

## *Masses in perturbative QCD*

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based on work in collaboration with

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Theorieseminar Mainz

# OUTLINE

- ❶ OVERVIEW AND MOTIVATION
- ❷ HEAVY FLAVOR SCHEMES
- ❸ ONE-PARTICLE INCLUSIVE PRODUCTION IN A GM-VFNS
- ❹ FRAGMENTATION FUNCTIONS IN A GM-VFNS
- ❺ APPLICATIONS
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# Overview and Motivation

# BIRD'S EYE VIEW

## QUANTUM CHROMODYNAMICS

### QCD: A QFT for the strong interactions



- Statement: Hadronic matter is made of spin-1/2 quarks [ $\leftrightarrow SU(3)_{\text{fl}}$ ]
- Baryons like  $\Delta^{++} = |u^\dagger u^\dagger u^\dagger\rangle$  forbidden by Pauli exclusion/Fermi-Dirac stat.  
**Need additional colour degree of freedom!**
- Local  $SU(3)$ -color gauge symmetry:

$$\mathcal{L}_{\text{QCD}} = \sum_{q=u,d,s,c,b,t} \bar{q}(i\partial - m_q)q - g\bar{q}\mathcal{G}q - \frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu} + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{ghost}}$$

- Fundamental d.o.f.: quark and gluon fields
- Free parameters:
  - gauge coupling:  $g$
  - quark masses:  $m_u, m_d, m_s, m_c, m_b, m_t$

# BIRD'S EYE VIEW

## QUANTUM CHROMODYNAMICS

### QCD: A QFT for the strong interactions

Properties:

- **Confinement and Hadronization:**

- Free quarks and gluons have not been observed:
  - A) They are **confined** in color-neutral hadrons of size  $\sim 1 \text{ fm}$ .
  - B) They **hadronize** into the observed hadrons.
- Hadronic energy scale: a few hundred MeV [ $1 \text{ fm} \leftrightarrow 200 \text{ MeV}$ ]
- Strong coupling large at long distances ( $\gtrsim 1 \text{ fm}$ ): '**IR-slavery**'
- Hadrons and hadron masses enter the game



- **Asymptotic freedom:**

- Strong coupling small at short distances: **perturbation theory**
- Quarks and gluons behave as free particles at asymptotically large energies

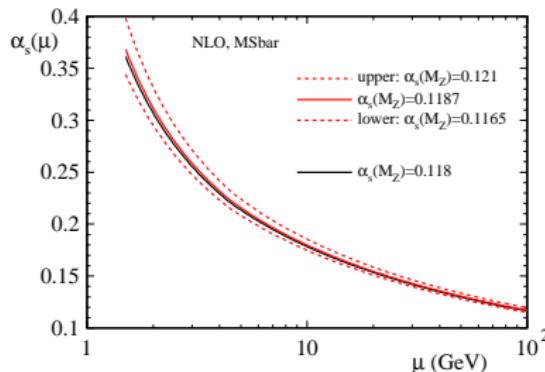
# BIRD'S EYE VIEW

## ASYMPTOTIC FREEDOM

Renormalization of UV-divergences:

Running coupling constant  $a_s := \alpha_s/(4\pi)$

$$a_s(\mu) = \frac{1}{\beta_0 \ln(\mu^2/\Lambda^2)}$$



- Gross, Wilczek ('73); Politzer ('73)



Non-abelian gauge theories:  
negative beta-functions

$$\frac{da_s}{d \ln \mu^2} = -\beta_0 a_s^2 + \dots$$

$$\text{where } \beta_0 = \frac{11}{3} C_A - \frac{2}{3} n_f$$

⇒ asympt. freedom:  $a_s \searrow$  for  $\mu \nearrow$

- Nobel Prize 2004

# BIRD'S EYE VIEW

## PERTURBATIVE QCD

Asymptotic freedom:

- pQCD directly applicable if **all** energy scales large (hard scales)
- However, usually long-distance contributions to the amplitudes present:
  - emission of soft gluons
  - emission of collinear gluons and quarks

pQCD still useful for two classes of observables:

- IR- and collinear-save observables (insensitive to soft or collinear branching)
- Factorizable observables

Factorization:

- Separate amplitudes into (quantum mechanically) **independent factors**:
  - Soft parts (long distances/small energies): **universal**
  - hard parts (short distances/large energies): **perturbatively calculable**
- Note: Soft parts non-perturbative but universal → **Predictive framework**

# BIRD'S EYE VIEW

## PARTON MODEL

Parton Model based on QCD factorization theorems:

$$d\sigma = \text{PDF} \otimes d\hat{\sigma} + \text{remainder}$$

- PDF:
  - Proton composed of partons = quarks, gluons
  - Structure of proton described by parton distribution functions (PDF)
  - Factorization theorems provide field theoretic definition of PDFs
  - PDFs **universal** → predictive power
- Hard part  $d\hat{\sigma}$ :
  - depends on the process
  - calculable order by order in **perturbation theory**
- Remainder suppressed by hard scale

Original factorization proofs considered massless partons

# BIRD'S EYE VIEW

## HEAVY QUARKS IN pQCD

Heavy Quarks:  $h = c, b, t$

- $m_u, m_d, m_s \lesssim \Lambda_{\text{QCD}} \ll m_c, m_b, m_t$
- $m_h \gg \Lambda_{\text{QCD}} \Rightarrow \alpha_s(m_h^2) \propto \ln^{-1}\left(\frac{m_h^2}{\Lambda_{\text{QCD}}^2}\right) \ll 1$  (asymptotic freedom)
- $m_h$  sets hard scale; acts as long distance cut-off  $\rightarrow$  pQCD
- Heavy quark production processes are:
  - Fundamental elementary particle processes
  - Important background to New Physics searches at the LHC

How to incorporate heavy quark masses into the pQCD formalism?

Requirements:

- (1)  $\mu \ll m$ : Decoupling of heavy degrees of freedom
- (2)  $\mu \gg m$ : IR-safety
- (3)  $\mu \sim m$ : Correct threshold behavior

# Heavy Flavor Schemes

# HEAVY FLAVOR SCHEMES

## Requirements:

- (1)  $\mu \ll m$ : Decoupling of heavy degrees of freedom
- (2)  $\mu \gg m$ : IR-safety
- (3)  $\mu \sim m$ : Correct threshold behavior

## Problem:

- Multiple hard scales:  $m_c, m_b, m_t, \mu$
- Mass-independent factorization/renormalization schemes like  $\overline{\text{MS}}$
- A single  $\overline{\text{MS}}$  scheme cannot meet requirements (1) and (3) (is unphysical).

## Way out: Patchwork of $\overline{\text{MS}}$ schemes $S^{n_f, n_R}$

- Variable Flavor-Number Scheme (VFNS):  $S^{3,3} \rightarrow S^{4,4} \rightarrow S^{5,5}$
- Fixed Flavor-Number Scheme (FFNS):  $S^{3,3} \rightarrow S^{3,4} \rightarrow S^{3,5}$  (3-FFNS)
- **Masses reintroduced** by backdoor: threshold corrections (=matching conditions)

# HEAVY FLAVOR SCHEMES

FFNS OR VFNS

## 3-FFNS

- charm is not a parton, appears only in final state
- no collinear divergences from  $c \rightarrow c + g$   
but terms  $\propto \log(\mu/m)$  with  $\mu = Q, p_T, \dots$  the hard scale
- Collinear logarithms  $\log(\mu/m)$  kept in **fixed order perturbation theory**
- + correct threshold behavior
- + finite charm mass terms  $m/\mu$  exactly taken into account
- not IR-safe: does not meet requirement (2)
- How to include possible intrinsic charm?

The 3-FFNS should fail when  $\alpha_s \ln(\mu/m)$  becomes large [or  $a_s \ln(\mu/m)$ ?]

**Phenomenological question:** When need to resum collinear log's?

→ Not unambiguously answered yet! A lot of handwaving ...

# HEAVY FLAVOR SCHEMES

FFNS OR VFNS

## VFNS

- charm is a parton for  $\mu \gtrsim m$
- mass singularities absorbed in PDFs (and FFs)
  - if  $m = 0$ :  $1/\varepsilon$  poles  $\rightarrow \overline{\text{MS}}$  subtraction
  - if  $m \neq 0$ :  $\log(\mu/m)$  + finite  $\rightarrow \overline{\text{MS}}$  subtraction
- QCD prediction: DGLAP (RG) evolution resums large logarithms  $\log(\mu/m)$
- + Requirements (1), (2) satisfied
- + finite mass terms  $m/\mu$  can be taken into account: massive VFNS (GM-VFNS) (otherwise: massless VFNS (ZM-VFNS) which is the original parton model)
- Requirement (3) **problematic point**:
  - In DIS slow-rescaling prescriptions (ACOT- $\chi$ ) good approximation of exact threshold kinematics:  $c(x) \rightarrow c(\chi)$  where  $\chi = x(1 + 4m^2/Q^2)$
  - What to do in hadron–hadron collisions?
  - What to do in 1-particle inclusive production?
- Intrinsic charm natural to incorporate

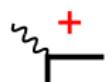
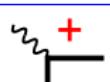
# HEAVY FLAVOR SCHEMES

VFNS: MORE

## VFNS

- Factorization proof with massive quarks for inclusive DIS: Collins '98  
Remainder  $\sim \mathcal{O}(\Lambda^2/Q^2)$  not  $\sim \mathcal{O}(m^2/Q^2)$
- Many incarnations of VFNS (ACOT, ACOT- $\chi$ , TR): Freedom to shift finite  $m$ -terms without spoiling IR-safety
- S-ACOT scheme: incoming heavy quarks massless ( $\leftrightarrow$  scheme choice)  
more complex at NNLO
- Massive quarks can be described by **massless** evolution kernels ( $\leftrightarrow$  scheme choice)
- Matching  $n \rightarrow n+1$ : PDFs,  $\alpha_s$ , masses
- At NLO matching continuous at  $\mu = m$ :  $f_i^{n_f} = f_i^{n_f+1}$
- At higher orders matching discontinuous:
  - for PDFs discontinuous at  $\mathcal{O}(\alpha_s^2)$
  - for  $\alpha_s$  discontinuous at  $\mathcal{O}(\alpha_s^3)$
- Observable discontinuous:  $\sigma^{n_f} = \sigma^{n_f+1} + \mathcal{O}(\alpha_s^{K+1})$

# SCHEMES USED IN GLOBAL ANALYSES OF PDFS

ACOT type schemes		constant term	TR type schemes		constant term
$Q < m_H$	$Q > m_H$		$Q < m_H$	$Q > m_H$	
LO	$\emptyset$		$\emptyset$		$Q = m_H$
NLO			$\emptyset$		$Q = m_H$
NNLO			$\emptyset$		$Q = m_H$

# One-particle inclusive production in a GM-VFNS

# OVERVIEW

- One-particle inclusive production of heavy hadrons  $H = D, B, \Lambda_c, \dots$
- General-Mass Variable Flavour Number Scheme (**GM-VFNS**):
  - Collinear logarithms of the heavy-quark mass  $\ln \mu/m_h$  are subtracted and resummed [1]
  - Finite non-logarithmic  $m_h/Q$  terms are kept in the hard part/taken into account
  - Scheme guided by the factorization theorem of **Collins** with heavy quarks [2]

Ongoing effort to compute all relevant processes in the **GM-VFNS** at NLO:

- Available:
  - $e^+ + e^- \rightarrow (D^0, D^+, D^{*+}) + X$ : FFs [3]
  - $\gamma + \gamma \rightarrow D^{*+} + X$ : direct process [4]
  - $\gamma + \gamma \rightarrow D^{*+} + X$ : single-resolved process [5]
  - $\gamma + p \rightarrow D^{*+} + X$ : direct process [6]
  - $\gamma + p \rightarrow D^{*+} + X$ : resolved process [7]
  - $p + \bar{p} \rightarrow (D^0, D^+, D^{*+}, D_s^+, \Lambda_c^+, B^0, B^+) + X$  [1]

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[1] Kniehl,Kramer,IS,Spiesberger, PRD71(2005)014018; EPJC41(2005)199;  
PRL96(2006)012001; PRD77(2008)014011; arXiv:0901.4130[hep-ph], PRD (in press)

