Heavy quark production in pp at the LHC

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OUTLINE

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Heavy quarks in pQCD
A quark $h$ is heavy $\iff m_h \gg \Lambda_{\text{QCD}} \sim 250$ MeV

- $m_h \gg \Lambda_{\text{QCD}} \Rightarrow \alpha_s(m_h^2) \propto \ln^{-1}\left(\frac{m_h^2}{\Lambda_{\text{QCD}}^2}\right) \ll 1$ (asymptotic freedom)
- $m_h$ sets hard scale; acts as long distance cut-off
- Perturbation theory (pQCD) applicable

charm: $m_c \sim 1.5$ GeV \hspace{1cm} \Lambda_{\text{QCD}}/m_c \sim 0.17 \hspace{1cm} \alpha_s(m_c^2) \sim 0.34$
bottom: $m_b \sim 5$ GeV \hspace{1cm} \Lambda_{\text{QCD}}/m_b \sim 0.05 \hspace{1cm} \alpha_s(m_b^2) \sim 0.21$
top: $m_t \sim 175$ GeV \hspace{1cm} \Lambda_{\text{QCD}}/m_t \sim 0.001 \hspace{1cm} \alpha_s(m_t^2) \sim 0.11$

- The smaller the ratio $\Lambda_{\text{QCD}}/m_h$, the smaller effects of non-perturbative QCD (such as hadronization)
- Top quark decays before it could hadronize due to its large mass ($\Gamma \propto m_t^3$):
  $$\Gamma \simeq \Gamma(t \to bW) \simeq \frac{G_F m_t^3}{8\pi \sqrt{2}} |V_{tb}|^2 \simeq 1.76 \text{ GeV} \left(\frac{m_t}{175 \text{ GeV}}\right)^3$$
Requirements:

1. $\mu \ll m$: Decoupling of heavy degrees of freedom
2. $\mu \gg m$: IR-safety
3. $\mu \sim m$: Correct threshold behavior

Problems:

- Multiple hard scales: $m_c, m_b, m_t, \mu$
- Mass-independent factorization/renormalization schemes like $\overline{\text{MS}}$
- A single $\overline{\text{MS}}$ scheme cannot meet requirements (1) and (3) (is unphysical).

Way out: Patchwork of $\overline{\text{MS}}$ schemes $S_{n_f, n_R}^{n_f, n_R}$

- Variable Flavor-Number Scheme (VFNS): $S_{3,3}^3 \rightarrow S_{4,4}^4 \rightarrow S_{5,5}^5$
- Fixed Flavor-Number Scheme (FFNS): $S_{3,3}^3 \rightarrow S_{3,4}^3 \rightarrow S_{3,5}^3$ (3-FFNS)
- **Masses reintroduced** by backdoor: threshold corrections (=matching conditions)
3-FFNS/Fixed Order:

- No charm PDF! Of course need exp. Input for $u, d, s, g$ PDFs at scale $Q_0^{(3)}$
- finite collinear logs $\ln Q/m_c$ arise → are kept in hard part (unresummed, in fixed order)
- Requirement (3) naturally satisfied
- Not IR-safe, does not meet requirement (2):
  - Not valid for $Q >> m_c$
  - Can we quantify? Valid for $Q < m_c, 3m_c, 5m_c$?
Variable Flavour Number Scheme (VFNS):

- often large ratios of scales involved: multi-scale problems
  For $Q \gg m_c$: write $\ln Q/m_c = \ln \mu/m_c + \ln Q/\mu$, subtract $\ln \mu/m_c$ and resum $\ln \mu/m_c$ by introducing charm PDF at $Q_0^{(4)} \approx m_c$ using a perturbative boundary condition

- $Q < Q_0: n_f = 3$ no charm PDF, $Q \geq Q_0: n_f \rightarrow n_f + 1$, charm PDF without fit parameters

- IR-safe, satisfies requirement (2); resums colliner logarithms

- Problem: original ZM-VFNS (=massless parton model) only valid for $Q >> m$ (Quantify?)

