrelated errors
 - Tension in NuTeV data, high χ^2 in fit to NuTeV data alone
 - NOMAD data can help to decide
- **If confirmed, important consequences for:**
 - global analyses of proton and nuclear PDF; impact on strange PDF?
 - models explaining the nuclear effects
 - precision observables in the neutrino sector
- **Possible explanations**
 - Non-universal nuclear effects (breaking of factorization)
 - Twist-2 factorization valid but nuclear-enhanced higher-twist effects

Backup

PDF Uncertainties

Sources:

- **Experimental Errors** to be propagated to the PDFs
- **Theoretical Uncertainties**
- **Details** of the Global Fits
- **Inconsistencies** in the use of the PDFs/application of the theoretical framework

There are known Unknowns ...



Errors of experimental data

Methods: to propagate exp. errors to PDFs

- **Hesse Matrix**
 - Eigenvector PDFs
 - Quadratic approximation
 - Simple computation of correlations
- **Lagrange Multipliers**
 - No quadratic approximation
 - Time consuming
- **Monte Carlo Methods**
 - generate N data samples by varying data within errors
 - N fits to the N samples -> Estimate uncertainty

Hessian method:

Assume only one fit parameter a --> Expand $\chi^2(a)$ around Minimum a_0

$$\chi^2(a) = \chi^2(a_0) + \frac{1}{2} \chi^{2''}(a_0)(a - a_0)^2 + \dots$$

Eigenvalue of
Hessian 'matrix'

Determine Tolerance T <--> 1-sigma uncertainty: $T = \Delta \chi^2$

--> 1- σ uncertainty range for parameter a such that:

$$\chi^2(a) = \chi^2(a_0) + \Delta \chi^2 \Rightarrow \Delta a = T \sqrt{2 / \chi^{2''}(a_0)}$$

--> best fit PDF: a_0 , two 'Eigenvector' PDFs: $a_0 + \Delta a, a_0 - \Delta a$

1- σ uncertainty for **Observable X**:

$$\Delta X = \frac{X(\text{PDF}[a_0 + \Delta a]) - X(\text{PDF}[a_0 - \Delta a])}{2} \propto \Delta a \propto T$$

Generalization
to n parameters:
add in quadrature

Details of a global analysis

'Internal choices':

- **Choice/Weight of data sets** used
- **Assumptions on PDFs** (replace uncertainty!)
- Choice of Nuclear corrections to be applied to data taken with nuclear targets (D, Fe)
- Estimate/Choice of **tolerance T** corresponding to 1-sigma uncertainties
- Choice of the **input scale**
- Choice of the **functional form** of the PDFs at the input scale
- Scale evolution: x-space or n-space, spurious terms, soft-gluon resummation (evolution)

Details of a global analysis

'Public choices':

- Perturbative Order (LO, NLO, NNLO)
- Parameters: m_c , m_b , $\alpha_s(M_Z)$
- Factorization Scheme
- Heavy Flavour Scheme
- Central Factorization/Renormalization Scales
- Include?
 - Resummations (hard part)
 - Target Mass Corrections (TMC), Higher Twist
 - QED-effects

Remarks:

- 'Public choices' are choices also to be made by the user of the PDFs.
- For each public choice need in principle consistent set of PDFs
- Note: **Changes in the “details” may lead to results which lie outside previous error bands!**
- Certain items on the list become relevant due to the ever increasing demand for precision

Conclusion: Useful and necessary to have several different global analyses of PDFs.