[2] Collins, PRD58(1998)094002

[3] Kneesch,Kniehl,Kramer,IS, NPB799(2008)34

[4] Kramer,Spiesberger, EPJC22(2001)289; [5] EPJC28(2003)495; [6] EPJC38(2004)309

[7] Kniehl,Kramer,IS,Spiesberger, arXiv:0902.3166[hep-ph], EPJC (in press)

# OVERVIEW -CONTINUED-

Input for the computation: Fragmentation Functions (FFs) into heavy hadrons  $H$

- FFs from fits to  $e^+e^-$  data from  $Z$  factories
- Include also  $B$  factories → Switch from ZM to GM
- Use initial scale  $\mu_0 = m$  (instead of  $\mu_0 = 2m$ ) for consistency with PDFs → important for gluon fragmentation

$H$	Data	Scheme	Reference
$D^{*+}$	ALEPH,OPAL	ZM $2m$	BKK, PRD58(1998)014014
$D^0, D^+, D_s^+, \Lambda_c^+$	OPAL	ZM $2m$	KK, PRD71(2005)094013
$D^0, D^+, D^{*+}, D_s^+, \Lambda_c^+$	OPAL	ZM $m$	KK, PRD74(2006)037502
$D^0, D^+, D_s^+$	Belle,CLEO,ALEPH,OPAL	GM $m$	KKKSc, NPB799(2008)34
$B^0, B^+$	OPAL	ZM $2m$	BKK, PRD58(1998)034016
$B^0, B^+$	ALEPH,OPAL,SLD	ZM $m$	KKScSp, PRD77(2008)014011

Goal:

- Test pQCD formalism, scaling violations and universality of FFs in as many processes as possible

$$A + B \rightarrow H + X: \quad d\sigma = \sum_{i,j,k} f_i^A(x_1) \otimes f_j^B(x_2) \otimes d\sigma(ij \rightarrow kX) \otimes D_k^H(z)$$

sum over all possible subprocesses  $i + j \rightarrow k + X$

Parton distribution functions:	Hard scattering cross section:	Fragmentation functions:
$f_i^A(x_1, \mu_F), f_j^B(x_2, \mu_F)$		$D_k^H(z, [\mu'_F])$
non-perturbative input		non-perturbative input
long distance		long distance
universal	(perturbatively computable short distance coefficient functions)	universal

Accuracy:

light hadrons:  $\mathcal{O}((\Lambda/p_T)^p)$  with  $p_T$  hard scale,  $\Lambda$  hadronic scale,  $p = 1, 2$   
 heavy hadrons: if  $m_h$  is neglected in  $d\sigma$ :  $\mathcal{O}((m_h/p_T)^p)$

Details (subprocesses, PDFs, FFs; mass terms) depend on  
 the Heavy Flavour Scheme

## LIST OF SUBPROCESSES: GM-VFNS

Only light lines

- ①  $gg \rightarrow qX$
- ②  $gg \rightarrow gX$
- ③  $qg \rightarrow gX$
- ④  $qg \rightarrow qX$
- ⑤  $q\bar{q} \rightarrow gX$
- ⑥  $q\bar{q} \rightarrow qX$
- ⑦  $qg \rightarrow \bar{q}X$
- ⑧  $qg \rightarrow \bar{q}'X$
- ⑨  $qg \rightarrow q'X$
- ⑩  $qq \rightarrow gX$
- ⑪  $qq \rightarrow qX$
- ⑫  $q\bar{q} \rightarrow q'X$
- ⑬  $q\bar{q}' \rightarrow gX$
- ⑭  $q\bar{q}' \rightarrow qX$
- ⑮  $qq' \rightarrow gX$
- ⑯  $qq' \rightarrow qX$

⊕ charge conjugated processes

Heavy quark initiated ( $m_Q = 0$ )

- ① -
- ② -
- ③  $Qg \rightarrow gX$
- ④  $Qg \rightarrow QX$
- ⑤  $Q\bar{Q} \rightarrow gX$
- ⑥  $Q\bar{Q} \rightarrow QX$
- ⑦  $Qg \rightarrow \bar{Q}X$
- ⑧  $Qg \rightarrow \bar{q}X$
- ⑨  $Qg \rightarrow qX$
- ⑩  $QQ \rightarrow gX$
- ⑪  $QQ \rightarrow QX$
- ⑫  $Q\bar{Q} \rightarrow qX$
- ⑬  $Q\bar{q} \rightarrow gX, q\bar{Q} \rightarrow gX$
- ⑭  $Q\bar{q} \rightarrow QX, q\bar{Q} \rightarrow qX$
- ⑮  $Qq \rightarrow gX, qQ \rightarrow gX$
- ⑯  $Qq \rightarrow QX, qQ \rightarrow qX$

Mass effects:  $m_Q \neq 0$

- ①  $gg \rightarrow QX$
- ② -
- ③ -
- ④ -
- ⑤ -
- ⑥ -
- ⑦ -
- ⑧  $qg \rightarrow \bar{Q}X$
- ⑨  $qg \rightarrow QX$
- ⑩ -
- ⑪ -
- ⑫  $q\bar{q} \rightarrow QX$
- ⑬ -
- ⑭ -
- ⑮ -
- ⑯ -

Mass terms contained in the hard scattering coefficients:

$$d\hat{\sigma}(\mu_F, \mu_{F'}, \alpha_s(\mu_R), \frac{m}{p_T})$$

Two ways to derive them:

- (1) Compare **massless limit** of a massive fixed-order calculation with a massless  $\overline{\text{MS}}$  calculation to determine subtraction terms

OR

- (2) Perform **mass factorization** using partonic PDFs and FFs

## (1) SUBTRACTION TERMS FOR THE GM-VFNS FROM MASSLESS LIMIT

- Compare limit  $m \rightarrow 0$  of the massive calculation (Merebashvili et al., Ellis, Nason; Smith, van Neerven; Bojak, Stratmann; ...) with massless  $\overline{\text{MS}}$  calculation (Aurenche et al., Aversa et al., ...)

$$\lim_{m \rightarrow 0} d\tilde{\sigma}(m) = d\hat{\sigma}_{\overline{\text{MS}}} + \Delta d\sigma$$

⇒ Subtraction terms

$$d\sigma_{\text{sub}} \equiv \Delta d\sigma = \lim_{m \rightarrow 0} d\tilde{\sigma}(m) - d\hat{\sigma}_{\overline{\text{MS}}}$$

- Subtract  $d\sigma_{\text{sub}}$  from **massive** partonic cross section while **keeping mass terms**

$$d\hat{\sigma}(m) = d\tilde{\sigma}(m) - d\sigma_{\text{sub}}$$

→  $d\hat{\sigma}(m)$  short distance coefficient including  $m$  dependence

→ allows to use PDFs and FFs with  $\overline{\text{MS}}$  factorization  $\otimes$  **massive** short distance cross sections

- Treat contributions with charm in the initial state with  $m = 0$
- Massless limit: technically non-trivial, map from phase-space slicing to subtraction method

## Mass factorization

Subtraction terms are associated to mass singularities:

can be described by

**partonic PDFs and FFs** for collinear splittings  $a \rightarrow b + X$

- initial state:  $f_{g \rightarrow Q}^{(1)}(x, \mu^2) = \frac{\alpha_s(\mu)}{2\pi} P_{g \rightarrow q}^{(0)}(x) \ln \frac{\mu^2}{m^2}$   
 $f_{Q \rightarrow Q}^{(1)}(x, \mu^2) = \frac{\alpha_s(\mu)}{2\pi} C_F \left[ \frac{1+z^2}{1-z} \left( \ln \frac{\mu^2}{m^2} - 2 \ln(1-z) - 1 \right) \right]_+$   
 $f_{g \rightarrow g}^{(1)}(x, \mu^2) = -\frac{\alpha_s(\mu)}{2\pi} \frac{1}{3} \ln \frac{\mu^2}{m^2} \delta(1-x)$
- final state:  $d_{g \rightarrow Q}^{(1)}(z, \mu^2) = \frac{\alpha_s(\mu)}{2\pi} P_{g \rightarrow q}^{(0)}(z) \ln \frac{\mu^2}{m^2}$   
 $d_{Q \rightarrow Q}^{(1)}(z, \mu^2) = C_F \frac{\alpha_s(\mu)}{2\pi} \left[ \frac{1+z^2}{1-z} \left( \ln \frac{\mu^2}{m^2} - 2 \ln(1-z) - 1 \right) \right]_+$
- Other partonic distribution functions are zero to order  $\alpha_s$

Mele, Nason; Kretzer, Schienbein; Melnikov, Mitov

(2) SUBTRACTION TERMS VIA  $\overline{\text{MS}}$  MASS FACTORIZATION:  $a(k_1)b(k_2) \rightarrow Q(p_1)X$  [1]

Sketch of kinematics:

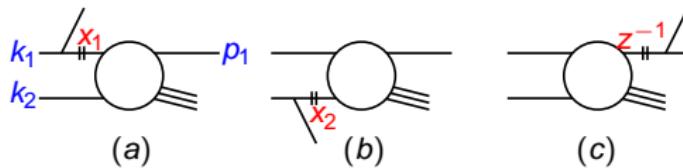


Fig. (a):  $d\sigma^{\text{sub}}(ab \rightarrow QX) = \int_0^1 dx_1 f_{a \rightarrow i}^{(1)}(x_1, \mu_F^2) d\hat{\sigma}^{(0)}(ib \rightarrow QX)[x_1 k_1, k_2, p_1]$

$$\equiv f_{a \rightarrow i}^{(1)}(x_1) \otimes d\hat{\sigma}^{(0)}(ib \rightarrow QX)$$

Fig. (b):  $d\sigma^{\text{sub}}(ab \rightarrow QX) = \int_0^1 dx_2 f_{b \rightarrow j}^{(1)}(x_2, \mu_F^2) d\hat{\sigma}^{(0)}(aj \rightarrow QX)[k_1, x_2 k_2, p_1]$

$$\equiv f_{b \rightarrow j}^{(1)}(x_2) \otimes d\hat{\sigma}^{(0)}(aj \rightarrow QX)$$

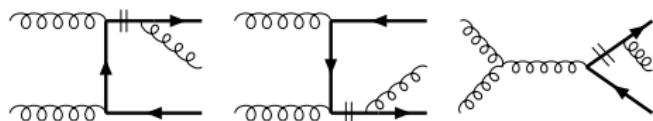
Fig. (c):  $d\sigma^{\text{sub}}(ab \rightarrow QX) = \int_0^1 dz d\hat{\sigma}^{(0)}(ab \rightarrow kX)[k_1, k_2, z^{-1} p_1] d_{k \rightarrow Q}^{(1)}(z, \mu_F'^2)$

$$\equiv d\hat{\sigma}^{(0)}(ab \rightarrow kX) \otimes d_{k \rightarrow Q}^{(1)}(z)$$

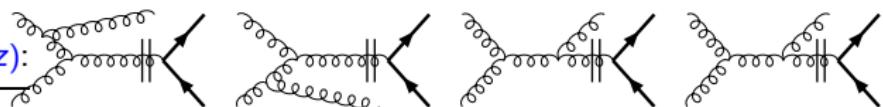

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# GRAPHICAL REPRESENTATION OF SUBTRACTION TERMS FOR $gg \rightarrow Q\bar{Q}g$

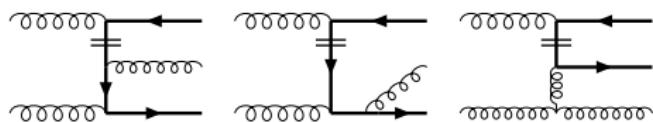
$d\hat{\sigma}^{(0)}(gg \rightarrow Q\bar{Q}) \otimes d_{Q \rightarrow Q}^{(1)}(z)$ :



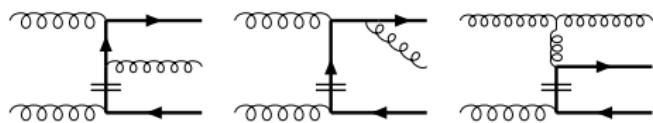
$d\hat{\sigma}^{(0)}(gg \rightarrow gg) \otimes d_{g \rightarrow Q}^{(1)}(z)$ :



$f_{g \rightarrow Q}^{(1)}(x_1) \otimes d\hat{\sigma}^{(0)}(Qg \rightarrow Qg)$ :