- GM-VFNS: need extra work to satisfy requirement (3) but then valid for all scales $Q$
  approaches FFNS for $Q \sim m$, approaches ZM-VFNS for $Q >> m$
HEAVY FLAVOUR SCHEMES AND PDFS

- GM-VFNS essential for $W, Z$ cross sections at the LHC [see talk by M. Guzzi]

- Most of the most recent global analyses of proton PDFs use a version of a GM-VFNS
  - MSTW08: TR scheme
  - CTEQ6.6/CT10: S-ACOT$_\chi$
  - NNPDF2.1: FONLL
  - HERANPDF1.0: same as MSTW08
  - GJR08, JR09: as GRV in a FFNS
  - CTEQ5, CTEQ6.1, NNPDF2.0,$\ldots$ and older: ZM-VFNS

- The various GM-VFN schemes are ’tuned to’ the DIS structure functions $F_2^c, F_L^c$

IF THESE SCHEME ARE NOT JUST COOKING RECIPES BUT PQCD FORMALISMS WITH HEAVY QUARKS, THEY SHOULD BE APPLICABLE TO OTHER PROCESSES AS WELL
Hado production of heavy quarks: Theory
Factorization for 1-particle inclusive reactions $A + B \rightarrow H + X$

$$A + B \rightarrow H + X: \quad d\sigma = \sum_{i,j,k} f_i^A(x_1) \otimes f_j^B(x_2) \otimes d\sigma(ij \rightarrow kX) \otimes D_k^H(z)$$

sum over all possible subprocesses $i + j \rightarrow k + X$

Parton distribution functions:
\[ f_i^A(x_1, \mu_F), \quad f_j^B(x_2, \mu_F) \]
non-perturbative input
long distance
universal

Hard scattering cross section:
\[ d\sigma(\mu_F, \mu'_F, \alpha_s(\mu_R), [\frac{m_h}{p_T}]) \]
perturbatively computable
short distance
(coefficient functions)

Fragmentation functions:
\[ D_k^H(z, [\mu'_F]) \]
non-perturbative input
long distance
universal

Accuracy:
light hadrons: $O((\Lambda/p_T)^p)$ with $p_T$ hard scale, $\Lambda$ hadronic scale, $p = 1, 2$

heavy hadrons: if $m_h$ is neglected in $d\sigma$: $O((m_h/p_T)^p)$

Details (subprocesses, PDFs, FFs; mass terms) depend on the Heavy Flavour Scheme
FFNS/Fixed Order
Start with FFNS = Fixed Order:

- NLO calculation more than 20 years old, very well tested
- Allows to predict the total cross section
- Allows to compute $p_T$ spectrum if $(p_T \gg m)$
  (up to inclusion of a non-perturbative FF which is very hard)

Compare data with best FFNS prediction!

- Find $p_T$-range where FFNS applicable. Guess: $p_T = 5m$ still ok.
- When need for resummation of $\ln m$ terms visible?
  (Apart from smaller uncertainty band in resummed theory)

As described before: GM-VFNS $\rightarrow$ FFNS for $p_T \sim m$
Leading order subprocesses:

1. $gg \rightarrow Q\bar{Q}$

2. $q\bar{q} \rightarrow Q\bar{Q}$ \((q = u, d, s)\)

- The $gg$-channel is dominant at the LHC \((\sim 85\% \text{ at } \sqrt{S} = 14 \text{ TeV})\).
- The total production cross section for heavy quarks is finite. The minimum virtuality of the t-channel propagator is $m^2$. Sets the scale in $\alpha_s$. Perturbation theory should be reliable.
- Note: For $m^2 \rightarrow 0$ total cross section would diverge.

[See M. Mangano, hep-ph/9711337; Textbook by Ellis, Stirling and Webber]
Next-to-leading order (NLO) subprocesses:

1. $gg \rightarrow Q\bar{Q}g$
2. $q\bar{q} \rightarrow Q\bar{Q}g \quad (q = u, d, s)$
3. $gq \rightarrow Q\bar{Q}q, g\bar{q} \rightarrow Q\bar{Q}\bar{q} \quad [\text{new at NLO}]$
4. Virtual corrections to $gg \rightarrow Q\bar{Q}$ and $q\bar{q} \rightarrow Q\bar{Q}$

NLO corrections for $\sigma_{\text{tot}}$ and differential cross sections $d\sigma/dp_Tdy$ known since long:

- Nason, Dawson, Ellis, NPB303(1988)607; Beenakker, Kuif, van Neerven, Smith, PRD40(1989)54 [$\sigma_{\text{tot}}$]
- NDE, NPB327(1989)49; (E)B335(1990)260; Beenakker et al., NPB351(1991)507 [$d\sigma/dp_Tdy$]

Well tested by recalculation and zero-mass limit:

- Bojak, Stratmann, PRD67(2003)034010 [$d\sigma/dp_Tdy$ (un)polarized]
- Kniehl, Kramer, Spiesberger, IS, PRD71(2005)014018 [$m \rightarrow 0$ limit of diff. x-sec]
- Czakon, Mitov, NPB824(2010)111 [$\sigma_{\text{tot}}$, fully analytic]
HEAVY QUARK HADROPRODUCTION: SOME FIXED ORDER RESULTS

- $d\sigma/dp_T$ for the process $pp \rightarrow B^+ X$; Fragmentation $b \rightarrow B$ via Peterson-FF
- CTEQ6.1 PDFs (slightly inconsistent)
- Prediction in NLO perturbation theory

\[ p p \rightarrow B^+ X, \quad \sqrt{S} = 1.96 \text{ TeV} \]

\[ -1 \leq y \leq 1 \]

FFN; $\epsilon = 0.0001$

$\mu_i = \xi_i m_T$

$m_b = 4.5 \text{ GeV}$

\[ d\sigma/dp_T \text{ (nb/GeV)} \]

\[ p p \rightarrow B^+ X, \quad \sqrt{S} = 14 \text{ TeV} \]

\[ -2.5 \leq y \leq 2.5 \]

FFN; $\epsilon = 0.0001$

$\mu_i = \xi_i m_T$

$m_b = 4.5 \text{ GeV}$

\[ d\sigma/dp_T \text{ (nb/GeV)} \]
\[ p \bar{p} \rightarrow B^+ X, \quad \sqrt{S} = 1.96 \text{ TeV} \]
\[-1 \leq y \leq 1 \]

FFN; \( \mu_i = m_T \)
- \( \varepsilon = 0.001; m_b = 4.5 \)
- \( \varepsilon = 0.001; m_b = 4.75 \)
- \( \varepsilon = 0.0001; m_b = 4.5 \)

CTEQ6AB PDFs
- \( \alpha_s(m_Z) = 0.128 \)
- \( \alpha_s(m_Z) = 0.118 \)
- \( \alpha_s(m_Z) = 0.110 \)
Remarks:

- Fixed order theory in reasonable agreement with Tevatron data up to $p_T \sim 5 m_b$
- At $p_T \lesssim m_b$ factorization less obvious. Depends on definition of convolution variable $z$: $p_B = z p_b$ or $p^B_T = z p^b_T$ or $p^+_B = z p^+_b$ or $\tilde{p}_B = z \tilde{p}_b$
- Less hadronization effects than originally believed:
  - $\epsilon$-parameter small corresponding to a hard fragmentation function. Harder FF $\rightarrow$ harder $p_T$-spectrum
- Larger $\alpha_s(M_Z) \rightarrow$ harder $p_T$-spectrum
- Mass dependence important for $p_T \lesssim m$ (peak) $\rightarrow \sigma_{tot}$
- Only the 4th or 5th Mellin-moment of the FF is relevant for large $p_T$ [M. Mangano]:
  
  $d\sigma^b/dp_T(b) \sim A/p_T(b)^n$ with $n \simeq 4, \ldots, 5$ [see talk by F. Arleo]

$$d\sigma^B/dp_T(B) = \int dz/z D(z) d\sigma^b/dp_T(b)[p_T(b) = p_T(B)/z] = A/p_T(B)^n \times \int dz \, z^{n-1} \, D(z)$$
ZM-VFNS/RS (RS: Resummed)
Next ZM-VFNS/RS which is the baseline for $p_T >> m$

- Again NLO calculation more than 20 years old, very well tested
- Allows to compute $p_T$ spectrum if $p_T >> m$
- Needs scale-dependent FFs for quarks and gluons $D_q^H(z, \mu_F'), D_g^H(z, \mu_F')$
- Same theory used for the computation of inclusive $\pi$ or $K$ production.

Compare data with best ZM-VFNS prediction!

