Recent progress on nuclear CTEQ PDFs

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based on work in collaboration with

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Outline

1. From protons to nuclei
2. Analysis of neutrino data
3. Analysis of charged lepton DIS and DY data
4. Analysis of νA, ℓA and DY data

From protons to nuclei

From protons to nuclei

Starting point: (CTEQ) global analysis framework for free nucleons

Make sure it can be applied to the case of PDFs for nuclear targets (A, Z)

- Variable: $0 < x_N < A$
- Evolution equations
- Sum rules
- Observables

Apart from the validity of factorization which is (possibly up to precision effects) a working assumption and to be verified phenomenologically

DIS ON NUCLEAR TARGETS

Consider deep inelastic lepton–nucleon collisions: $l(k) + A(p_A) \to l'(k') + X$

Introduce the usual DIS variables: $q \equiv k - k'$, $\quad Q^2 \equiv -q^2$, $\quad x_A \equiv \frac{Q^2}{2p_A \cdot q}$

Hadronic tensor: $W^A_{\mu\nu} \propto \langle A(p_A)|\, J_\mu J^\dagger_\nu \,|A(p_A)\rangle = \sum_i \, a^{(i)}_{\mu\nu} \, \tilde{F}^A_i(x_A, Q^2)$,
where $a^{(i)}_{\mu\nu}$ are Lorentz-tensors composed out of the 4-vectors q and p_A and the metric $g_{\mu\nu}$

Express structure functions in the QCD improved parton model in terms of NPDFs

$$\tilde{\mathcal{F}}^A_k(x_A, Q^2) = \int_{x_A}^1 \frac{\mathrm{d}y_A}{y_A} \, \tilde{f}^A_i(y_A, Q^2) C_{k,i}(x_A/y_A) + \tilde{\mathcal{F}}^{A,\tau\geq 4}_k(x_A, Q^2)$$

NPDFs: Fourier transforms of matrix elements of twist-two operators composed out of the quark and gluon fields:

$$\tilde{f}^A_i(x_A, Q^2) \propto \langle A(p_A)|\, O_i \,|A(p_A)\rangle$$

Definitions of $\tilde{F}^A_i(x_A, Q^2)$, $\tilde{f}^A_i(x_A, Q^2)$, and the varibale $0 < x_A < 1$ carry over one-to-one from the well-known free nucleon case

Evolution equations and sum rules

DGLAP as usual:

$$\frac{\mathrm{d}\tilde{f}_i^A(x_A,Q^2)}{\mathrm{d}\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_{x_A}^1 \frac{\mathrm{d}y_A}{y_A} P_{ij}(y_A)\, \tilde{f}_j^A(x_A/y_A,Q^2)\,,$$

$$= \frac{\alpha_s(Q^2)}{2\pi}\int_{x_A}^1 \frac{\mathrm{d}y_A}{y_A} P_{ij}(x_A/y_A)\, \tilde{f}_j^A(y_A,Q^2)\,,$$

Sum rules:

$$\int_0^1 \mathrm{d}x_A\, \tilde{u}_v^A(x_A,Q^2) = 2Z+N\,,$$

$$\int_0^1 \mathrm{d}x_A\, \tilde{d}_v^A(x_A,Q^2) = Z+2N\,,$$

and the momentum sum rule

$$\int_0^1 \mathrm{d}x_A\, x_A\, \left[\tilde{\Sigma}^A(x_A,Q^2)+\tilde{g}^A(x_A,Q^2)\right] = 1\,,$$

where $N=A-Z$ and $\tilde{\Sigma}^A(x_A)=\sum_i(\tilde{q}_i^A(x_A)+\tilde{\bar{q}}_i^A(x_A))$ is the quark singlet combination

Rescaled definitions

Problem: average momentum fraction carried by a parton $\propto A^{-1}$
since there are 'A-times more partons' which have to share the momentum

- Different nuclei (A, Z) not directly comparable
- Functional form for x-shape would change drastically with A
- Need to rescale!

PDFs are number densities: $\tilde{f}_i^A(x_A)\, \mathrm{d}x_A$ is the number of partons carrying a momentum fraction in the interval $[x_A, x_A + \mathrm{d}x_A]$

Define rescaled NPDFs $f_i^A(x_N)$ with $0 < x_N := Ax_A < A$:

$$\boxed{f_i^A(x_N)\, \mathrm{d}x_N := \tilde{f}_i^A(x_A)\, \mathrm{d}x_A}$$

The variable x_N can be interpreted as parton momentum fraction w.r.t. the **average** nucleon momentum $\bar{p}_N := p_A/A$

RESCALED EVOLUTION EQUATIONS AND SUM RULES

Evolution:

$$\begin{aligned}
\frac{\mathrm{d} f_i^A(x_N, Q^2)}{\mathrm{d} \ln Q^2} &= \frac{\alpha_s(Q^2)}{2\pi} \int_{x_N/A}^{1} \frac{\mathrm{d} y_A}{y_A} \, P(y_A) \, f_i^A(x_N/y_A, Q^2) \, , \\
&= \frac{\alpha_s(Q^2)}{2\pi} \int_{x_N}^{A} \frac{\mathrm{d} y_N}{y_N} \, P(x_N/y_N) \, f_i^A(y_N, Q^2) \, .
\end{aligned}$$