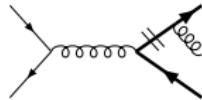


$f_{g \rightarrow Q}^{(1)}(x_2) \otimes d\hat{\sigma}^{(0)}(gQ \rightarrow Qg)$ :

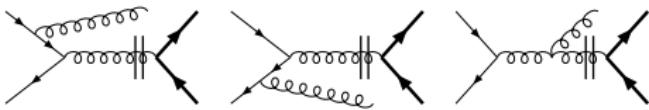


# GRAPHICAL REPRESENTATION OF SUBTRACTION TERMS FOR $q\bar{q} \rightarrow Q\bar{Q}$ AND $gq \rightarrow Q\bar{Q}q$

$d\hat{\sigma}^{(0)}(q\bar{q} \rightarrow Q\bar{Q}) \otimes d_{Q \rightarrow Q}^{(1)}(z)$ :



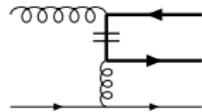
$d\hat{\sigma}^{(0)}(q\bar{q} \rightarrow gg) \otimes d_{g \rightarrow Q}^{(1)}(z)$ :



$d\hat{\sigma}^{(0)}(gq \rightarrow gq) \otimes d_{g \rightarrow Q}^{(1)}(z)$ :



$f_{g \rightarrow Q}^{(1)}(x_1) \otimes d\hat{\sigma}^{(0)}(Qq \rightarrow Qq)$ :



# Fragmentation Functions in a GM-VFNS

# $D^0, D^+, D^{*+}$ FFs WITH FINITE-MASS CORRECTIONS [1]

## FORMALISM

- $e^+ + e^- \rightarrow (\gamma, Z) \rightarrow H + X, \quad H = D^0, D^+, D^{*+}, \dots$
- $x = 2(p_H \cdot q)/q^2 = 2E_H/\sqrt{s} \quad \sqrt{\rho_H} \leq x \leq 1 \quad (\rho_H = 4m_H^2/s)$

$$\frac{d\sigma}{dx}(x, s) = \sum_a \int_{y_{\min}}^{y_{\max}} \frac{dy}{y} \frac{d\sigma_a}{dy}(y, \mu, \mu_f) D_a \left( \frac{x}{y}, \mu_f \right)$$

$d\sigma_a/dy$  at NLO with  $m_q = 0$  [2] and  $m_q \neq 0$  [1,3]

- $x_p = p/p_{\max} = \sqrt{(x^2 - \rho_H)/(1 - \rho_H)} \quad 0 \leq x_p \leq 1$

$$\frac{d\sigma}{dx_p}(x_p) = (1 - \rho_H) \frac{x_p}{x} \frac{d\sigma}{dx}(x)$$

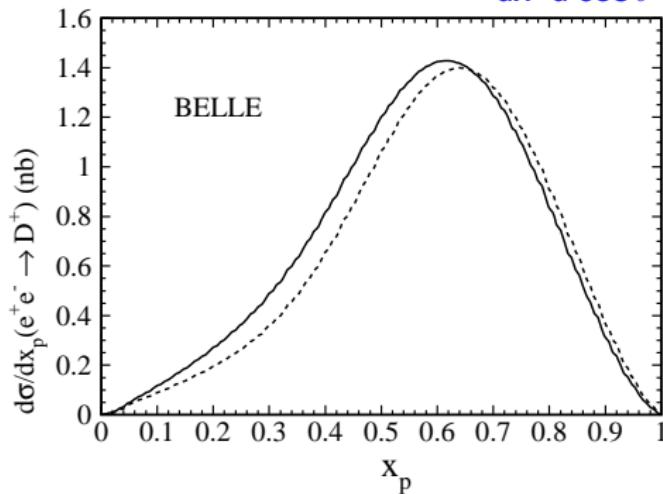
[1] Kneesch,B.K.,Kramer,Schienbein, NPB799(2008)34

[2] Baier, Fey, ZPC2(1979)339; Altarelli et al. NPB160(1979)301

[3] Nason, Webber, NPB421(1994)473

- Use radiator  $D_{e^\pm}$  [1]

$$\frac{d\sigma_{\text{ISR}}}{dx}(x, s) = \int dx_+ dx_- dx' d\cos\theta' \delta(x - x(x_+, x_-, x', \cos\theta')) \\ \times D_{e^+}(x_+, s) D_{e^-}(x_-, s) \frac{d^2\sigma}{dx' d\cos\theta'}(x', \cos\theta', x_+ x_- s)$$



[1] Kuraev, Fadin, SJNP41(1985)466; Nicrosini, Trentadue, PLB196(1987)551

- Experimental data

Type	$\sqrt{s}$ [GeV]	H	Collaboration
$d\sigma/dx_p$	10.52	$D^0, D^+, D^{*+}$	Belle 06
$d\sigma/dx_p$	10.52	$D^0, D^+, D^{*+}$	CLEO 04
$(1/\sigma_{\text{tot}})d\sigma/dx$	91.2	$D^{*+}$	ALEPH 00
$(1/\sigma_{\text{tot}})d\sigma/dx$	91.2	$D^0, D^+, D^{*+}$	OPAL 96,98

- Theoretical input

- $m_c = 1.5$  GeV,  $m_b = 5.0$  GeV,  $\alpha(m_T) = 1/132$ ,  
 $\alpha_s(m_Z) = 0.1176 \rightsquigarrow \Lambda_{\text{QCD}}^{(5)} = 221$  MeV
- Bowler ansatz [1]

$$D_Q^{H_c}(z, \mu_0) = N z^{-(1+\gamma^2)} (1-z)^a e^{-\gamma^2/z}$$

---

[1] Bowler, ZPC11(1981)169

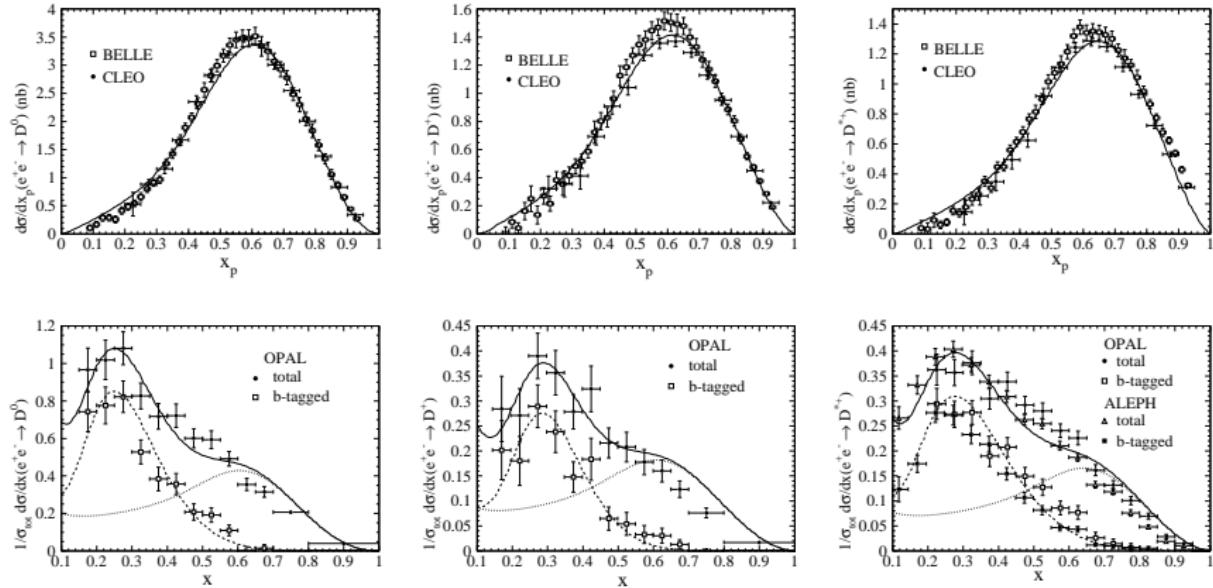
## RESULTS: GOODNESS

- $\chi^2/\text{d.o.f.}$

$H$	VFNS	Belle/CLEO	ALEPH/OPAL	Global
$D^0$	GM	3.15	0.794	4.03
	ZM	3.25	0.789	4.66
$D^+$	GM	1.30	0.509	1.99
	ZM	1.37	0.507	2.21
$D^{*+}$	GM	3.74	2.06	6.90
	ZM	3.69	2.04	7.64

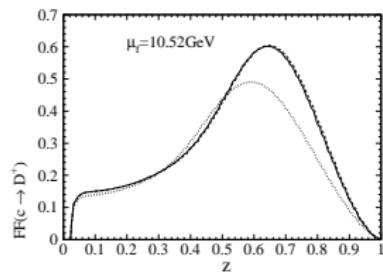
- Quark mass effects improve global fits and Belle/CLEO fits for  $D^0, D^+$ , but have no impact on ALEPH/OPAL fits.
- Belle and CLEO data on  $D^0, D^{*+}$  moderately compatible.
- OPAL fits for  $D^0, D^+$  excellent; ALEPH and OPAL data on  $D^{*+}$  moderately compatible.
- Tension between Belle/CLEO and ALEPH/OPAL data.

## RESULTS: GLOBAL FITS

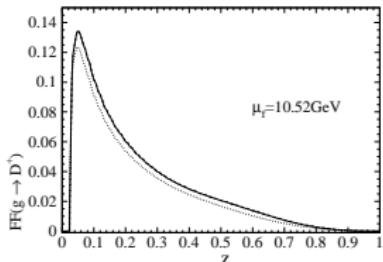


- Belle/CLEO data push  $\langle z \rangle_c(m_Z)$  up by 0.03–0.04

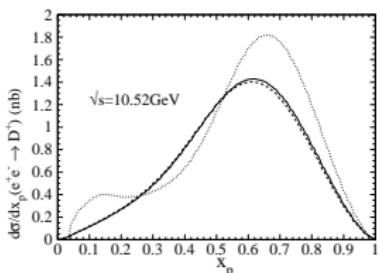
## RESULTS: QUARK AND HADRON MASS EFFECTS



$c \rightarrow D^+$  FF



$g \rightarrow D^+$  FF



$d\sigma/dx_p$  w/ Belle/CLEO-GM FFs

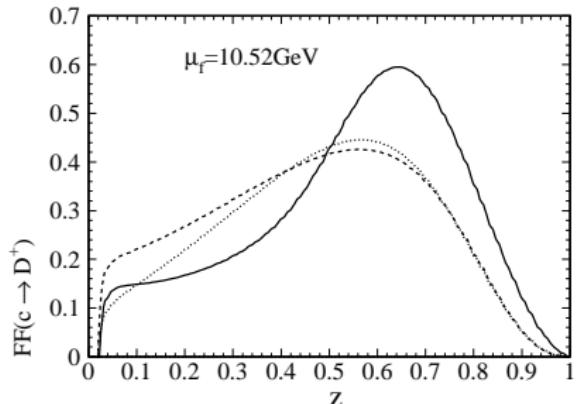
dotted:  $m_c = m_H = 0$

dashed:  $m_c = 0 \neq m_H$  (ZM-VFNS)

solid:  $m_c \neq 0 \neq m_H$  (GM-VFNS)

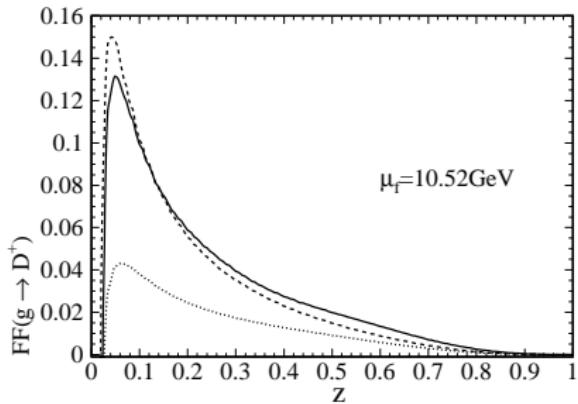
- Hadron mass effects on FFs important, quark mass effects marginal

## RESULTS: COMPARISONS W/ PREVIOUS FFs



$c \rightarrow D^+$  FF

- dotted:  $m_c = 0 = m_H$     $\mu_0 = 2m_c$
- dashed:  $m_c = 0 = m_H$     $\mu_0 = m_c$
- solid:       $m_c \neq 0 \neq m_H$     $\mu_0 = m_c$