- Find smallest $p_T$ where ZM-VFNS applicable.
- $m/p_T$ terms neglected.
- Is there an overlapping region where both, FFNS and ZM-VFNS are valid?

As said before: GM-VFNS $\rightarrow$ ZM-VFNS for $p_T >> m$
List of Subprocesses: ZM-VFNS

NLO calculation: [Aversa, Chiappetta, Greco, Guillet, NPB327(1989)105]

1. $gg \rightarrow qX$
2. $gg \rightarrow gX$
3. $qg \rightarrow gX$
4. $qg \rightarrow qX$
5. $qg \rightarrow qX$
6. $q\bar{q} \rightarrow qX$
7. $qg \rightarrow gX$
8. $qg \rightarrow gX$
9. $qg \rightarrow gX$
10. $qq \rightarrow gX$
11. $qq \rightarrow qX$
12. $q\bar{q} \rightarrow q'X$
13. $q\bar{q'} \rightarrow gX$
14. $q\bar{q'} \rightarrow qX$
15. $qq' \rightarrow gX$
16. $qq' \rightarrow qX$

⊕ charge conjugated processes
One-particle inclusive production in a GM-VFNS
FONLL = FO+NLL \[1\]

\[ FONLL = FO + (RS - FOM0)G(m, p_T) \]

\[ FO \]: Fixed Order; \[ FOM0 \]: Massless limit of \[ FO \]; \[ RS \]: Resummed

\[ G(m, p_T) = \frac{p_T^2}{p_T^2 + 25m^2} \]

\[ \Rightarrow FONLL = \begin{cases} 
  FO \quad : \quad p_T \lesssim 5m \\
  RS \quad : \quad p_T \gtrsim 5m 
\end{cases} \]

Fragmentation functions:

- \( D^H_i(z, \mu_F^{'}) = D^Q_i \otimes D^H_Q \) where:
  - \( D^Q_i(z, \mu_F^{'}) \): perturbative fragmentation functions of \( i = q, g, Q \) into an on-shell heavy quark \( Q \)
  - \( D^H_Q(z) \): scale-independent, non-perturbative FF describing transition of heavy quark to heavy hadron

- Non-perturbative FF fitted to \( e^+ e^- \rightarrow DX, BX \) data
Applications available for:

- $\gamma^* + p \rightarrow D^{*,0,+,+} + X$
  photoproduction
  [JHEP0103(2001)006]

- $p + \bar{p} \rightarrow (D^0, D^{*\pm}, D^\pm, D_s^\pm, \Lambda_c^\pm) + X$
  good description of Tevatron data
  [JHEP05(1998)007]

- $p + \bar{p} \rightarrow B + X$
  good description of Tevatron data

- $p + p \rightarrow D, B + X$
  good description of RHIC data
  [PRL95(2005)122001]
**Factorization Formula:**

\[
d\sigma(p\bar{p} \rightarrow D^* X) = \sum_{i,j,k} \int dx_1 \, dx_2 \, dz \, f^p_i(x_1) \, f^{\bar{p}}_j(x_2) \times d\hat{\sigma}(ij \rightarrow kX) \, D^*_k(z) + O(\alpha_s^{n+1}, (\frac{\Lambda}{Q})^p)
\]

\[Q:\text{ hard scale, } p = 1, 2\]

- \(d\hat{\sigma}(\mu_F, \mu'_F, \alpha_s(\mu_R), \frac{m_h}{p_T})\): hard scattering cross sections free of long-distance physics \(\rightarrow m_h\) kept
- PDFs \(f^p_i(x_1, \mu_F), f^{\bar{p}}_j(x_2, \mu_F)\): \(i, j = g, q, c\) \([q = u, d, s]\)
- FFs \(D^*_k(z, \mu'_F)\): \(k = g, q, c\)

\(\Rightarrow\) need short distance coefficients including heavy quark masses

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# List of Subprocesses: GM-VFNS