Assume that $f_i^A(x_N) = 0$ for $x_N > 1$, then **original, symmetrical** form recovered:

$$\frac{\mathrm{d} f_i^A(x_N, Q^2)}{\mathrm{d} \ln Q^2} = \begin{cases} \frac{\alpha_s(Q^2)}{2\pi} \int_{x_N}^{1} \frac{\mathrm{d} y_N}{y_N} \, P(y_N) \, f_i^A(x_N/y_N, Q^2) & : 0 < x_N \leq 1 \\ 0 & : 1 < x_N < A, \end{cases}$$

Sum rules for the rescaled PDFs:

$$\begin{aligned}
\int_0^A \mathrm{d} x_N \, u_v^A(x_N) &= 2Z + N \, , \\
\int_0^A \mathrm{d} x_N \, d_v^A(x_N) &= Z + 2N \, ,
\end{aligned}$$

and

$$\int_0^A \mathrm{d} x_N \, x_N \, \left[\Sigma^A(x_N) + g^A(x_N) \right] = A \, ,$$

RESCALED STRUCTURE FUNCTIONS

The rescaled structure functions can be defined as

$$x_N \mathcal{F}_i^A(x_N) := x_A \tilde{\mathcal{F}}_i^A(x_A) \ ,$$

with $\mathcal{F}_{1,2,3}(x) = \{F_1(x), F_2(x)/x, F_3(x)\}$.

More explicitly:

$$\begin{aligned} F_2^A(x_N) &:= \tilde{F}_2^A(x_A) \ , \\ x_N F_1^A(x_N) &:= x_A \tilde{F}_1^A(x_A) \ , \\ x_N F_3^A(x_N) &:= x_A \tilde{F}_3^A(x_A) \ . \end{aligned}$$

This leads to consistent results in the parton model using the rescaled PDFs.

PDFs of bound nucleons

Further decompose the NPDFs $f_i^A(x_N)$ in terms of effective parton densities for **bound** protons, $f_i^{p/A}(x_N)$, and neutrons, $f_i^{n/A}(x_N)$, inside a nucleus A:

$$f_i^A(x_N, Q^2) = Z\, f_i^{p/A}(x_N, Q^2) + N\, f_i^{n/A}(x_N, Q^2)$$

- The bound proton PDFs have the **same** evolution equations and sum rules as the free proton PDFs **provided** we neglect any contributions from the region $x_N > 1$
- Neglecting the region $x_N > 1$, is consistent with the DGLAP evolution
- The region $x_N > 1$ is expected to have a minor influence on the sum rules of less than one or two percent (see also [PRC73(2006)045206])
- Isospin symmetry: $u^{n/A}(x_N) = d^{p/A}(x_N)$, $d^{n/A}(x_N) = u^{p/A}(x_N)$

An observable $\mathcal{O}^A$ is then given by:

$$\mathcal{O}^A = Z\, \mathcal{O}^{p/A} + N\, \mathcal{O}^{n/A}$$

In conclusion: the free proton framework can be used to analyse nuclear data

Analysis of neutrino data

I.S.,Yu,Keppel,Morfin,Olness,Owens,PRD77(2008)054013

Why neutrino DIS?

- **Flavor separation:**
  Neutrino sfs depend on different combinations of PDFs

- **Dimuon production:**
  - Main source of information on the strange sea
  - Large uncertainty on $s(x, Q^2)$ has signifi cant influence on the W and Z benchmark processes at the LHC

- **Data interesting for proton PDF and NPDF**

- **For proton PDF: need nuclear corrections**

- **EW precision measurements:**
  Paschos-Wolfenstein analysis: extraction of $\sin^2 \theta_W$

- **LBL precision neutrino experiments:**
  Need good understanding of neutrino–nucleus cross sections

NEUTRINO DATA SETS

NuTeV (CCFR):

- sign selected beam: νFe and $\bar{\nu} Fe$ cross section data
- very high statistics, about 2000 points
- dimuon data: essential for constraining strange quark PDF

CHORUS:

- sign selected beam: νPb and $\bar{\nu} Pb$ cross section data
- about 800 data points

Other:

- CDHSW: not used, about 800 points
- Nomad: not yet available

Analysis of NuTeV data

- **NuTeV cross section data:**
  - More than 1000 neutrino cross section data
  - More than 1000 anti-neutrino cross section data

- **NuTeV/CCFR dimuon data (172 pts):** fix strange sea

- **Idea:** Analyse iron data only and extract iron PDFs
  - Advantage: No nuclear A-dependence needs to be modeled
  - Disadvantage: Only two observables from one (high-statistics) experiment. Not all PDFs constrained. Need to be careful.