$g \rightarrow D^+$  FF

- |          |                 |          |
|----------|-----------------|----------|
| Peterson | OPAL            | KK 05    |
| Peterson | OPAL            | KK 06    |
| Bowler   | Belle,CLEO,OPAL | KKKSc 08 |

- Strong pull of Belle/CLEO data on  $c \rightarrow D^+$  FF
- Reduction in  $\mu_0$  increases  $g \rightarrow D^+$  FF

# Applications

Applications available for

- $\gamma + \gamma \rightarrow D^{*\pm} + X$   
direct and resolved contributions
- $\gamma^* + p \rightarrow D^{*\pm} + X$   
photoproduction
- $p + \bar{p} \rightarrow (D^0, D^{*\pm}, D^\pm, D_s^\pm, \Lambda_c^\pm) + X$   
good description of Tevatron data
- $p + \bar{p} \rightarrow B + X$   
works for Tevatron data at large  $p_T$
- work in progress for  $e + p \rightarrow D + X$

EPJC22, EPJC28

EPJC38, arXiv:0902.3166 [EPJC]

PRD71, PRL96, arXiv:0901.4130

PRD77

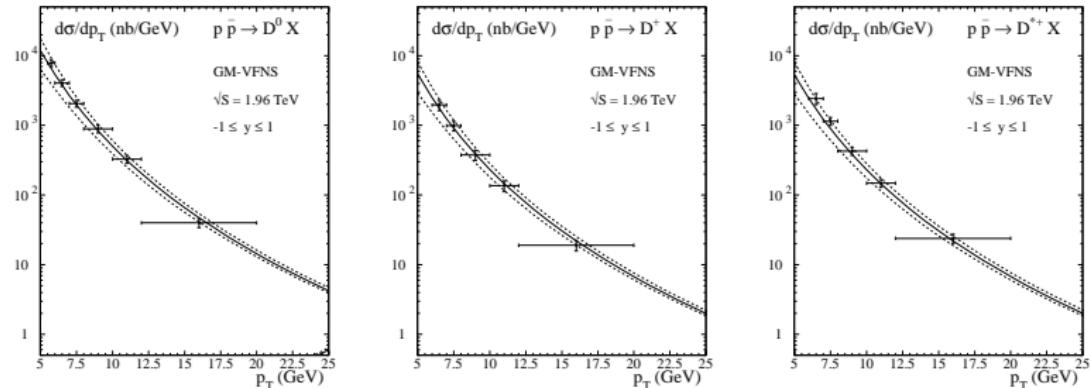
# NUMERICAL RESULTS

## Input parameters:

- $\alpha_s(M_Z) = 0.1181$
- $m_c = 1.5 \text{ GeV}$ ,  $m_b = 5 \text{ GeV}$
- PDFs: CTEQ6M (NLO)
- FFs: NLO FFs from fits to LEP-OPAL data,  
initial scale for evolution:  $\mu_0 = m_c$  ( $D$ -mesons) resp.  $\mu_0 = m_b$  ( $B$ -mesons)
- Default scale choice:  $\mu_R = \mu_F = \mu'_F = m_T$  where  $m_T = \sqrt{p_T^2 + m^2}$

# HADROPRODUCTION OF $D^0$ , $D^+$ , $D^{*+}$ , $D_s^+$

GM-VFNS RESULTS w/ KKKSc FFs [1]



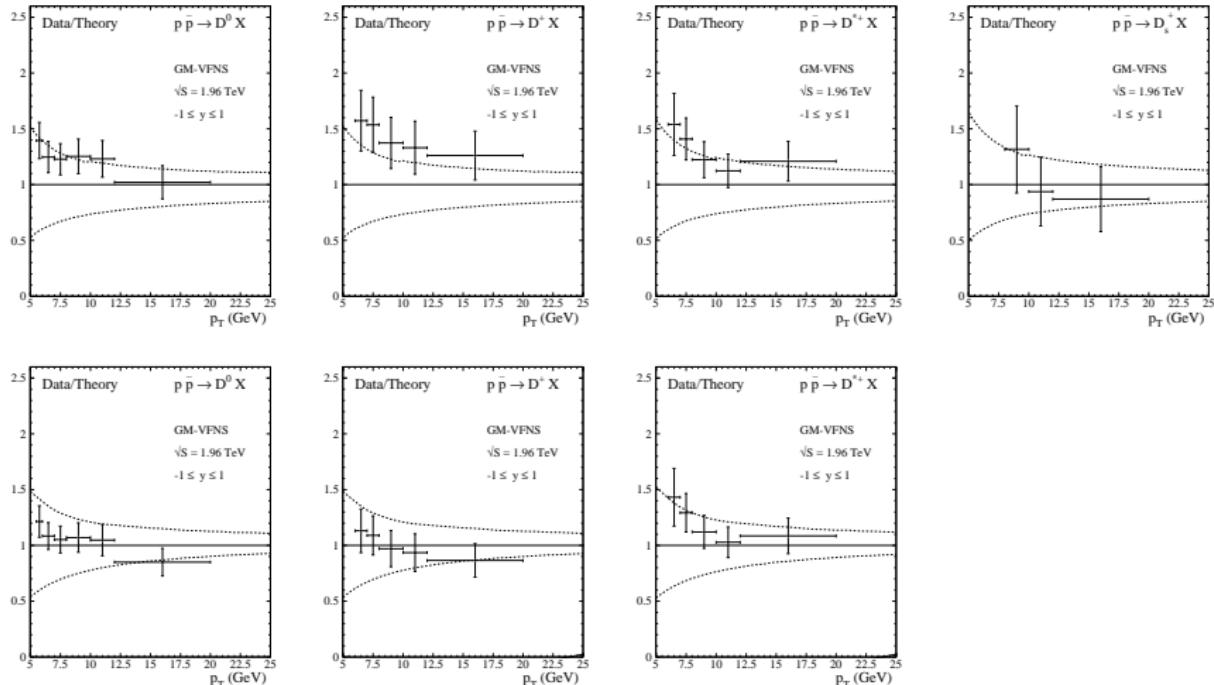
- $d\sigma/dp_T$  [nb/GeV]  $|y| \leq 1$  prompt charm
- Uncertainty band:  $1/2 \leq \mu_R/m_T, \mu_F/m_T \leq 2$  ( $m_T = \sqrt{p_T^2 + m_c^2}$ )
- CDF data from run II [2]
- GM-VFNS describes data within errors

---

[1] Kniehl, Kramer, IS, Spiesberger, arXiv:0901.4130[hep-ph], PRD(to appear)

[2] Acosta et al., PRL91(2003)241804

# COMPARISON W/ PREVIOUS KK FFs [1]



- New KKKSc FFs improve agreement w/ CDF data.

[1] Kniehl,Kramer, PRD74(2006)037502

# INTRINSIC CHARM IN THE PROTON

## GENERALITIES

- $c, \bar{c}, b, \bar{b}$  PDFs relatively weakly constrained by global QCD analyses
- Knowledge important
  - inherently: fundamental structure of the nucleon
  - phenomenologically: significant for physics at Tevatron II and LHC, e.g. charm, bottom, single-top, Higgs production etc.
- Global QCD analyses usually adopt *radiatively generated* HQ PDFs:
  - $f_Q(x, \mu_0) \equiv 0$  at  $\mu_0 = m_Q$  for  $Q = c, b$
  - completely determined by gluon and light-quark d.o.f. via QCD evolution
  - properly resums collinear logs of  $m_Q$  appearing in fixed-order pQCD
  - theoretical: heavy-quark d.o.f. perturbatively calculable
  - practical: lack of clearly identifiable experimental constraints

# INTRINSIC CHARM IN THE PROTON

## GENERALITIES (CONT.)

- Room for additional, genuinely non-perturbative, intrinsic component with  $f_Q(x, \mu_0) \neq 0$
- Especially for charm because  $m_c \gtrsim m_p \rightsquigarrow$  *intrinsic charm (IC)*
- Constrain/determine IC through general global analysis with  $m_Q \neq 0$  and comprehensive experimental inputs, such as extension of CTEQ6.5 [1]
- Consider 3 representative IC models: BHPS, meson-cloud, sealike
- Open charm hadroproduction as a laboratory to probe IC [2]

---

[1] Pumplin,Lai,Tung, PRD75(2007)054029

[2] Kniehl,Kramer,IS,Spiesberger, arXiv:0901.4130[hep-ph], PRD(to appear)

# INTRINSIC CHARM IN THE PROTON

## IC MODELS

### ① BHPS model [1]

- Invokes light-cone Fock space picture of nucleon structure
- states with heavy quarks suppressed by off-shell distance  $(p_T^2 + m^2)/x \rightsquigarrow$  large  $x$  preferred
- Predicts  $c(x) = \bar{c}(x)$
- $c(x, \mu_0) = \bar{c}(x, \mu_0) = Ax^2[6x(1+x)\ln x + (1-x)(1+10x+x^2)]$
- $A$  controls magnitude of IC, characterized by

$$\langle x \rangle_{c+\bar{c}} = \int_0^1 dx x[c(x) + \bar{c}(x)]$$

### ② Meson-cloud model [2]

- Another light-cone model  $\rightsquigarrow$  large  $x$  preferred
- IC from virtual  $uudc\bar{c}$  components, e.g.  $\overline{D}^0 \Lambda_c^+$
- Predicts  $c(x) \neq \bar{c}(x)$
- $c(x, \mu_0) \approx Ax^{1.897}(1-x)^{6.095}, \quad \bar{c}(x, \mu_0) \approx \bar{A}x^{2.511}(1-x)^{4.929}$
- $A/\bar{A}$  determined by quark number sum rule

$$\int_0^1 dx x[c(x) - \bar{c}(x)] = 0$$

# INTRINSIC CHARM IN THE PROTON

## IC MODELS (CONT.)

### ③ Sealike model [3]

- Purely phenomenological scenario
- Assume  $c(x, \mu_0) = \bar{c}(x, \mu_0) \propto \bar{u}(x, \mu_0) + \bar{d}(x, \mu_0)$  at  $\mu_0 = m_c$  w/ overall mass suppression
- IC interchangeable w/ light sea-quark components  $\rightsquigarrow$  softer  $x$  spectrum

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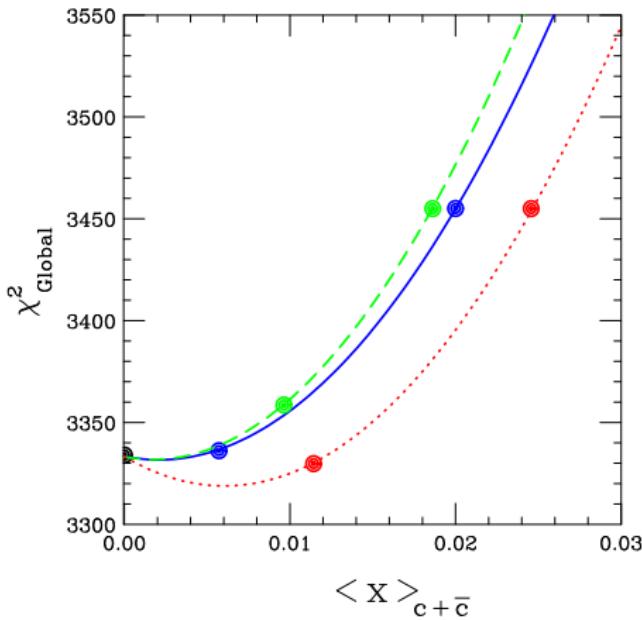
[1] Brodsky, Hoyer, Peterson, Sakai, PLB93(1980)451

[2] Navarra et al., PRD54(1996)842; Melnitchouk, Thomas, PLB414(1997)134

[3] Pumplin, Lai, Tung, PRD75(2007)054029

# INTRINSIC CHARM IN THE PROTON

## IC FROM QTEQ6.5 GLOBAL ANALYSIS [1]



CTEQ.6.5Cn

$n$	IC model	$\langle x \rangle_{c + \bar{c}}$
0	Zero IC	0
1	BHPS	0.57%
2		2.0%
3	Meson-cloud	0.96%
4		1.8%
5	Sealike	1.1%
6		2.4%

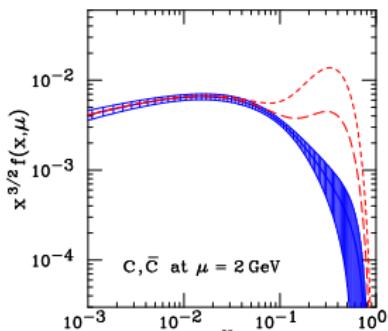
[1] Pumplin, Lai, Tung, PRD75(2007)054029

# INTRINSIC CHARM IN THE PROTON

## IC FROM QTEQ6.5 GLOBAL ANALYSIS (CONT.)

After evolution from  $\mu_0 = 1.3 \text{ GeV}$  to  $\mu = 2 \text{ GeV}$ .