Inconsistencies

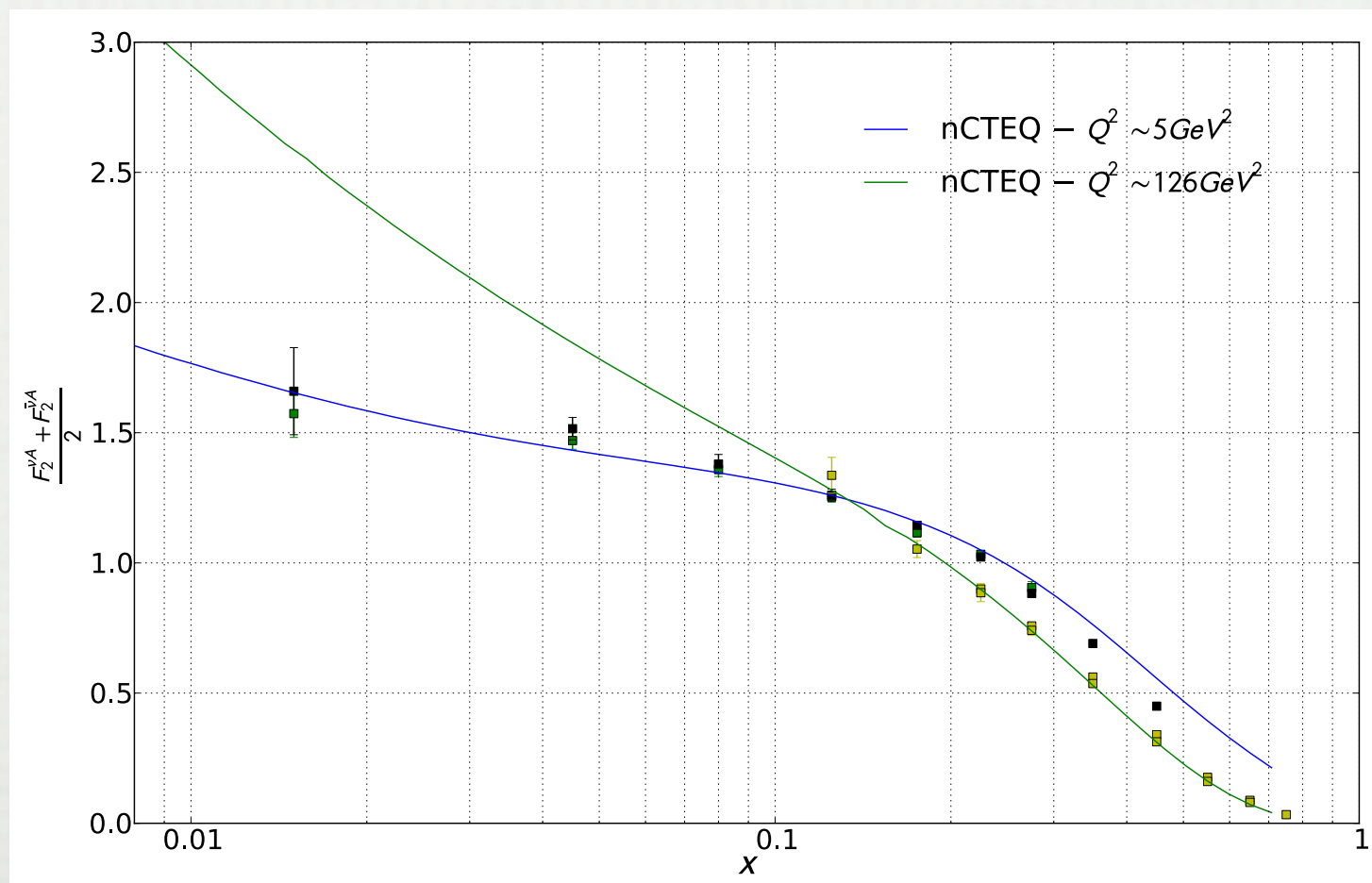
Examples:

- Use NLO PDFs with LO cross sections
- Use LO PDFs with NLO cross sections
- Use different schemes for PDFs and hard scattering cross sections
- Use different m_c , m_b , α_s than utilized in the global fit
- Use intrinsic k_T