THEORETICAL FRAMEWORK

Framework as in CTEQ6M proton fi t:

- $Q_0 = m_c = 1.3\,\mathrm{GeV}$, $m_b = 4.5\,\mathrm{GeV}$, $\alpha_s^{NLO,\overline{\mathrm{MS}}}(M_Z) = 0.118$
- Heavy quark treatment:
  - $\overline{\mathrm{MS}}$ scheme
  - ACOT scheme (including target mass corrections)
- Cuts:
  - loose: $Q > m_c$ and $W > M_p + m_\pi$
  - **standard CTEQ cuts:** $Q > 2\,\mathrm{GeV}$ and $W > 3.5\,\mathrm{GeV}$
- Functional form for bound proton PDFs inside an iron nucleus:

$$\begin{aligned} x\, f_k(x, Q_0) &= c_0 x^{c_1}(1-x)^{c_2} e^{c_3 x}(1+e^{c_4}x)^{c_5} \\ k &= u_v, d_v, g, \bar{u}+\bar{d}, s, \bar{s}\,, \\ \bar{d}(x, Q_0)/\bar{u}(x, Q_0) &= c_0 x^{c_1}(1-x)^{c_2} + (1+c_3 x)(1-x)^{c_4} \end{aligned}$$

- Reasonable **assumptions:**
  - Gluon NPDF not constrained: Fix gluon to free proton gluon (supported by results of de Florian, Sassot (nDS'04))
  - Assume nuclear corrections to $\bar{d}$ similar to corrections to $\bar{u}$ at moderate and small x
- Perform many 'sample fits' (more than 50)
  - start from different initial conditions
  - iterate these fits including/excluding additional paramters
- Result: band of fits of comparable quality providing approximate measure of constraining power of the data

EXPERIMENTAL INPUT

Consider only standard cuts: $Q > 2\ \mathrm{GeV}$ and $W > 3.5\ \mathrm{GeV}$

- NuteV cross section data (after cuts):
  - νFe: 1170 points
  - $\bar{\nu} Fe$: 966
  - In total: 2136 $(\nu + \bar{\nu})$ points
- Correlated errors
- Radiative corrections (table obtained from NuTeV collab.) applied to data
- with and without isoscalar corrections applied to data

- NuteV/CCFR dimuon data: 174 points

Results

Scheme	Cuts	Data	# points	χ^2	χ^2/pts	Name
ACOT	$Q > 1.3$ GeV	$\nu + \bar{\nu}$	2691	3678	1.37	A
	no W_{cut}	ν	1459	2139	1.47	Aν
		$\bar{\nu}$	1232	1430	1.16	A$\bar{\nu}$
ACOT	$Q > 2$ GeV	$\nu + \bar{\nu}$	**2310**	**3111**	**1.35**	A2
	$W > 3.5$ GeV	ν	**1258**	**1783**	**1.42**	A2ν
		$\bar{\nu}$	**1052**	**1199**	**1.14**	A2$\bar{\nu}$
$\overline{\mathrm{MS}}$	$Q > 1.3$ GeV	$\nu + \bar{\nu}$	2691	3732	1.39	M
	no W_{cut}	ν	1459	2205	1.51	Mν
		$\bar{\nu}$	1232	1419	1.15	M$\bar{\nu}$
$\overline{\mathrm{MS}}$	$Q > 2$ GeV	$\nu + \bar{\nu}$	2310	3080	1.33	M2
	$W > 3.5$ GeV	ν	1258	1817	1.44	M2ν
		$\bar{\nu}$	1052	1201	1.14	M2$\bar{\nu}$

Nuclear correction factors

Be $\mathcal{O}$ an **observable** calculable in the parton model

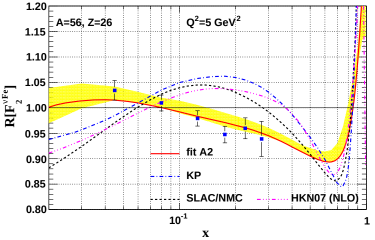
Define **nuclear correction factor**:

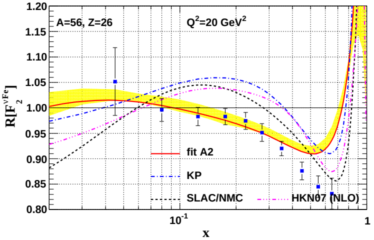
$$R\,[\mathcal{O}] := \frac{\mathcal{O}\,[\text{NPDF}]}{\mathcal{O}\,[\text{PDF}]} \qquad \text{or for data} \qquad R\,[\mathcal{O}] := \frac{\mathcal{O}^{\text{exp}}}{\mathcal{O}\,[\text{PDF}]}$$

- Factor needed to correct data to the free nucleon level
- Note: different observables $\Rightarrow$ different correction factors
- In particular, correction factor for $F_3^{\nu A}$ could be quite different from $F_2^{\nu A}$!
- Also $R[F_2^{\ell A}]$, $R[F_2^{\nu A}]$, $R[F_2^{\bar{\nu} A}]$, $R[d^2\sigma^{\nu A}/dxdQ^2]$, ... are all (more or less) different **even for universal nPDFs**

Note: the term “nuclear effects” is less precise and (mis-)used in the literature for a lot of different things

NUCLEAR CORRECTION FACTOR $R[F_2^{\nu Fe}]$





- Are nuclear corrections in charged-lepton and neutrino DIS different?
- Obviously the PDFs from fi ts to ℓA + DY data do not describe the neutrino DIS data.
- However, a better flavor decomposition could be possible resulting from a global analysis of ℓA, DY and νA data.