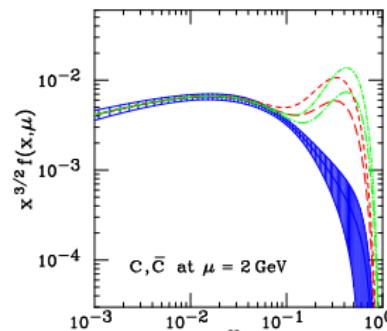
BHPS



$n = 0$  blue

$n = 1(2)$  down (up)

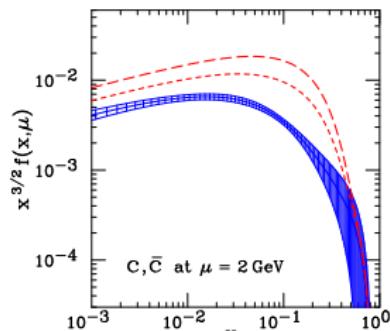
Meson-cloud



$c(\bar{c})$  red (green)

$n = 3(4)$  down (up)

Sealike

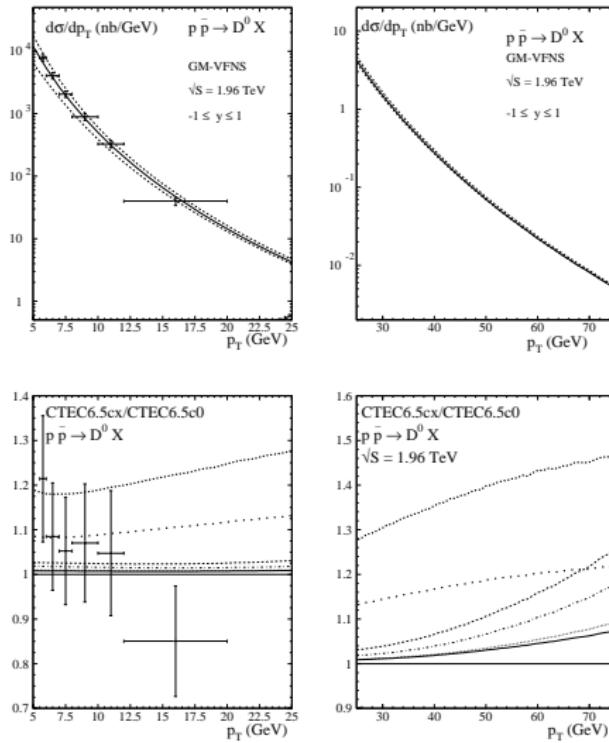


$n = 5(6)$  down (up)

Enhancements get washed out as  $\mu$  increases.

# INTRINSIC CHARM IN THE PROTON

## $D$ -MESONS AT THE TEVATRON

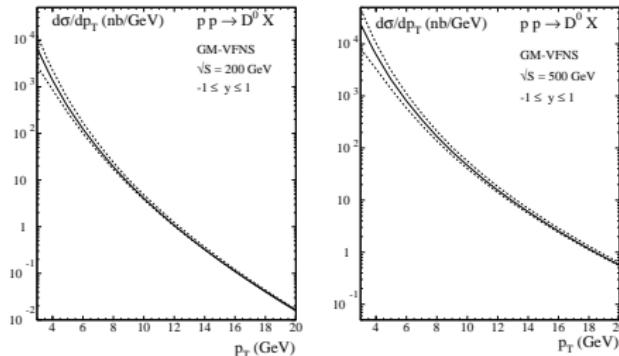


Acosta et al. (CDF Collaboration),  
PRL91(2003)241804  
 $(d\sigma/dp_T)(p\bar{p} \rightarrow D^0 + X)$   
 $\sqrt{s} = 1.96$  TeV  
 $|y| < 1$

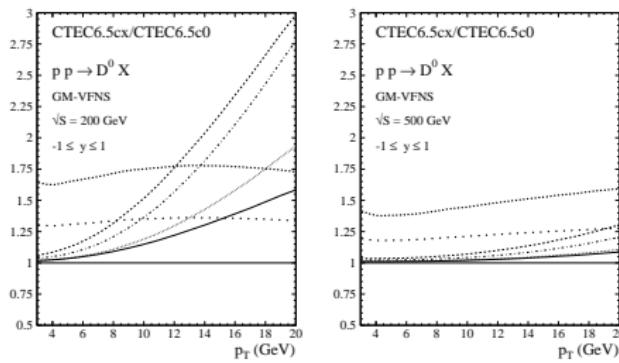
IC Model	moderate	marginal
BHPS	solid	dashed
Meson-cloud	densely dotted	dot-dashed
Sealike	scarsely dotted	dotted

# INTRINSIC CHARM IN THE PROTON

## D-MESONS AT RHIC



$(d\sigma/dp_T)(pp \rightarrow D^0 + X)$   
 $\sqrt{s} = 200, 500$  GeV  
 $|y| < 1$



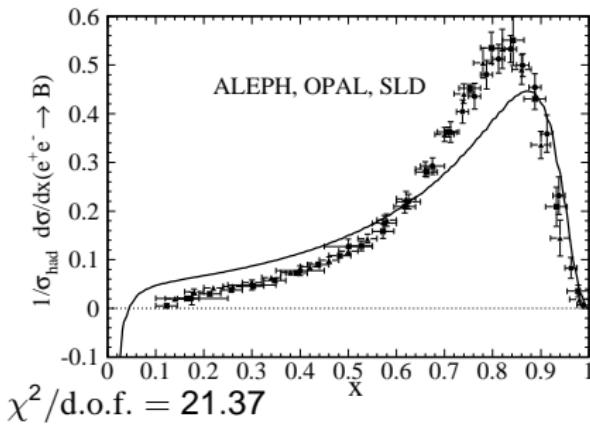
IC Model	moderate	marginal
BHPS	solid	dashed
Meson-cloud	densely dotted	dot-dashed
Sealike	scarcely dotted	dotted

# HADROPRODUCTION OF $B^0, B^+$ [1]

NEW FFs FROM LEP1/SLC DATA [2]

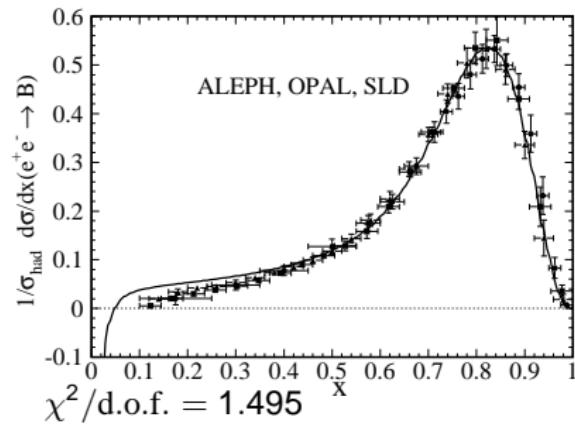
Petersen

$$D(x, \mu_0^2) = N \frac{x(1-x)^2}{[(1-x)^2 + \epsilon x]^2}$$



Kartvelishvili-Likhoded

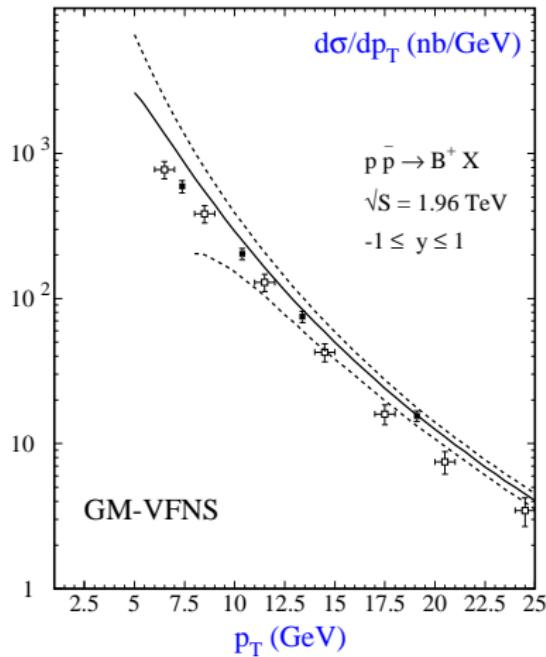
$$D(x, \mu_0^2) = Nx^\alpha(1-x)^\beta$$



[1] Kniehl,Kramer,IS,Spiesberger,PRD77(2008)014011

[2] ALEPH, PLB512(2001)30; OPAL, EPJC29(2003)463; SLD, PRL84(2000)4300; PRD65(2002)092006

## GM-VFNS PREDICTION VS. CDF II [1,2]

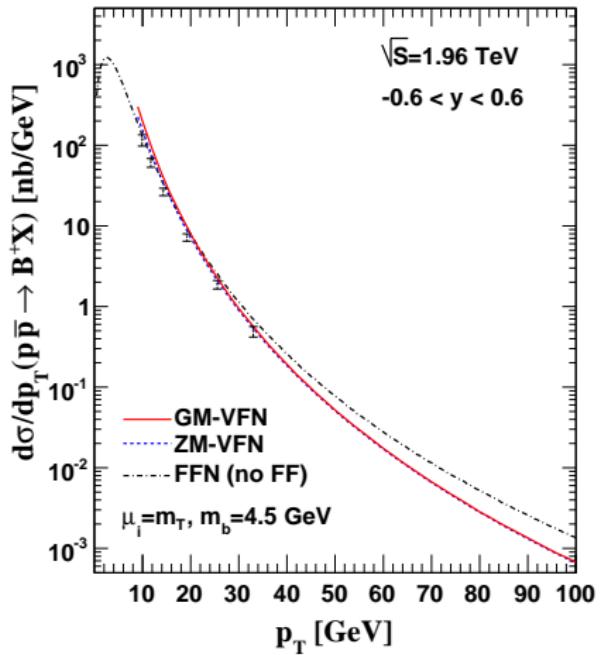


- CDF (1.96 TeV):
  - open squares  $J/\psi X$  [1]
  - solid squares  $J/\psi K^+$  [2]
- CTEQ6.1M PDFs
- $m_b = 4.5 \text{ GeV}$
- $\Lambda_{\overline{\text{MS}}}^{(5)} = 227 \text{ MeV} \rightsquigarrow \alpha_s^{(5)} = 0.1181$
- $1/2 \leq \mu_R/m_T, \mu_F/m_T, \mu_R/\mu_F \leq 2$   
 $(m_T = \sqrt{p_T^2 + m_b^2})$

[1] CDF, PRD71(2005)032001

[2] CDF, PRD75(2007)012010

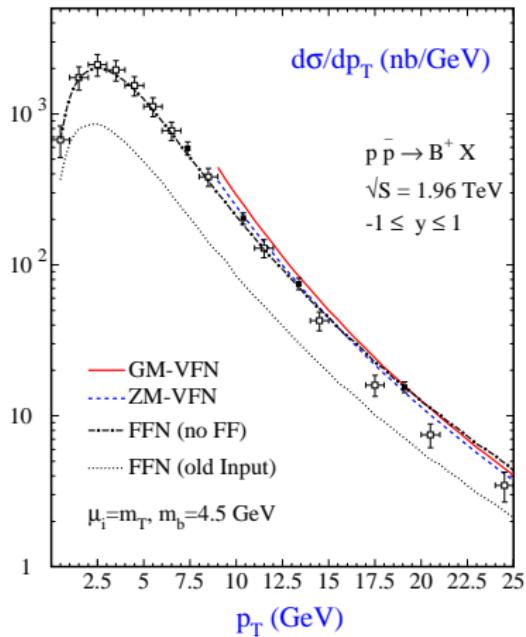
## GM-VFNS PREDICTION VS. CDF II [1]



- CDF II (preliminary) [1]
- $\mu_R = \mu_F = m_T$
- for  $p_T \gg m_b$ :
  - GM-VFN merges w/ ZM-VFN
  - FFN breaks down
- data point in bin [29,40] favors GM-VFN