NEUTRINO DIS

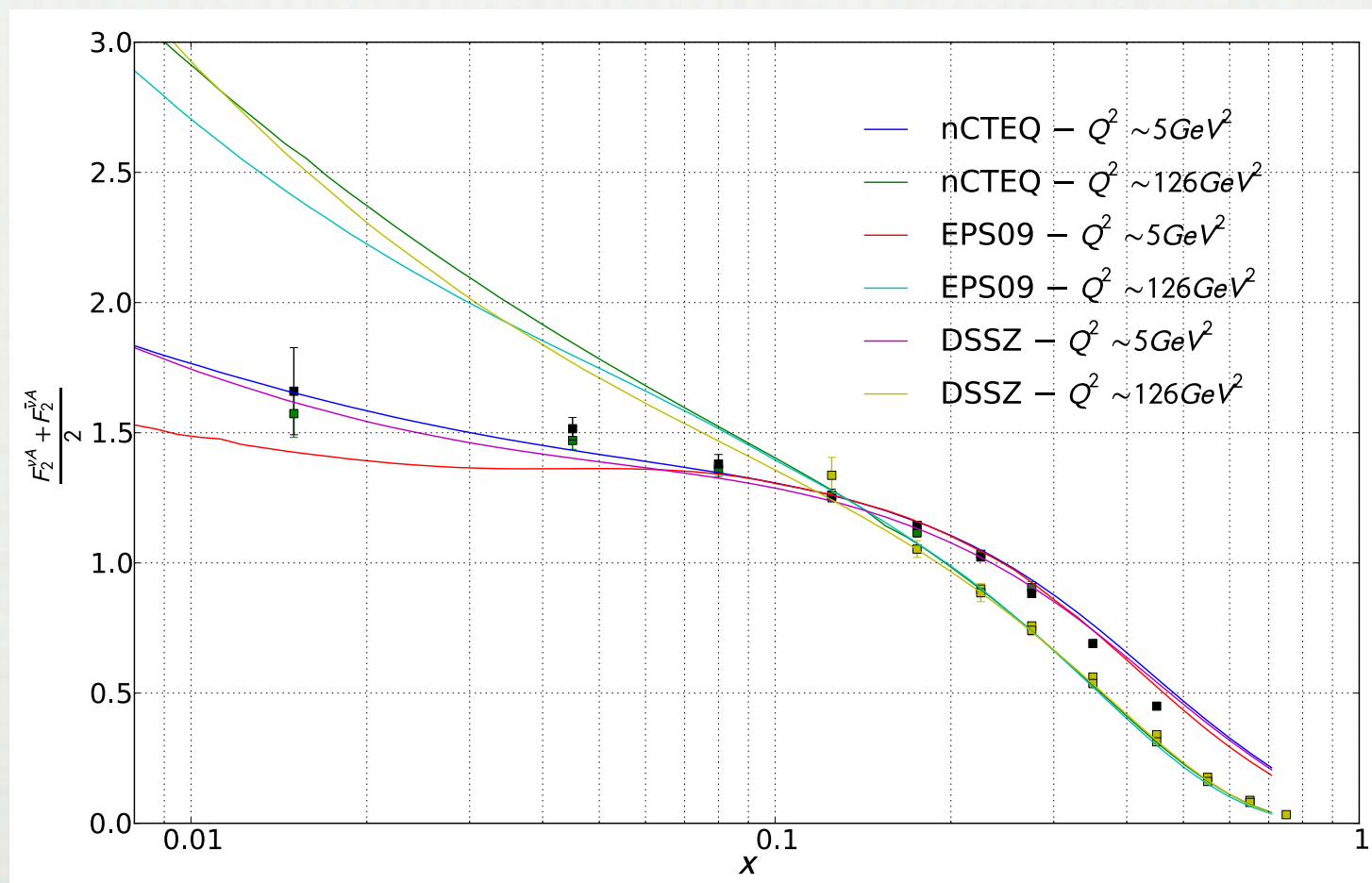
● NLO QCD calculation of $\frac{F_2^{\nu A} + F_2^{\bar{\nu} A}}{2}$ in the ACOT-VFN scheme

- comparison against extracted NuTeV data at different Q^2
- identical theory predictions for different nPDF



NEUTRINO DIS

- NLO QCD calculation of $\frac{F_2^{\nu A} + F_2^{\bar{\nu} A}}{2}$ in the ACOT-VFN scheme
 - comparison against extracted NuTeV data at different Q^2
 - identical theory predictions for different nPDF (nCTEQ-neutrino, EPS09, DSSZ prelim.)