Note: $x_{\min} = 0.02$ in these figures.

CONCLUSIONS I

- Nuclear correction factor $R[F_2^{\nu Fe}]$ different from predictions based on charged lepton DIS data
- Global analyses of proton PDFs need nuclear corrections in order to use heavy target data
- Analysis based on two observables from one experiment is delicate! Needs to be validated:
  - Introduce A-dependent parametrization
  - Reproduce results from ℓA DIS + DY data in this framework
  - Perform global analysis of ℓA DIS + DY + νA DIS data

Analysis of charged lepton DIS and DY data

I.S.,Yu,Kovarik,Keppel,Morfin,Olness,Owens,PRD80(2009)094004

Modeling the A dependence

x-dependence of our input distributions always the same:

$$
\begin{aligned}
x\, f_k(x,Q_0) &= c_0 x^{c_1}(1-x)^{c_2} e^{c_3 x}(1+e^{c_4} x)^{c_5} \\
k &= u_v, d_v, g, \bar{u}+\bar{d}, s, \bar{s}\,, \\
\bar{d}(x,Q_0)/\bar{u}(x,Q_0) &= c_0 x^{c_1}(1-x)^{c_2} + (1+c_3 x)(1-x)^{c_4}
\end{aligned}
$$

Introduce A-dependent fi t parameters:

$$
c_k \to c_k(A) \equiv c_{k,0} + c_{k,1}(1 - A^{-c_{k,2}}), \quad k = 1, \ldots, 5
$$

- Note: In the limit $A \to 1$ we have $c_k(A) \to c_{k,0}$
- $c_{k,0}$ are the coeffi cients of the free proton PDF

- DIS F_2^A/F_2^D data sets: 862 points (before cuts)
- DIS $F_2^A/F_2^{A'}$ data sets: 297 points (before cuts)
- DY data sets $\sigma_{\mathrm{DY}}^{pA}/\sigma_{\mathrm{DY}}^{pA'}$: 92 points (before cuts)

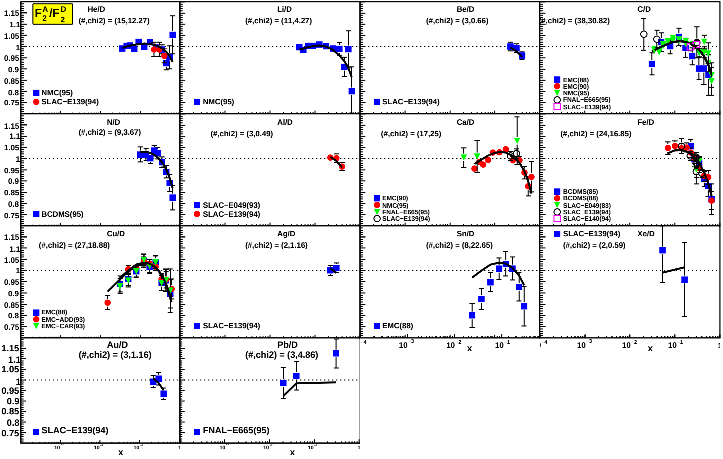
Table from Hirai et al.,arXiv:0909.2329

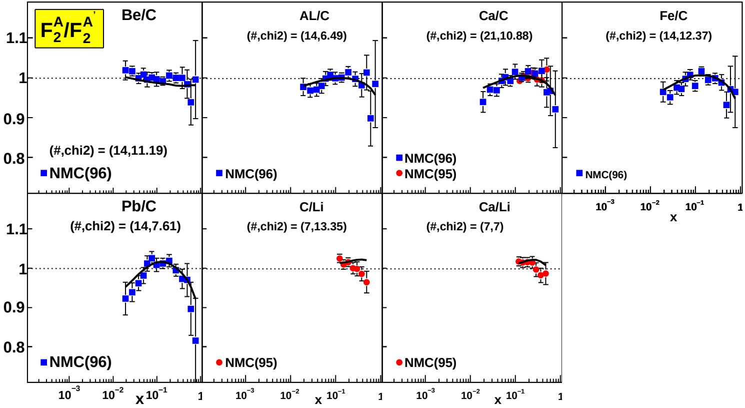
	R	Nucleus	Experiment	EPS09	HKN07	DS04
DIS		D/p	NMC		O	
	A/D	4He	SLAC E139	O	O	O
			NMC95	O (5)	O	O
		Li	NMC95	O	O	
		Be	SLAC E139	O	O	O
		C	EMC-88, 90		O	
			NMC 95	O	O	O
			SLAC E139	O	O	O
			FNAL-E665		O	
		N	BCDMS 85		O	
			HERMES 03		O	
		Al	SLAC E49		O	
			SLAC E139	O	O	O
		Ca	EMC 90		O	
			NMC 95	O	O	O
			SLAC E139	O	O	O
			FNAL-E665		O	
		Fe	SLAC E87		O	
			SLAC E139	O (15)	O	O
			SLAC E140		O	
			BCDMS 87		O	
		Cu	EMC 93	O	O	
		Kr	HERMES 03		O	
		Ag	SLAC E139	O	O	O
		Sn	EMC 88		O	
		Au	SLAC E139	O	O	O
			SLAC E140		O	
		Pb	FNAL-E665		O	
	A/C	Be	NMC 96	O	O	O
		Al	NMC 96	O	O	O
		Ca	NMC 95		O	
			NMC 96	O	O	O
		Fe	NMC 96	O	O	O
		Sn	NMC 96	O (10)	O	O
		Pb	NMC 96	O	O	O
	A/Li	C	NMC 95	O	O	
		Ca	NMC 95	O	O	
DY	A/D	C	FNAL-E772	O	O	O
		Ca		O (15)	O	O
		Fe		O (15)	O	O
		W		O (10)	O	O
	A/Be	Fe	FNAL E866	O	O	
		W		O	O	
π pro	dA/pp	Au	RHIC-PHENIX	O (20)		