<table>
<thead>
<tr>
<th>Only light lines</th>
<th>Heavy quark initiated ((m_Q = 0))</th>
<th>Mass effects: (m_Q \neq 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong> (gg \rightarrow qX)</td>
<td><strong>1</strong> -</td>
<td><strong>1</strong> (gg \rightarrow QX)</td>
</tr>
<tr>
<td><strong>2</strong> (gg \rightarrow gX)</td>
<td><strong>2</strong> -</td>
<td><strong>2</strong> -</td>
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<td><strong>3</strong> (qg \rightarrow gX)</td>
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<td><strong>5</strong> (q\bar{q} \rightarrow gX)</td>
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<td><strong>6</strong> (q\bar{q} \rightarrow qX)</td>
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<td><strong>7</strong> (qg \rightarrow \bar{q}X)</td>
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<td><strong>10</strong> (qq \rightarrow gX)</td>
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<td><strong>11</strong> (qq \rightarrow qX)</td>
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\(\oplus\) charge conjugated processes
Mass terms contained in the hard scattering coefficients:

\[ d\hat{\sigma}(\mu_F, \mu_{F'}, \alpha_s(\mu_R), \frac{m}{p_T}) \]

Two ways to derive them:

1. Compare massless limit of a massive fixed-order calculation with a massless \( \overline{\text{MS}} \) calculation to determine subtraction terms
   
   [Kniehl, Kramer, IS, Spiesberger, PRD71(2005)014018]

   OR

2. Perform mass factorization using partonic PDFs and FFs
   
   [Kniehl, Kramer, IS, Spiesberger, EPJC41(2005)199]
**Subtraction terms for the GM-VFNS from massless limit**

- Compare limit $m \to 0$ of the massive calculation (Merebashvili et al., Ellis, Nason; Smith, van Neerven; Bojak, Stratmann; ...) with massless $\overline{\text{MS}}$ calculation (Aurenche et al., Aversa et al., ...)

\[
\lim_{m \to 0} d\tilde{\sigma}(m) = d\tilde{\sigma}_{\overline{\text{MS}}} + \Delta d\sigma
\]

$\Rightarrow$ Subtraction terms

\[
d\sigma_{\text{sub}} \equiv \Delta d\sigma = \lim_{m \to 0} d\tilde{\sigma}(m) - d\tilde{\sigma}_{\overline{\text{MS}}}
\]

- Subtract $d\sigma_{\text{sub}}$ from massive partonic cross section while keeping mass terms

\[
d\tilde{\sigma}(m) = d\tilde{\sigma}(m) - d\sigma_{\text{sub}}
\]

$\rightarrow$ **$d\tilde{\sigma}(m)$ short distance coefficient including $m$ dependence**

$\rightarrow$ allows to use PDFs and FFs with $\overline{\text{MS}}$ factorization $\otimes$ massive short distance cross sections

- Treat contributions with charm in the initial state with $m = 0$

- Massless limit: technically non-trivial, map from phase-space slicing to subtraction method
Mass factorization

Subtraction terms are associated to mass singularities: can be described by partonic PDFs and FFs for collinear splittings \( a \to b + X \)

- **initial state:**
  
  \[
  f_{g \to Q}^{(1)}(x, \mu^2) = \frac{\alpha_s(\mu)}{2\pi} P_{g \to q}^{(0)}(x) \ln \frac{\mu^2}{m^2}
  \]
  \[
  f_{Q \to Q}^{(1)}(x, \mu^2) = \frac{\alpha_s(\mu)}{2\pi} C_F \left[ \frac{1+z^2}{1-z} \ln \frac{\mu^2}{m^2} - 2 \ln(1 - z) - 1 \right]_+
  \]
  \[
  f_{g \to g}^{(1)}(x, \mu^2) = -\frac{\alpha_s(\mu)}{2\pi} \frac{1}{3} \ln \frac{\mu^2}{m^2} \delta(1 - x)
  \]

- **final state:**
  
  \[
  d_{g \to Q}^{(1)}(z, \mu^2) = \frac{\alpha_s(\mu)}{2\pi} P_{g \to q}^{(0)}(z) \ln \frac{\mu^2}{m^2}
  \]
  \[
  d_{Q \to Q}^{(1)}(z, \mu^2) = C_F \frac{\alpha_s(\mu)}{2\pi} \left[ \frac{1+z^2}{1-z} \ln \frac{\mu^2}{m^2} - 2 \ln(1 - z) - 1 \right]_+
  \]

- Other partonic distribution functions are zero to order \( \alpha_s \)