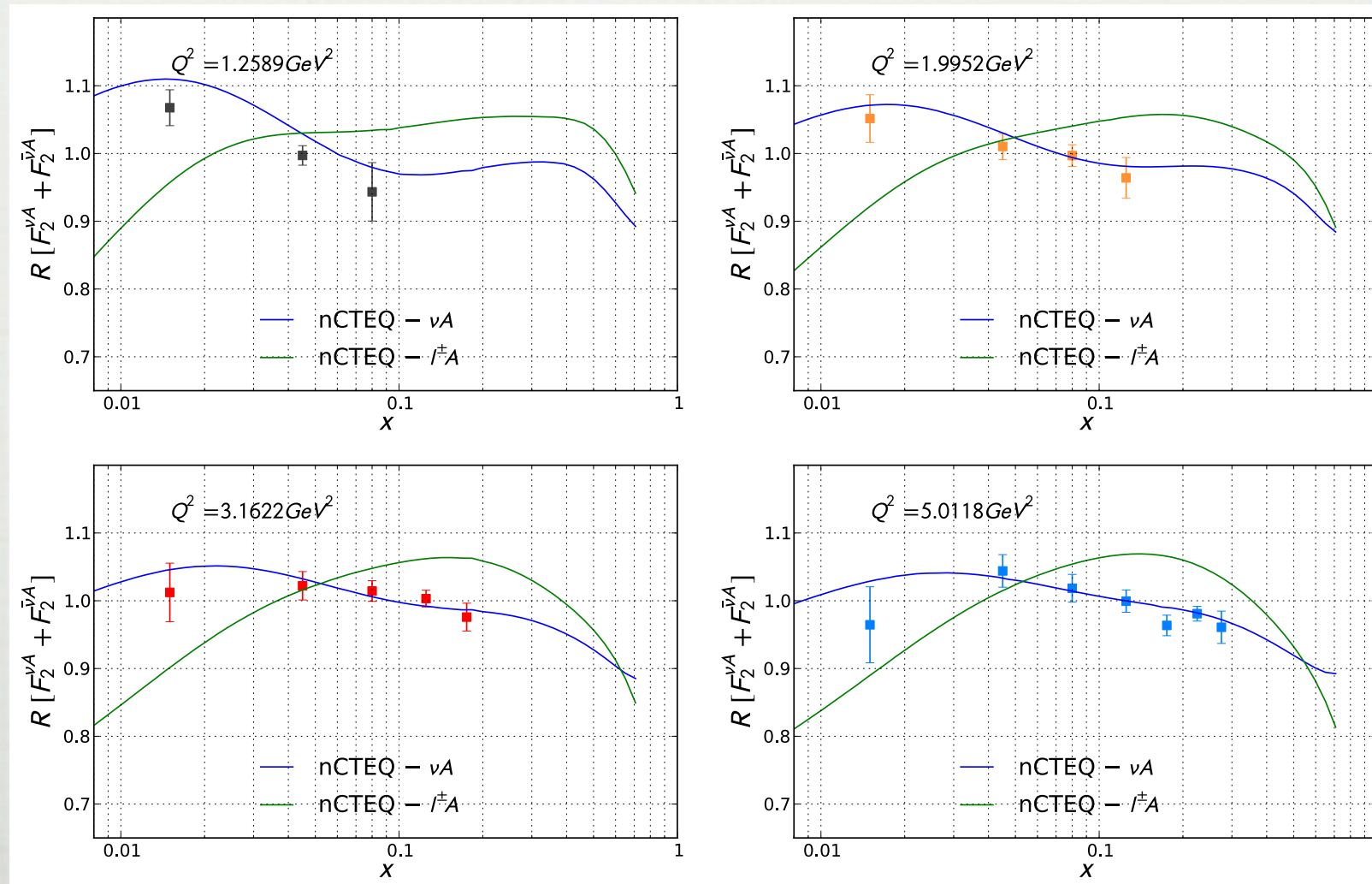


- general problems to fit data at high $x \sim 0.5$
- tension with charged lepton data at low $x \sim 0.01$

NEUTRINO DIS

● NLO QCD calculation of $\frac{F_2^{\nu A} + F_2^{\bar{\nu} A}}{2}$ in the ACOT-VFN scheme