[1] Kraus, FERMILAB-THESIS-2006-47; Annovi, FERMILAB-CONF-07-509-E

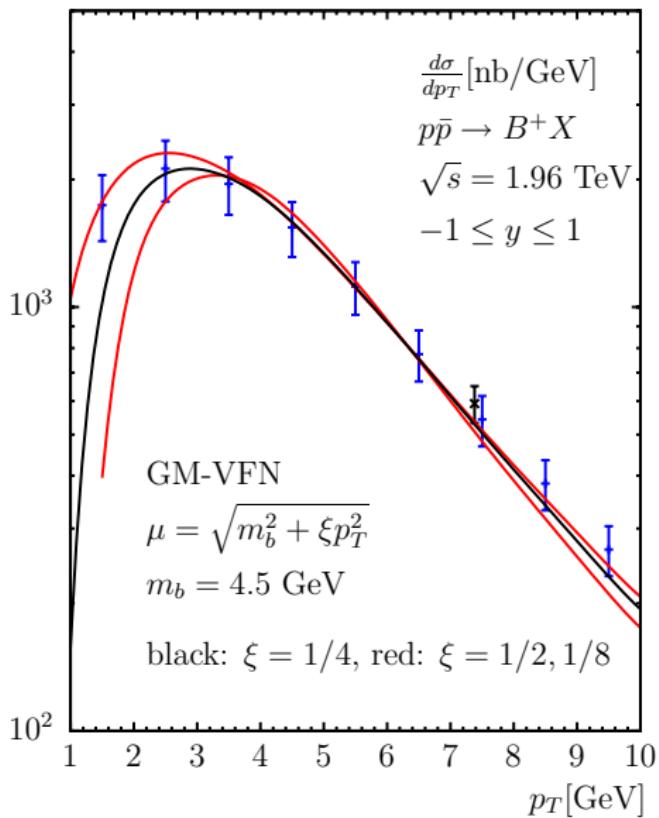
## FFN vs. CDF II [1]



- obsolete FFN as above
- up-to-date FFN evaluated with
  - CTEQ6.1M PDFs
  - $m_b = 4.5$  GeV
  - $\Lambda_{\overline{\text{MS}}}^{(5)} = 227$  MeV  $\leadsto \alpha_s^{(5)} = 0.1181$
  - $D(x) = B(b \rightarrow B)\delta(1-x)$  with  $B(b \rightarrow B) = 39.8\%$

[1] Kniehl,Kramer,IS,Spiesberger,PRD77(2008)014011

## LOW- $p_T$ IMPROVEMENT OF GM-VFNS [1]



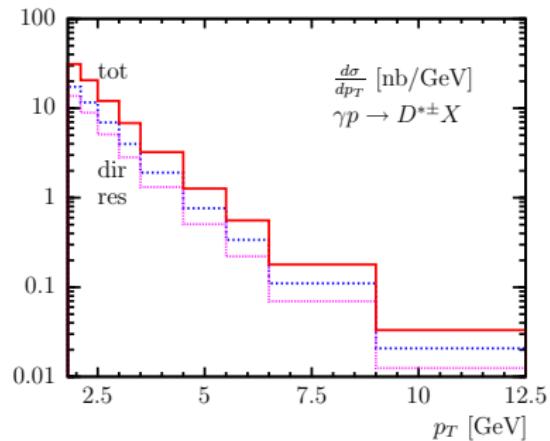
- evaluate  $d\hat{\sigma}_{\text{ZM}}^{(1)}(Q + g/q \rightarrow Q + X)$  @ LO to match  $f_{g \rightarrow Q}^{(1)} \otimes d\hat{\sigma}^{(0)}(Q + g/q \rightarrow Q + g/q)$
- evaluate  $d\hat{\sigma}^{(0)}(gg/q\bar{q} \rightarrow Q\bar{Q}) \otimes d\hat{\sigma}_{Q \rightarrow Q}^{(1)}$  w/  $m_Q \neq 0$  to match  $d\hat{\sigma}_{\text{GM}}^{(1)}(gg/q\bar{q} \rightarrow Q/\bar{Q} + X)$
- impose  $\theta(\hat{s} - 4m_Q^2)$  on massless kinematics
- choose  $\mu_F^2 = m_Q^2 + \xi p_T^2$  so that  $\mu_F \xrightarrow{p_T \rightarrow 0} m_Q = \mu_0$
- $G(m, p_T) \equiv 1$  in contrast to FONLL

[1] Kniehl,Kramer,IS,Spiesberger,in preparation

# NUMERICAL RESULTS

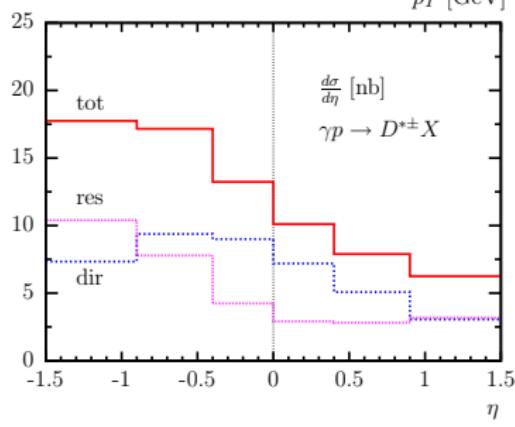
## Exemplify results for

- $p_T, \eta$  distributions
- photoproduction:  $Q^2 \leq 2 \text{ GeV}^2$   
 $1.5 \text{ GeV} \leq p_T \leq 12.5 \text{ GeV}$ ,  $|\eta| \leq 1.5$ ,  $100 \text{ GeV} \leq W_{\gamma p} \leq 285 \text{ GeV}$
- compare with H1 preliminary data: [H1prelim-08-073](#)
- $e^\pm p$  at low  $Q^2$ :  $0.05 < Q^2 < 0.7 \text{ GeV}^2$ ,  
 $1.5 < p_T < 9.0 \text{ GeV}$ ,  $|\eta| < 1.5$ ,  $0.02 < y < 0.85$
- compare with ZEUS data: [PLB649](#)
- charm mass:  $m = 1.5 \text{ GeV}$
- $\alpha_s$  at NLO with  $\Lambda_{N_f=4}^{\overline{\text{MS}}} = 0.328 \text{ GeV}$ , i.e.  $\alpha_s(M_Z^2) = 0.1180$
- independent choice of renormalization and factorization scales:  
 $\mu_i = \xi_i \sqrt{p_T^2 + m^2}$ ,  $i = R, F, F'$ , default:  $\xi_i = 1$
- PDFs: proton: CTEQ6.5, photon: GRV
- fragmentation functions: KKKS 2008

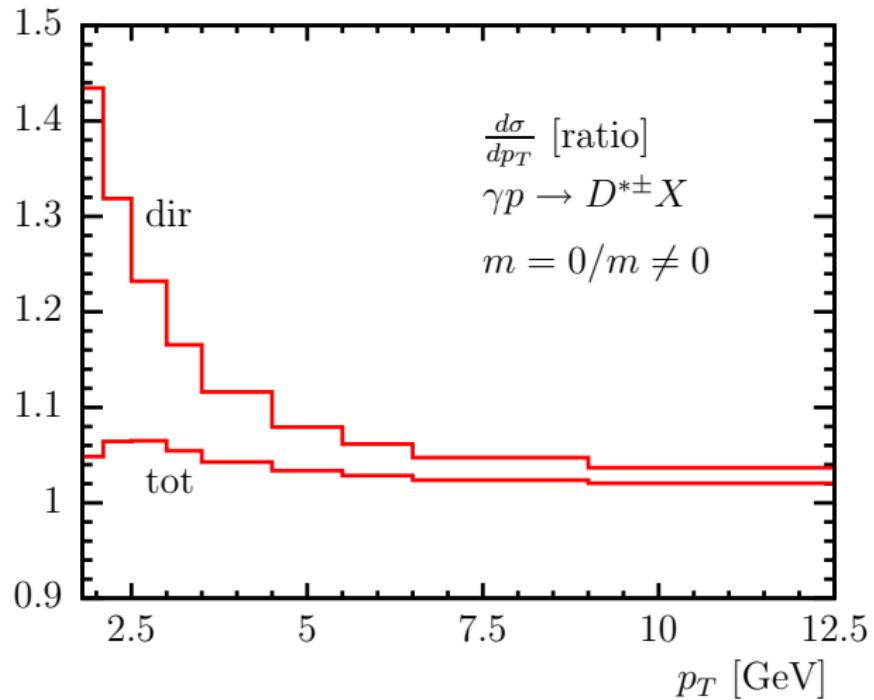


direct and resolved contributions:  
 $p_T$  distribution

resolved part  
dominated by  
charm PDF

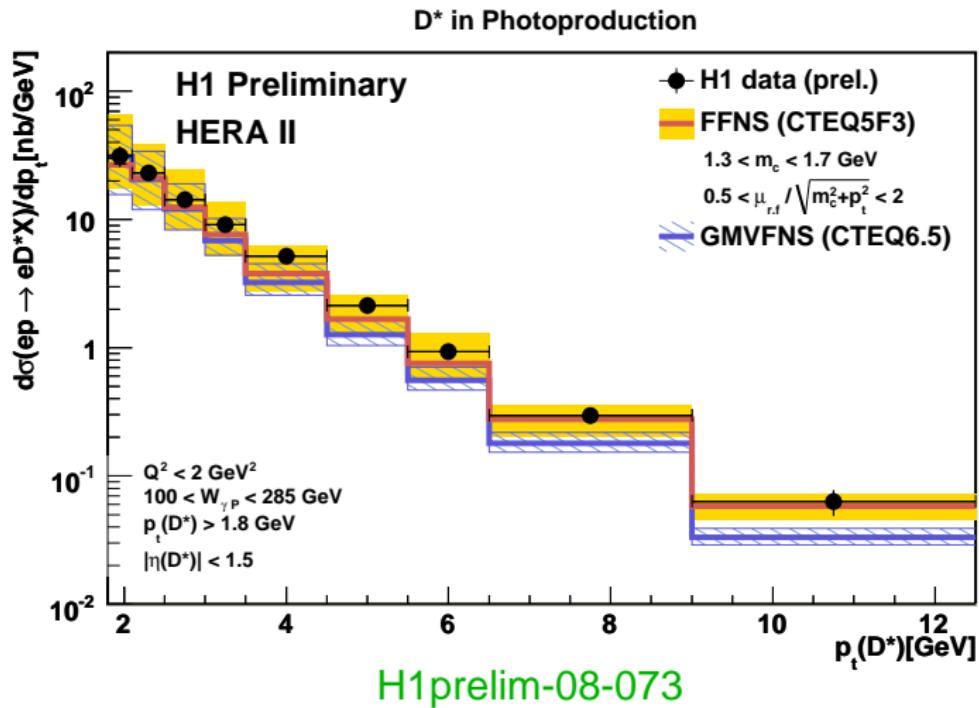


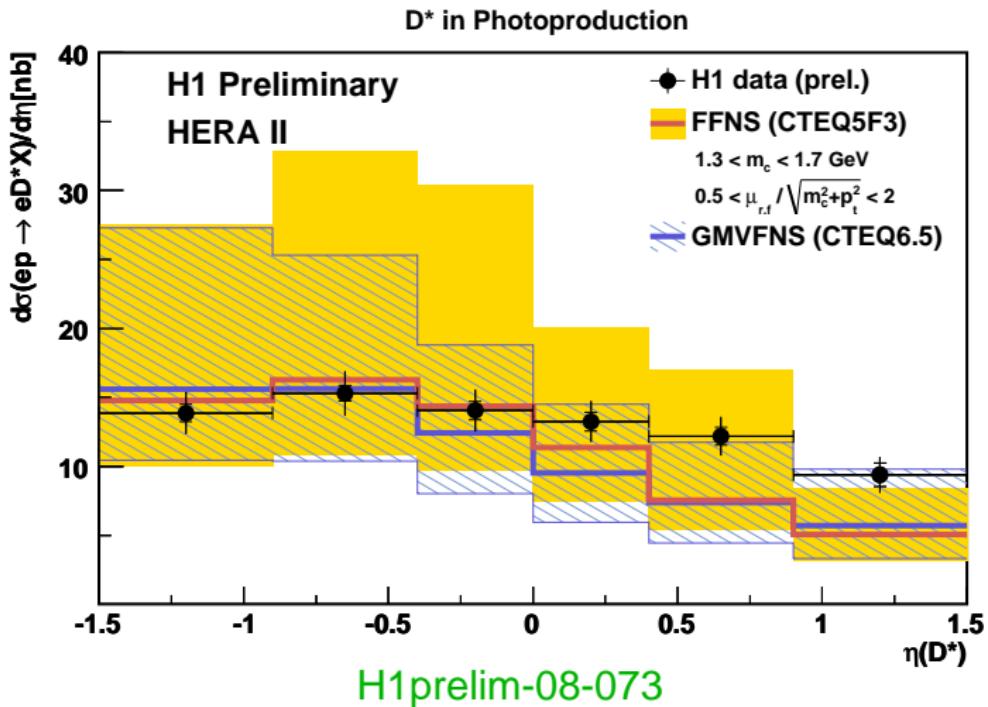
direct and resolved contributions:  
 $\eta$  distribution