- 708 (1233) data points after (before) cuts
- 32 free paramters; 675 d.o.f.
- Overall $\chi^2/\mathrm{d.o.f.} = 0.95$
- individually:
  - for F_2^A/F_2^D: $\chi^2/\mathrm{pt} = 0.92$
  - for $F_2^A/F_2^{A'}$: $\chi^2/\mathrm{pt} = 0.69$
  - for DY: $\chi^2/\mathrm{pt} = 1.08$
- **Our simple approach works!**

RESULTS: DECUT3 FIT

DIS DATA VS x

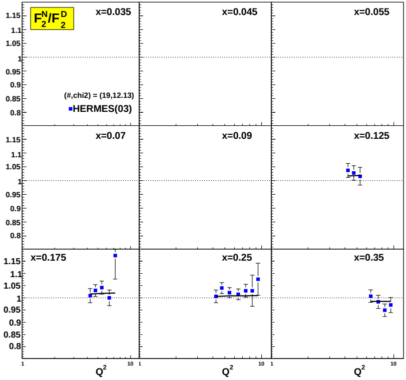


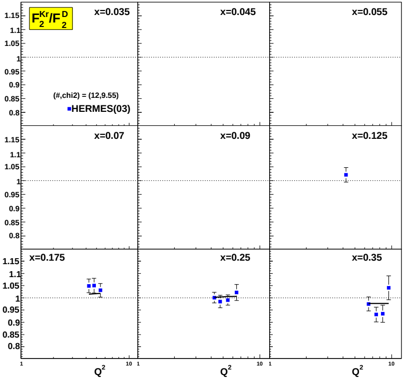


$F_2^A/F_2^{A'}$
Be/C
(#,chi2) = (14,11.19)
NMC(96)
AL/C
(#,chi2) = (14,6.49)
NMC(96)
Ca/C
(#,chi2) = (21,10.88)
NMC(96)
NMC(95)
Fe/C
(#,chi2) = (14,12.37)
NMC(96)
Pb/C
(#,chi2) = (14,7.61)
NMC(96)
C/Li
(#,chi2) = (7,13.35)
NMC(95)
Ca/Li
(#,chi2) = (7,7)
NMC(95)
1.1
1
0.9
0.8
10^{-3}
10^{-2}
10^{-1}
1
x