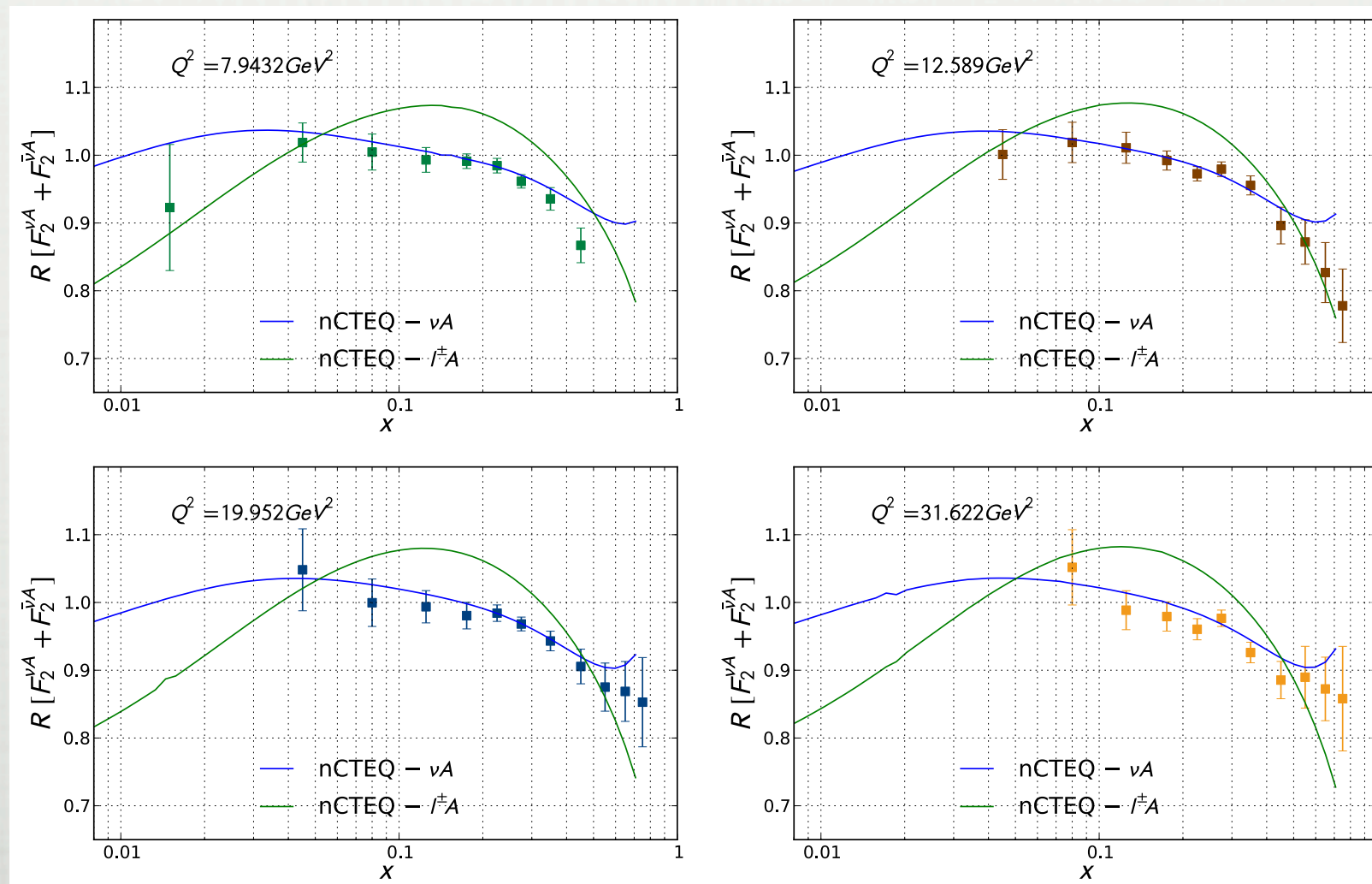
- comparison of nCTEQ - only neutrino fit against extracted NuTeV data at different Q^2
- charge lepton fit undershoots low-x data & overshoots mid-x data
- low- Q^2 and low-x data cause tension with the shadowing observed in charged lepton data



NEUTRINO DIS

● NLO QCD calculation of $\frac{F_2^{\nu A} + F_2^{\bar{\nu} A}}{2}$ in the ACOT-VFN scheme