[Mele, Nason; Kretzer, Schienbein; Melnikov, Mitov]
(2) Subtraction terms via $\overline{\text{MS}}$ mass factorization: $a(k_1)b(k_2) \to Q(p_1)X$ \[1\]

Sketch of kinematics:

Fig. (a):
$$d\sigma_{\text{sub}}(ab \to QX) = \int_0^1 dx_1 \, f_{a \to i}^{(1)}(x_1, \mu_F^2) \, d\hat{\sigma}^{(0)}(ib \to QX)[x_1 k_1, k_2, p_1]$$
$$\equiv f_{a \to i}^{(1)}(x_1) \otimes d\hat{\sigma}^{(0)}(ib \to QX)$$

Fig. (b):
$$d\sigma_{\text{sub}}(ab \to QX) = \int_0^1 dx_2 \, f_{b \to j}^{(1)}(x_2, \mu_F^2) \, d\hat{\sigma}^{(0)}(aj \to QX)[k_1, x_2 k_2, p_1]$$
$$\equiv f_{b \to j}^{(1)}(x_2) \otimes d\hat{\sigma}^{(0)}(aj \to QX)$$

Fig. (c):
$$d\sigma_{\text{sub}}(ab \to QX) = \int_0^1 dz \, d\hat{\sigma}^{(0)}(ab \to kX)[k_1, k_2, z^{-1} p_1] \, d_{k \to Q}^{(1)}(z, \mu_F^2)$$
$$\equiv d\hat{\sigma}^{(0)}(ab \to kX) \otimes d_{k \to Q}^{(1)}(z)$$

Graphical representation of subtraction terms for $gg \rightarrow Q\bar{Q}g$:

$d\hat{\sigma}^{(0)}(gg \rightarrow Q\bar{Q}) \otimes d'_{Q \rightarrow Q}(z)$:

$d\hat{\sigma}^{(0)}(gg \rightarrow gg) \otimes d'_{g \rightarrow Q}(z)$:

$f^{(1)}_{g \rightarrow Q}(x_1) \otimes d\hat{\sigma}^{(0)}(Qg \rightarrow Qg)$:

$f^{(1)}_{g \rightarrow Q}(x_2) \otimes d\hat{\sigma}^{(0)}(gQ \rightarrow Qg)$:
Graphical representation of subtraction terms for $q\bar{q} \rightarrow Q\bar{Q}g$ and $gq \rightarrow Q\bar{Q}q$

$d\hat{\sigma}^{(0)}(q\bar{q} \rightarrow Q\bar{Q}) \otimes d_{Q \rightarrow Q}^{(1)}(z)$:

$d\hat{\sigma}^{(0)}(q\bar{q} \rightarrow gg) \otimes d_{g \rightarrow Q}^{(1)}(z)$:

$d\hat{\sigma}^{(0)}(gq \rightarrow gq) \otimes d_{g \rightarrow Q}^{(1)}(z)$:

$f_{g \\ Q(x_1)}^{(1)} \otimes d\hat{\sigma}^{(0)}(Qq \rightarrow Qq)$:
Applications
Applications available for

- $\gamma + \gamma \rightarrow D^{*\pm} + X$
  direct and resolved contributions

- $\gamma^* + p \rightarrow D^{*\pm} + X$
  photoproduction

- $p + \bar{p} \rightarrow (D^0, D^{*\pm}, D^{\pm}, D_s^{\pm}, \Lambda_c^{\pm}) + X$
  good description of Tevatron data

- $p + \bar{p} \rightarrow B + X$
  works for Tevatron data at large $p_T$

- work in progress for $e + p \rightarrow D + X$

EPJC22, EPJC28

EPJC38, arXiv:0902.3166 [EPJC]

PRD71, PRL96, arXiv:0901.4130

PRD77
NUMERICAL RESULTS

Input parameters:

- \( \alpha_s(M_Z) = 0.1181 \)
- \( m_c = 1.5 \) GeV, \( m_b = 5 \) GeV
- PDFs: CTEQ6M (NLO)
- FFs: NLO FFs from fits to LEP-OPAL, Belle/CLEO data
  initial scale for evolution: \( \mu_0 = m_c \) (D-mesons) resp. \( \mu_0 = m_b \) (B-mesons)
- Default scale choice: \( \mu_R = \mu_F = \mu_F' = m_T \) where \( m_T = \sqrt{p_T^2 + m^2} \)
FF for $c \rightarrow D^*$ from fitting to $e^+e^-$ data