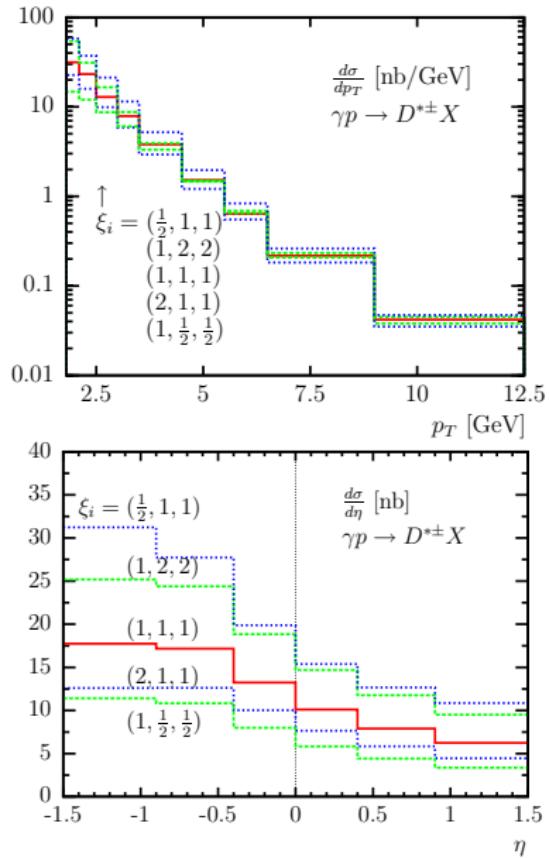
ratio of cross sections:  
 $\sigma(m = 0)/\sigma(m \neq 0)$

mass effects  
suppressed in  $\sigma_{\text{tot}}$





## SCALE DEPENDENCE



$$\mu_i = \xi_i \sqrt{p_T^2 + m^2}$$

for  $i = R, F, F'$

renormalization scale:  $R$

factorization scales:

$F$ : initial state (PDF)

$F'$ : final state (FF)

variation by factor 2 up/down:

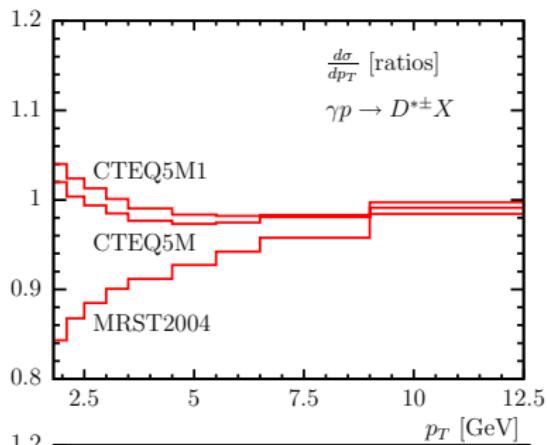
+84 / -53 % at low  $p_T$   
 +13 / -16 % at high  $p_T$

large scale uncertainties

at small  $p_T$

determines scale uncertainty for all  $\eta$

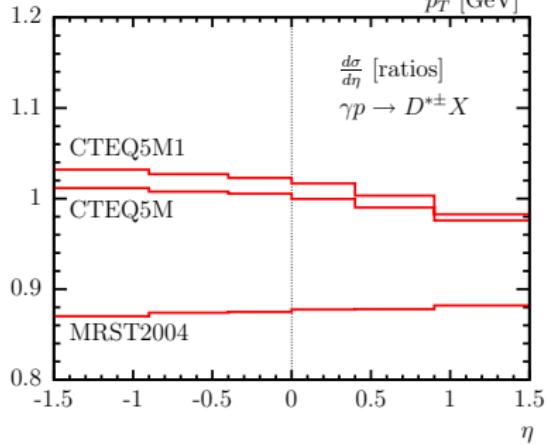
→ improvement: matching to  $N_f = 3$ ,  
 threshold for  $c$ -initiated  
 subprocesses

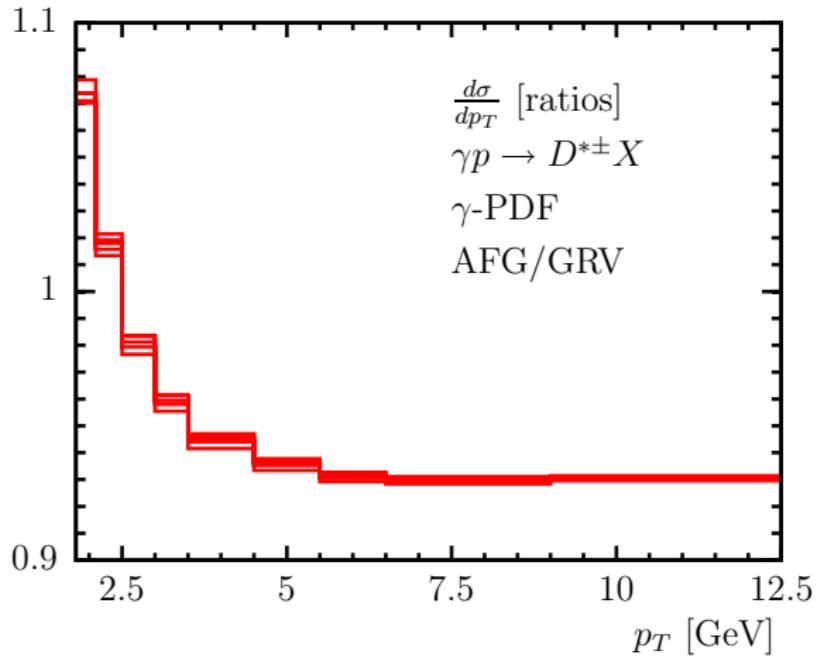


ratio of cross sections  
normalized to  
CTEQ6.5

largest influence  
from varying PDF input  
at small  $p_T$

but small compared to  
scale uncertainty

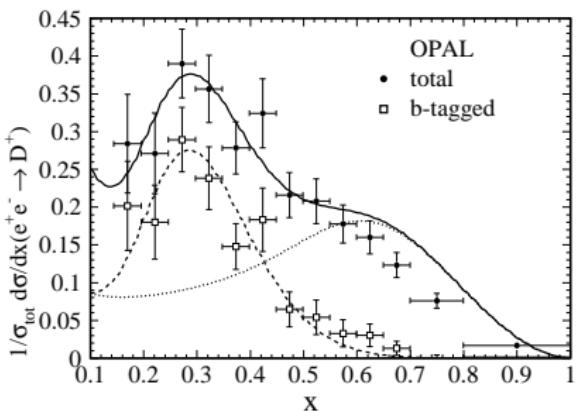
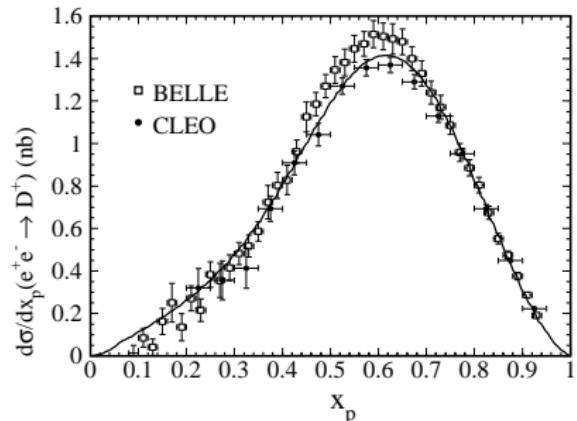




ratio of cross sections

uncertainties  
from  $\gamma$  PDF input  
slightly smaller

default: GRV  
compared with AFG:  
Aurenche, Fontannaz,  
Guillet, EPJC44 (2005)  
5 sets (low/high  $\mu_0^2$ , soft/hard  
non-perturbative gluon)



FF for  $c \rightarrow D^*$   
from fitting to  $e^+e^-$  data

2008 analysis based on GM-VFNS  
 $\mu_0 = m$

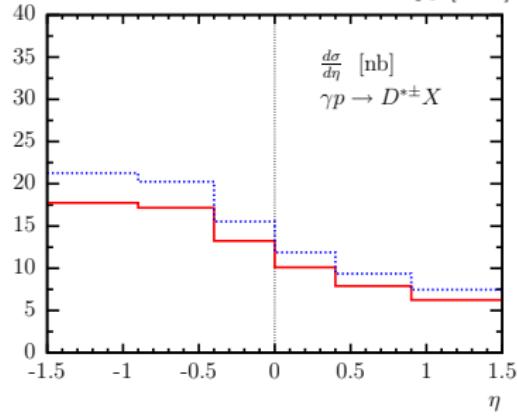
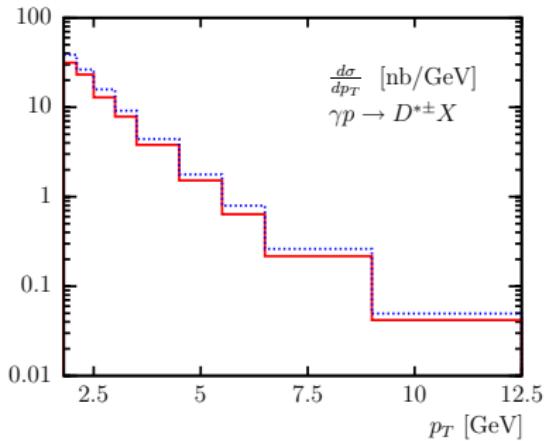
**global fit:** data from  
ALEPH, OPAL, BELLE, CLEO

**BELLE/CLEO fit**

**KKKS: Kneesch, Kramer, Kniehl, IS  
NPB799 (2008)**

tension between low and high energy  
data sets → speculations about non-  
perturbative (power-suppressed) terms

## FF INPUT



uncertainties from

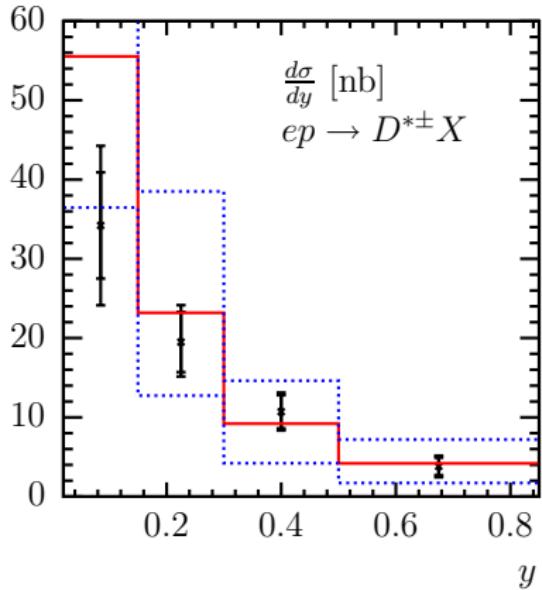
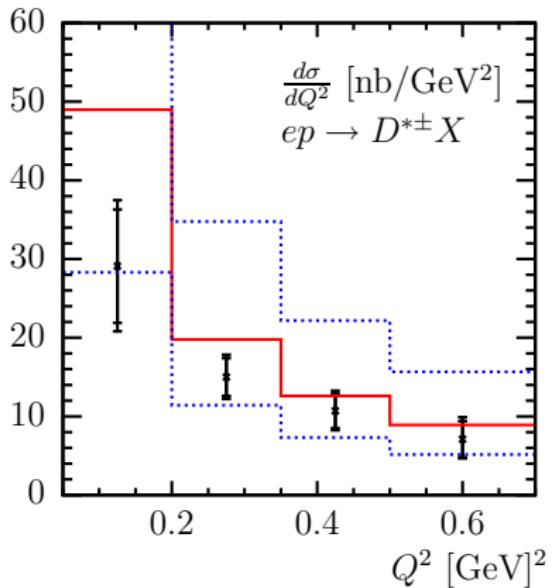
$c \rightarrow D^*$  FF:

**global fit**: data from

ALEPH, OPAL, BELLE, CLEO

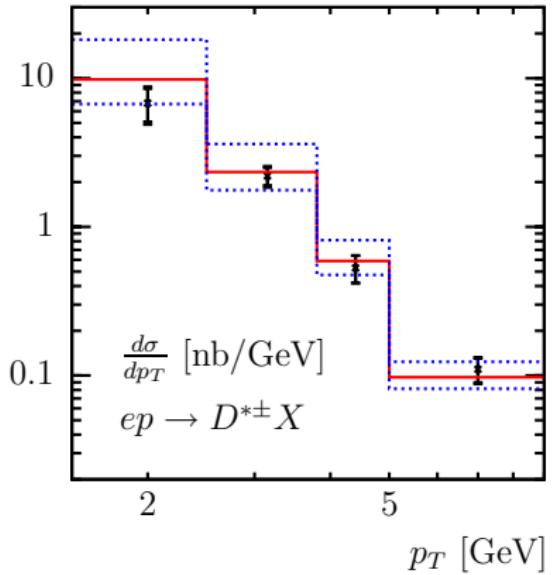
**BELLE/CLEO fit**

Kneesch, Kramer, Kniehl, IS  
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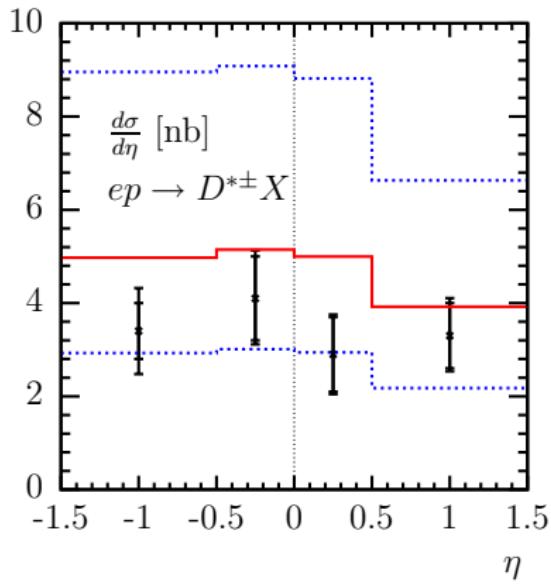
 $0.05 < Q^2 < 0.7 \text{ GeV}^2$ 

ZEUS PLB649

scales at  $\mu_{R,F} = \xi_{R,f} \sqrt{p_T^2 + m^2}$  varied by factor 2 up and down

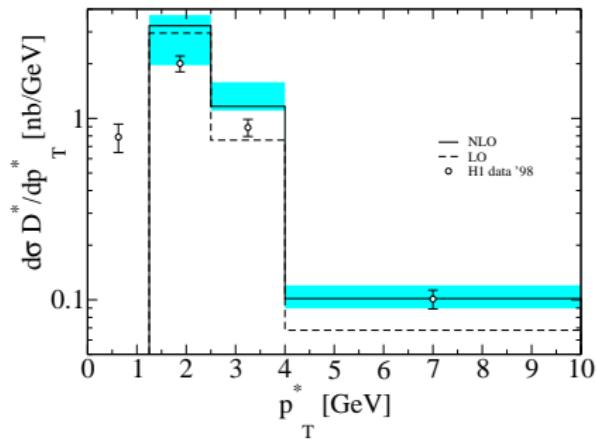
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ZEUS PLB649



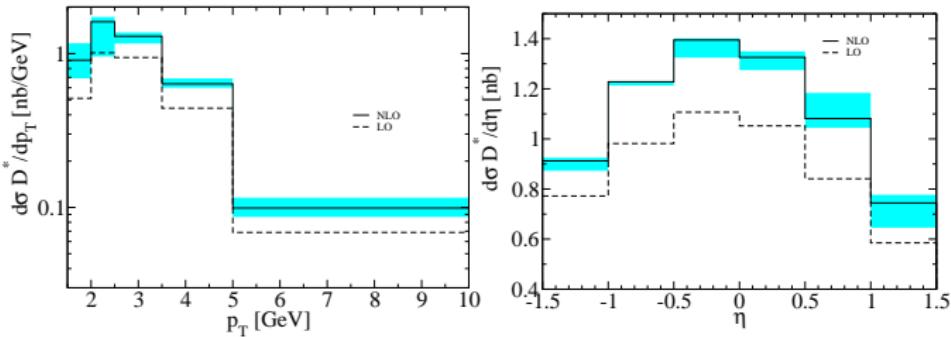
scales at  $\mu_{R,F} = \xi_{R,f} \sqrt{p_T^2 + m^2}$  varied by factor 2 up and down  
uncertainty for all  $\eta$  dominated by smallest  $p_T$

- Phase Space:  $2 < Q^2 < 100 \text{ GeV}^2$ ,  $0.05 < y < 0.7$ ,  
 $1.5 \leq p_{T,\text{Lab}}(D^*) \leq 15 \text{ GeV}$ ,  $|\eta_{\text{Lab}}(D^*)| < 1.5$ ,



- $\mu_R^2 = \mu_F^2 = \mu'_F{}^2 = \xi \frac{Q^2 + (p_T^*)^2}{2}$
- Uncertainty band: variation of  $\xi \in [1/2, 2]$

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► More distributions

- Discussion of 1-particle inclusive production of heavy quarks in a **massive VFNS** (GM-VFNS)
- Available at NLO in the GM-VFNS:
  - $\gamma\gamma \rightarrow HX$
  - $\gamma p \rightarrow HX$
  - $p\bar{p} \rightarrow HX$
- Work in progress:
  - $ep \rightarrow HX$
- Generell expectation:
  - Improvement at  $p_T \gg m_h$  due to **updated FFs** (and PDFs)
  - Mass effects: Improve agreement with (HERA) data for  $p_T \gtrsim m_h$
  - Mass effects: Reduced factorization scale dependence

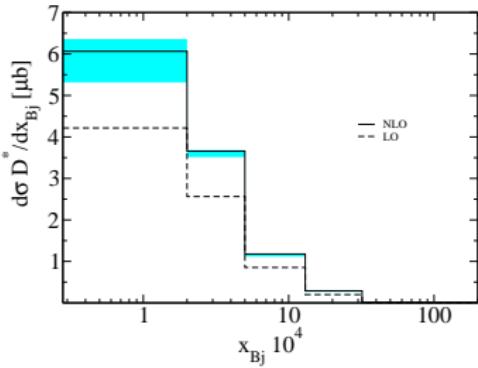
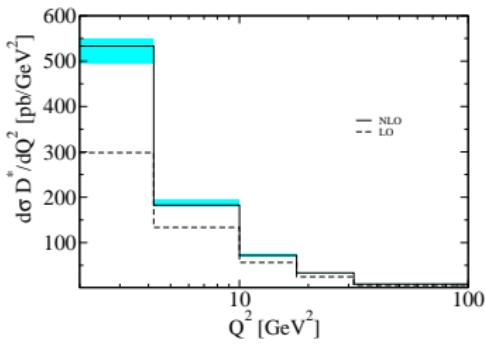
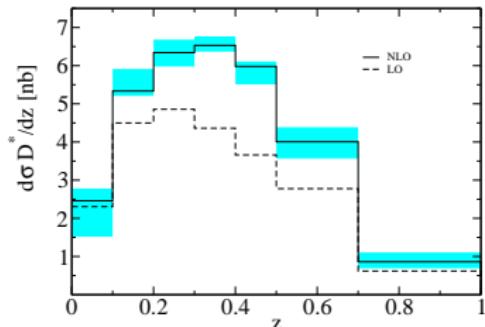
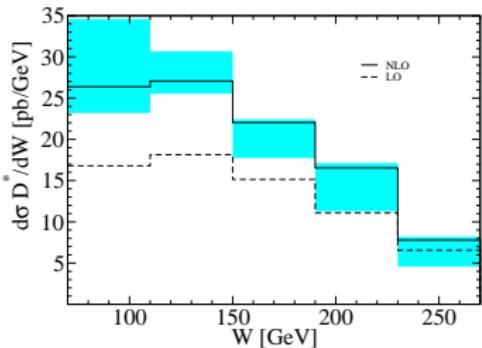
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# Backup Slides

# PREDICTIONS FOR $ep \rightarrow D^{*+}X$ IN THE ZM-VFNS (PROVIDED BY MARKOS MANIATIS)



[◀ go back](#)

$$\text{FONLL} = \text{FO} + (\text{RS} - \text{FOM0})G(m, p_T)$$

**FO:** Fixed Order; **FOM0:** Massless limit of FO; **RS:** Resummed

$$G(m, p_T) = \frac{p_T^2}{p_T^2 + 25m^2}$$

$$\Rightarrow \text{FONLL} = \begin{cases} \text{FO} & : p_T \lesssim 5m \\ \text{RS} & : p_T \gtrsim 5m \end{cases}$$

◀ back to schemes

$\text{FONLL} = \text{FO} + (\text{RS} - \text{FOM0})G(m, p_T)$  with

$$G(m, p_T) = \frac{p_T^2}{p_T^2 + 25m^2}$$

$\text{GM-VFNS} = \text{FO} + (\text{RS} - \text{FOM0})G(m, p_T)$  with

$$\tilde{G}(m, p_T) = 1$$

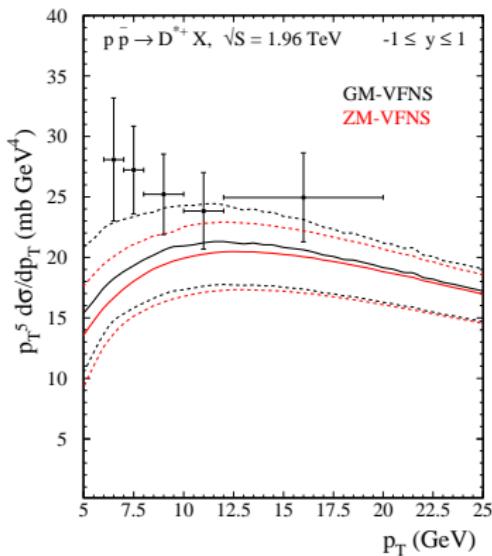
$\text{FO}$ : Fixed Order;  $\text{FOM0}$ : Massless limit of  $\text{FO}$ ;  $\text{RS} \equiv \text{ZM-VFNS}$ : Resummed

- Both approaches interpolate between  $\text{FO}$  and  $\text{ZM-VFNS}$ 
  - $\text{FONLL}$ : obvious;
  - $\text{GM-VFNS}$ : matching with  $\text{FO}$  at quark level (see  
Olness, Scalise, Tung, PRD59(1998)014506)
- Factor  $\tilde{G}(m, p_T)$  follows from calculation;  $\tilde{G}(m, p_T) = 1 \leftrightarrow \text{S-ACOT scheme}$
- Different point-of-view:  $\text{GM-VFNS}$  finally needs PDFs and FFs in this scheme.
- Numerical comparisons interesting and should be done!

# MASS EFFECTS: GM-VFNS vs. ZM-VFNS

$p\bar{p} \rightarrow D^{*+} X$

- Results with old FFs with initial scale  $\mu_0 = 2m_c$
- Uncertainty band: independent variation of  $\mu_R, \mu_F, \mu'_F = \xi m_T, \xi \in [1/2, 2]$



- In this example still  $p_T > 3m_c$
- Mass effects bigger for small  $\mu_R$  (large  $\alpha_s(\mu_R)$ )

# STRONG COUPLING CONSTANT

- PDG'04:  $\alpha_s(M_Z) = 0.1187 \pm 0.0020$
- CTEQ6M PDFs:  $\alpha_s(M_Z) = 0.118$ ; MRST03  $\alpha_s(M_Z) = 0.1165$ ;
- $d\sigma(p\bar{p} \rightarrow DX) \propto \alpha_s^2(1 + \alpha_s K)$