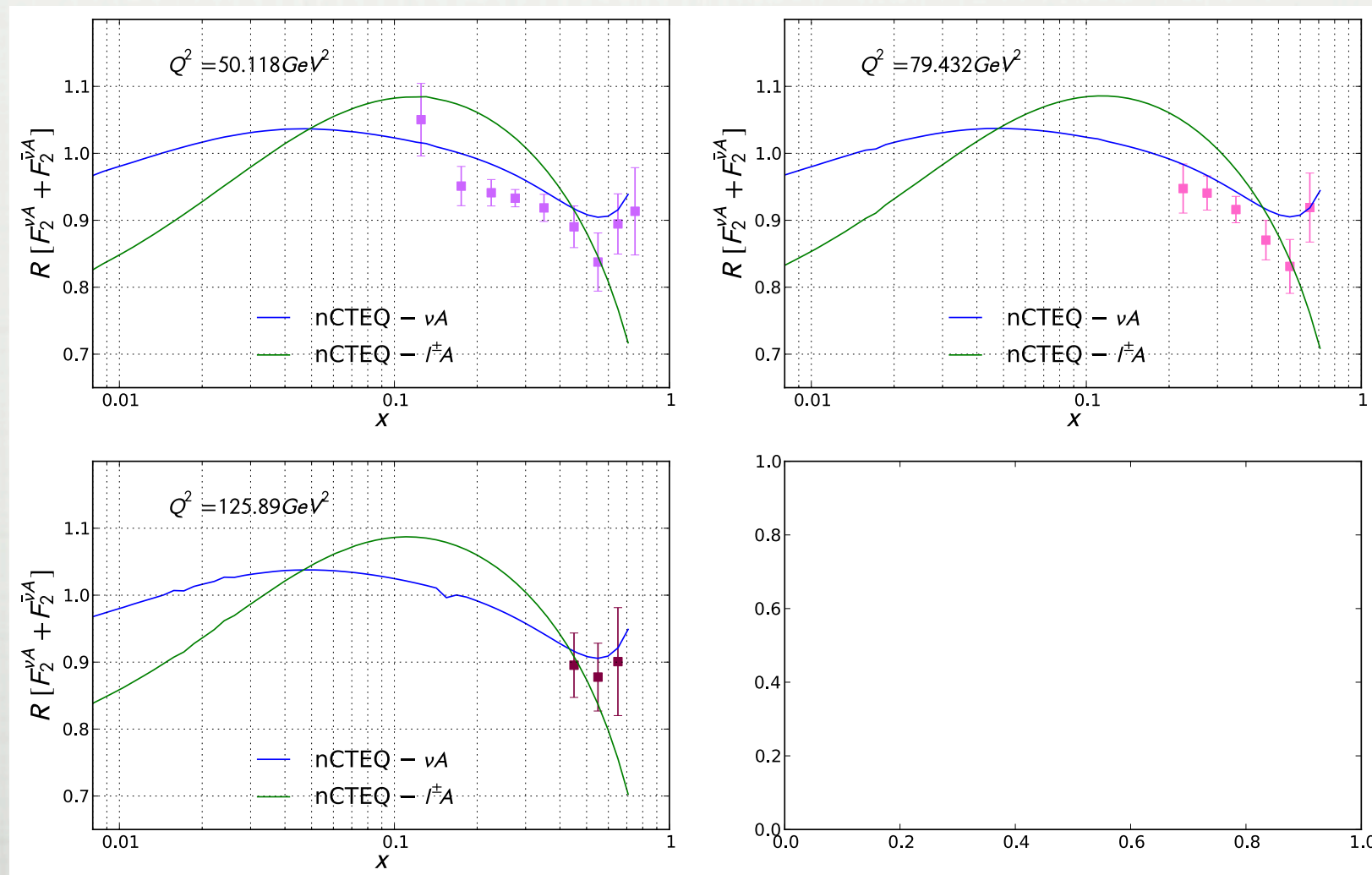
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NEUTRINO DIS