2008 analysis based on GM-VFNS

$\mu_0 = m$

global fit: data from ALEPH, OPAL, BELLE, CLEO

BELLE/CLEO fit

[KKKS: Kneesch, Kramer, Kniehl, IS NPB799 (2008)]

tension between low and high energy data sets $\rightarrow$ speculations about non-perturbative (power-suppressed) terms
**HADROPRODUCTION OF $D^0, D^+, D^{*+}, D_s^+$**

GM-VFNS results with KKKSc FFs [1]

- $d\sigma/dp_T \ [nb/GeV] \quad |y| \leq 1$ prompt charm
- Uncertainty band: $1/2 \leq \mu_R/m_T, \mu_F/m_T \leq 2 \quad (m_T = \sqrt{p_T^2 + m_c^2})$
- CDF data from run II [2]
- GM-VFNS describes data within errors

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• New KKKSc FFs improve agreement w/ CDF data.

**Hadroproduction of** $B^0$, $B^+$ [1]

**New FFs from LEP1/SLC data** [2]

Petersen

$$D(x, \mu_0^2) = N \frac{x(1-x)^2}{[(1-x)^2 + \epsilon x]^2}$$

Kartvelishvili-Likhoded

$$D(x, \mu_0^2) = N x^\alpha (1-x)^\beta$$

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PRD65(2002)092006
CDF (1.96 TeV):
- open squares $J/\psi X$ [1]
- solid squares $J/\psi K^+$ [2]

CTEQ6.1M PDFs
- $m_b = 4.5$ GeV
- $\Lambda^{(5)}_{\text{MS}} = 227$ MeV $\sim \alpha_s^{(5)} = 0.1181$
- $1/2 \leq \mu_R/m_T, \mu_F/m_T, \mu_R/\mu_F \leq 2$
  
  \[
  m_T = \sqrt{p_T^2 + m_b^2}
  \]

GM-VFNS PREDICTION VS. CDF II [1]

CDF II (preliminary) [1]
- $\mu_R = \mu_F = m_T$
- for $p_T \gg m_b$:
  - GM-VFN merges w/ ZM-VFN
  - FFN breaks down
- data point in bin [29,40] favors GM-VFN

 obsolete FFN as above

 up-to-date FFN evaluated with

 - CTEQ6.1M PDFs
 - \( m_b = 4.5 \text{ GeV} \)
 - \( \Lambda_{\overline{\text{MS}}}^{(5)} = 227 \text{ MeV} \sim \alpha_s^{(5)} = 0.1181 \)
 - \( D(x) = B(b \rightarrow B)\delta(1-x) \) with \( B(b \rightarrow B) = 39.8\% \)

Predictions for the LHC
GM-VFNS predictions for $D^0$, $D^{*\pm}$, $D^{\pm}$ production at ALICE

- $pp$ collisions, $\sqrt{S} = 7$ TeV
- CTEQ6.5 PDF, KKKSc FF, $m_c = 1.5$ GeV
- Results for $D^0 + \bar{D}^0$, $D^{*+} + D^{*-}$, $D^+ + D^-$
- Error bands: Varying $\mu_R$ by factors 2 up and down (Except for very small $p_T$ this gives maximal variation in the cross section)

- pQCD predictions (FONLL, GM-VFNS) compatible with data
**GM-VFNS predictions for** $D^0$, $D^{*\pm}$, $D^\pm$ **production at ATLAS**

- **$pp$ collisions, $\sqrt{S} = 7$ TeV**
- **CTEQ6.6 PDF, KKKS06 FF, $m_c = 1.5$ GeV**
- **Rapidity bins (top to bottom):**
  - $|\eta| < 0.2$, $0.2 < |\eta| < 0.5$, $0.5 < |\eta| < 0.8$, $0.8 < |\eta| < 1.3$, $1.3 < |\eta| < 2.1$
- **Results for average** $(D^0 + \bar{D}^0)/2$, $(D^{*+} + D^{*-})/2$, $(D^+ + D^-)/2$
GM-VFNS predictions for $D^0$, $D^{*\pm}$, $D^{\pm}$ production at ATLAS