RESULTS: DECUT3 FIT

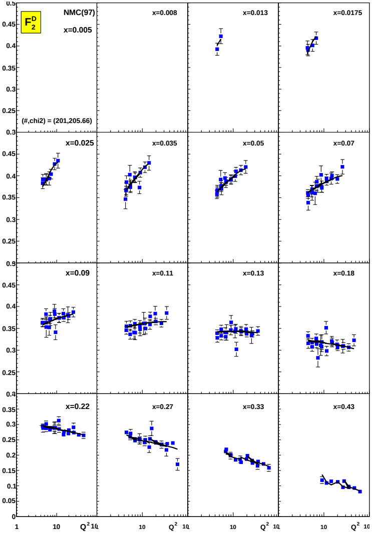
HERMES DATA VS Q^2

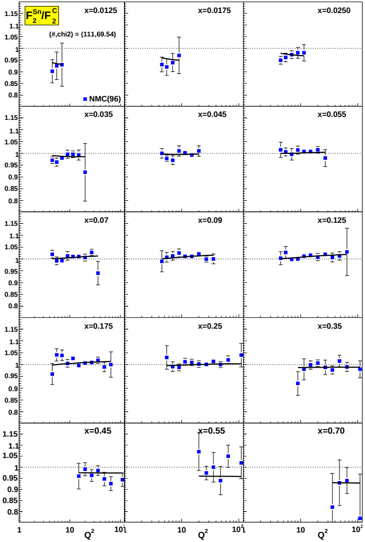


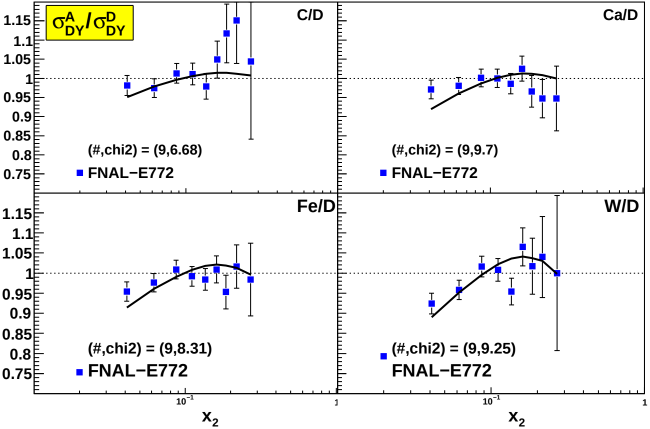


RESULTS: DECUT3 FIT

NMC DATA FOR D AND Sn/C VS Q^2



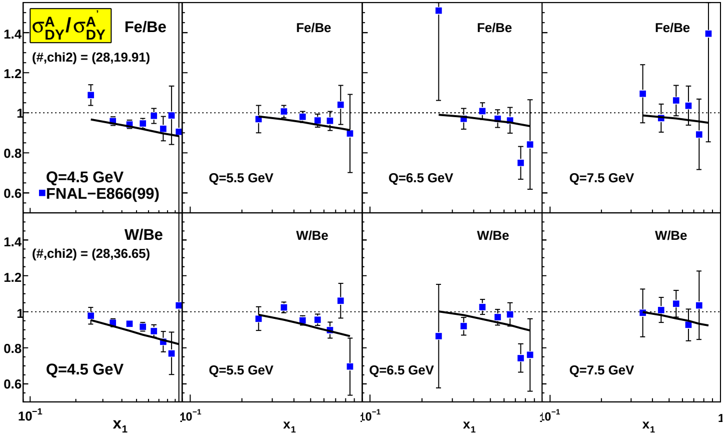




$\sigma^A_{DY}/\sigma^D_{DY}$
C/D
Ca/D
Fe/D
W/D
(#,chi2) = (9,6.68)
(#,chi2) = (9,9.7)
(#,chi2) = (9,8.31)
(#,chi2) = (9,9.25)
FNAL−E772
FNAL−E772
FNAL−E772
FNAL−E772
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
0.75
10^{-1}
1
x_2

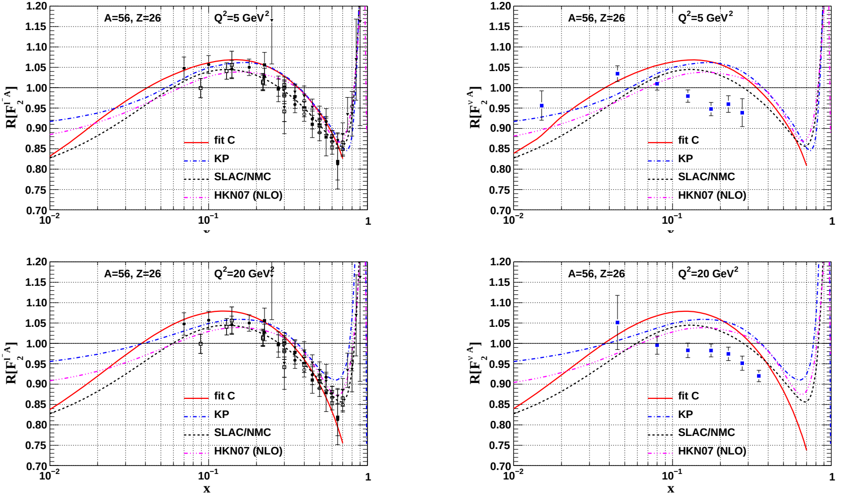
RESULTS: DECUT3 FIT

DRELL–YAN DATA



COMPARISON OF NUCLEAR CORRECTION FACTORS

$R[F_2^{\ell Fe}]$ (LEFT) VS $R[F_2^{\nu Fe}]$ (RIGHT)



Note: $x_{\min} = 0.01$ in these figures.

CONCLUSIONS II

- Our global fit to $\ell A +$ DY data is compatible with the literature. This validates our framework.
- First sets of CTEQ nPDFs will be released soon.

- Our fit does *not* describe the neutrino data
- Is there a compromise fit? A better flavor decomposition?
- Need global analysis of the combined ℓA DIS + DY + νA DIS data for definite conclusions

Analysis of νA, ℓA and DY data

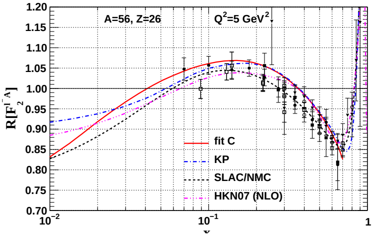
Kovarik,Yu,Keppel,Morfin,Olness,Owens,Schienbein,Stavreva, work in progress

Combining ℓA DIS, DY and νA DIS data

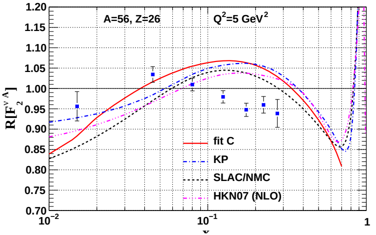
- ℓA and DY data sets as before
- 8 Neutrino data sets
  - NuTeV cross section data: νFe, $\bar{\nu} Fe$
  - CHORUS cross section data: νPb, $\bar{\nu} Pb$
  - NuTeV dimuon data: νFe, $\bar{\nu} Fe$
  - CCFR dimuon data: νFe, $\bar{\nu} Fe$
- Problem: Neutrino data sets have much higher statistics. Systematically study fi ts with different weights.