● Properties of neutrino fits

- CHORUS data are in good agreement with the charged lepton data

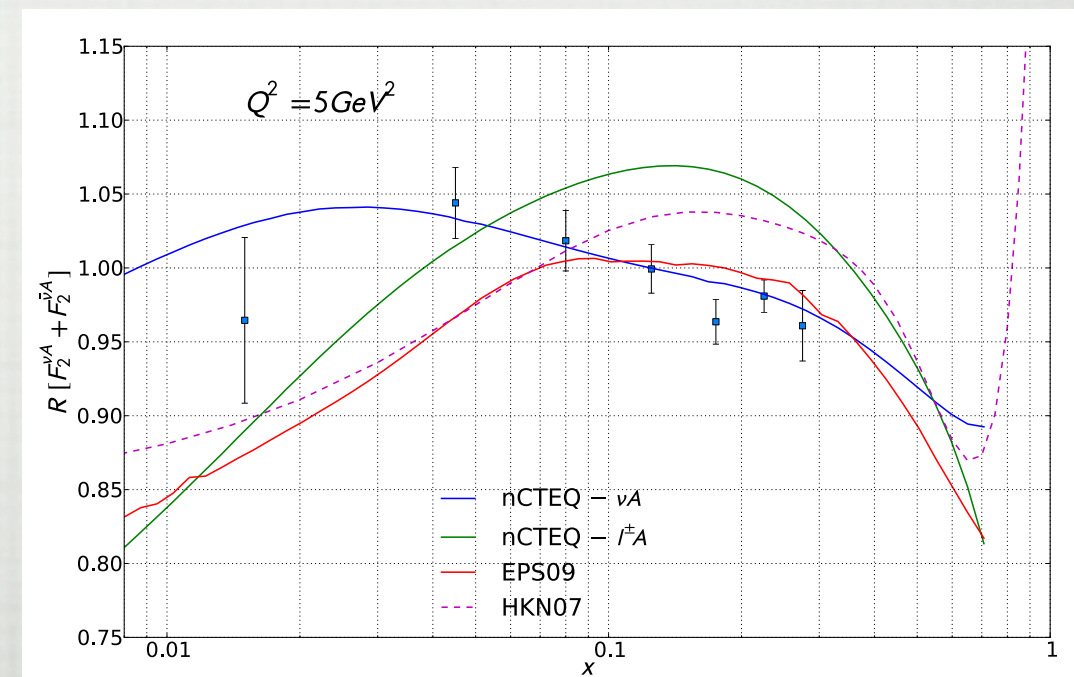
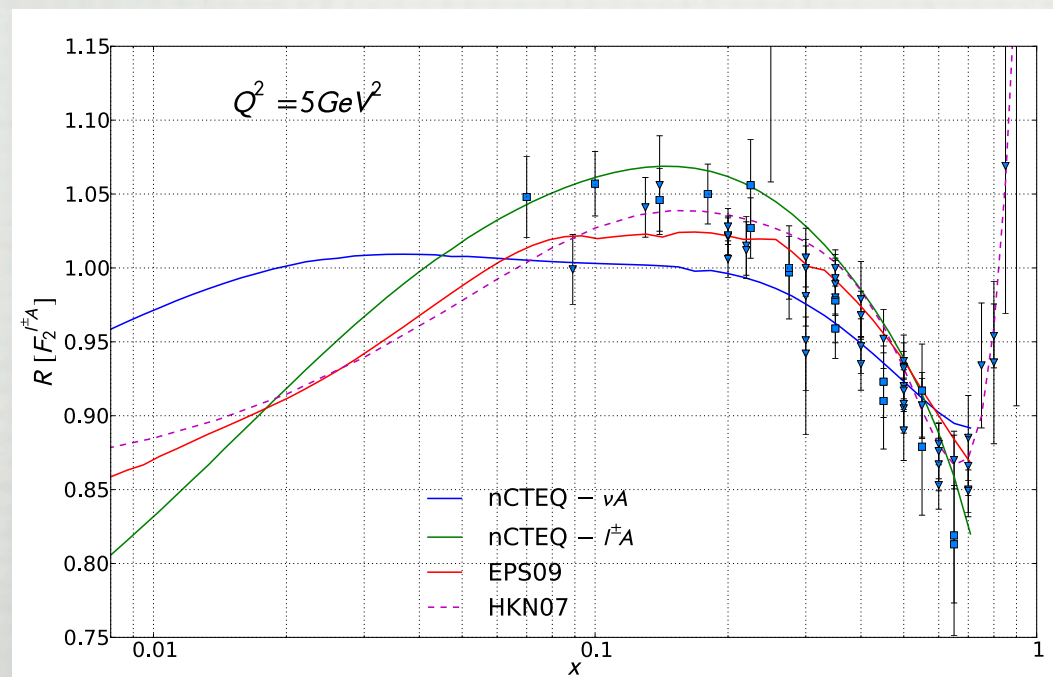
combined: $\chi^2/\text{pt}=1.03$

- NuTeV data (with correlated errors) difficult to fit alone or with the charged lepton data

alone: $\chi^2/\text{pt}=1.35$

combined: $\chi^2/\text{pt}=1.33$

- Neutrino data dominate the combined fit without re-weighting - final result depends from the weight chosen



CONCLUSIONS

- Incompatibility of neutrino DIS with charged lepton DIS (?)
 - conclusions heavily rely on only NuTeV data - most precise
 - incompatibility a "precision" effect - the result changes e.g. when using uncorrelated errors
 - tension in NuTeV data \rightarrow high χ^2 of the fit to NuTeV alone \rightarrow problem of NuTeV data ?
 - NOMAD data can help decide

- The impact of nuclear PDF from neutrino DIS on proton PDF
 - how does the incompatibility of neutrino DIS impact the uncertainty of strange quark PDF ?

NEUTRINO DIS