Figures provided by S. Head

- $pp$ collisions, $\sqrt{S} = 7$ TeV
- CTEQ6.6 PDF, KKKS06 FF, $m_c = 1.5$ GeV
- Left figure: $d\sigma/d\eta$ for $3.5 < p_T < 40$
- Right figure: $d\sigma/dp_T$ for $0 < \eta < 2.1$
- Results for sum $D^0 + \bar{D}^0$, $D^{*+} + D^{*-}$, $D^+ + D^-$
- Independent variation of $\mu_R$ and $\mu_F$ by a factor two up and down
LHCb: $D^0$ cross section (talk by P. Urquijo at LPCC, Dec. 2010)

- Prelim. results ($\mathcal{L} = 1.8 \text{ nb}^{-1}$), $D^0 \rightarrow K^- \pi^+$, Data: 12% correlated error not shown

- BAK et al. = GM-VFNS: B. Kniehl, G. Kramer, I. Schienbein, H. Spiesberger
- MC et al. = FONLL: M. Cacciari, S. Frixione, M. Mangano, P. Nason, G. Ridolfi
Prelim. results ($\mathcal{L} = 1.8\, \text{nb}^{-1}$), $D^+ \rightarrow K^- \pi^+ \pi^+$, Data: 14% correlated error not shown

- BAK et al. = GM-VFNS: B. Kniehl, G. Kramer, I. Schienbein, H. Spiesberger
- MC et al. = FONLL: M. Cacciari, S. Frixione, M. Mangano, P. Nason, G. Ridolfi
Preliminary ($\mathcal{L} = 1.8 \text{ nb}^{-1}$), $D^{*+} \rightarrow (D^{0} \rightarrow K^{-}\pi^{+})\pi^{+}$, Data: 14% corr. error not shown

- **BAK et al.** = GM-VFNS: B. Kniehl, G. Kramer, I. Schienbein, H. Spiesberger
- **MC et al.** = FONLL: M. Cacciari, S. Frixione, M. Mangano, P. Nason, G. Ridolfi
LHCb: $D_s$ cross section (talk by P. Urquijo at LPCC, Dec. 2010)

- Preliminary ($\mathcal{L} = 1.8 \text{ nb}^{-1}$), $D_s \rightarrow K^- K^+ \pi^+$, Data: 16% corr. error not shown

• BAK et al. = GM-VFNS: B. Kniehl, G. Kramer, I. Schienbein, H. Spiesberger
• MC et al. = FONLL: M. Cacciari, S. Frixione, M. Mangano, P. Nason, G. Ridolfi
- Presented an overview of theoretical approaches to hadroproduction of heavy quarks
- Main message: GM-VFNS predictions in good agreement with first LHC data
- Paper in preparation
  - More predictions (GM-VFNS, FFNS) for $D$ and $B$ mesons
  - Uncertainties
  - Matching to FFNS at small $p_T$
LOW-$p_T$ IMPROVEMENT OF GM-VFNS [1]

\[
\frac{d\sigma}{dp_T} [\text{nb/GeV}]
\]

\[p\bar{p} \rightarrow B^+ X\]

\[\sqrt{s} = 1.96 \text{ TeV}\]

\[-1 \leq y \leq 1\]

- evaluate \(d\hat{\sigma}^{(1)}_{ZM}(Q + g/q \rightarrow Q + X)\)
  @ LO to match \(f^{(1)}_{g\rightarrow Q} \otimes d\hat{\sigma}^{(0)}(Q + g/q \rightarrow Q + g/q)\)

- evaluate \(d\hat{\sigma}^{(0)}(gg/q\bar{q} \rightarrow Q\bar{Q}) \otimes d^{(1)}_{Q\rightarrow Q}\)
  w/ \(m_Q \neq 0\) to match \(d\hat{\sigma}^{(1)}_{GM}(gg/q\bar{q} \rightarrow Q/\bar{Q} + X)\)

- impose \(\theta(\hat{s} - 4m_Q^2)\) on massless kinematics

- choose \(\mu_F^2 = m_Q^2 + \xi p_T^2\) so that \(\mu_F \xrightarrow{p_T \rightarrow 0} m_Q = \mu_0\)

- \(G(m, p_T) \equiv 1\) in contrast to FONLL