Weight	Fit name	ℓ data	χ^2 (/pt)	ν data	χ^2 (/pt)	total χ^2 (/pt)
$w = 0$	decut3	708	639 (0.90)	-	-	639 (0.90)
$w = 1/7$	glofac1a	708	645 (0.91)	3134	4710 (1.50)	5355 (1.39)
$w = 1/4$	glofac1c	708	654 (0.92)	3134	4501 (1.43)	5155 (1.34)
$w = 1/2$	glofac1b	708	680 (0.96)	3134	4405 (1.40)	5085 (1.32)
$w = 1$	global2b	708	736 (1.04)	3134	4277 (1.36)	5014 (1.30)
$w = \infty$	nuanua1	-	-	3134	4192 (1.33)	4192 (1.33)

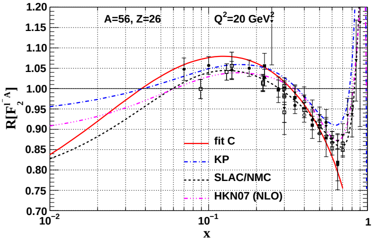
decut3 ($w = 0$)



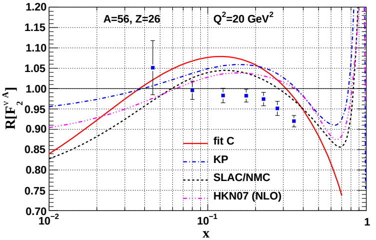
A=56, Z=26
Q^2=5 GeV^2
$R[F_2^{l\ A}]$
fit C
KP
SLAC/NMC
HKN07 (NLO)
x



A=56, Z=26
Q^2=5 GeV^2
$R[F_2^{\nu\ A}]$
fit C
KP
SLAC/NMC
HKN07 (NLO)
x

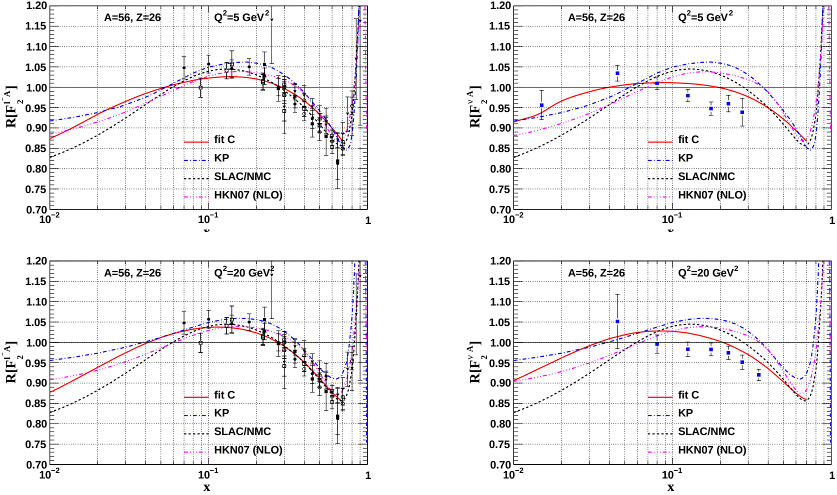


A=56, Z=26
Q^2=20 GeV^2
$R[F_2^{l\ A}]$
fit C
KP
SLAC/NMC
HKN07 (NLO)
x

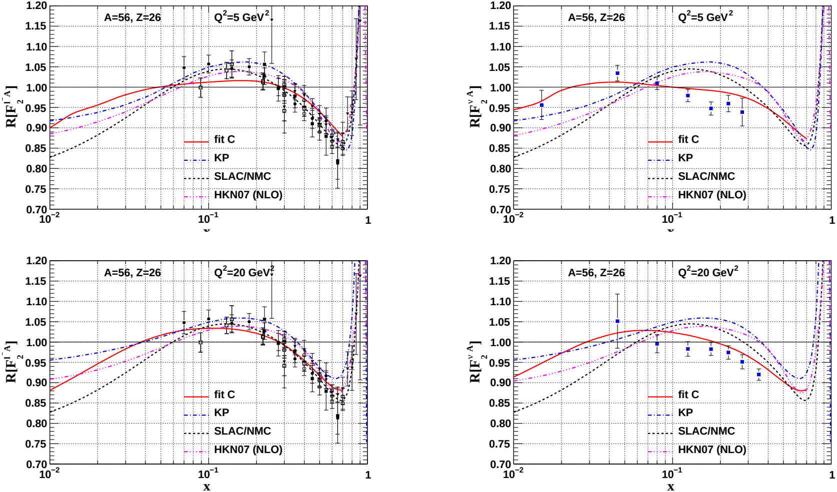


A=56, Z=26
Q^2=20 GeV^2
$R[F_2^{\nu\ A}]$
fit C
KP
SLAC/NMC
HKN07 (NLO)
x

glofac1a ($w = 1/7$)

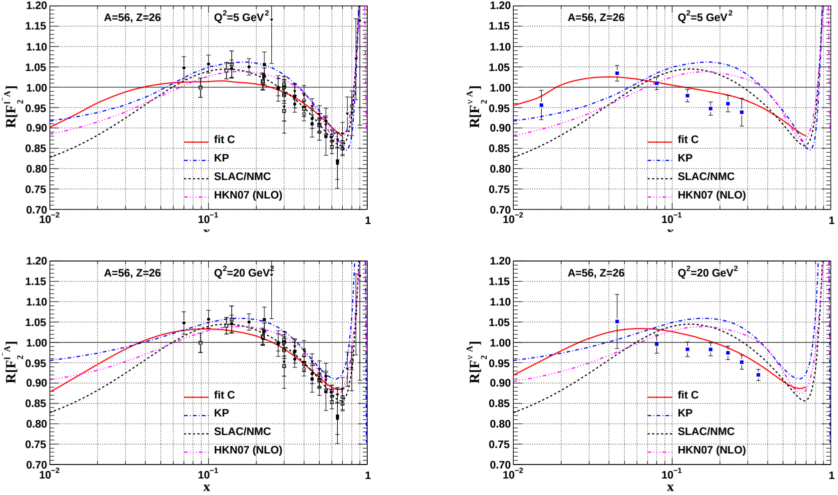


glofac1c ($w = 1/4$)



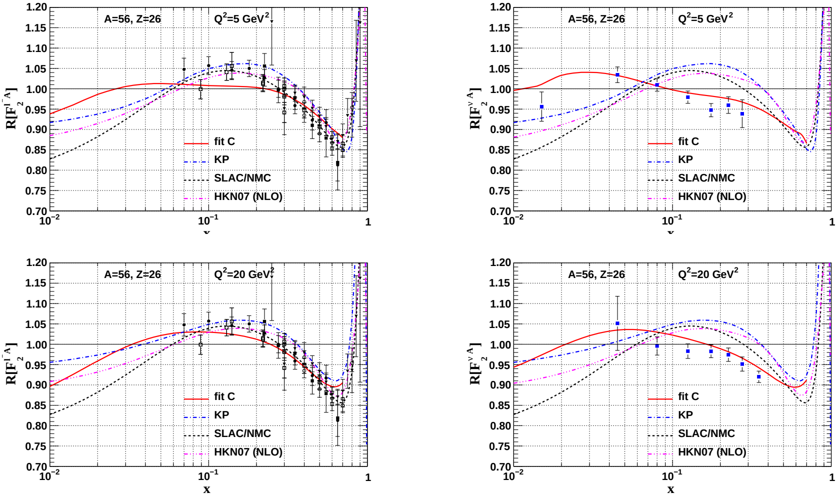
$R[F_2^{\ell Fe}]$ (LEFT) VS $R[F_2^{\nu Fe}]$ (RIGHT)

glofac1b ($w = 1/2$)

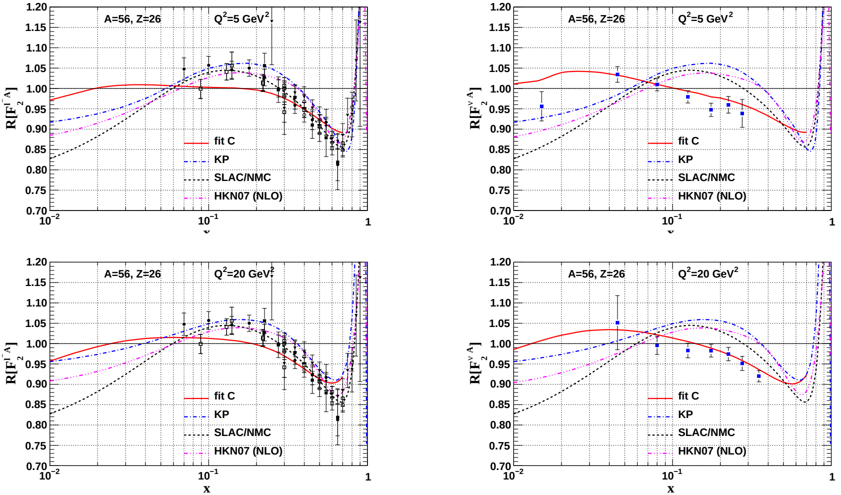


$R[F_2^{\ell Fe}]$ (LEFT) VS $R[F_2^{\nu Fe}]$ (RIGHT)

global2b ($w = 1$)



nuanua1 ($w = \infty$)



IS THERE A REASONABLE COMPROMISE FIT?

Weight	Fit name	ℓ data	χ^2 (/pt)	ν data	χ^2 (/pt)	total χ^2 (/pt)
$w = 0$	decut3	708	639 (0.90)	-	-	639 (0.90)
$w = 1/7$	glofac1a	708	645 (0.91)	3134	4710 (1.50)	5355 (1.39)
$w = 1/4$	glofac1c	708	654 (0.92)	3134	4501 (1.43)	5155 (1.34)
$w = 1/2$	glofac1b	708	680 (0.96)	3134	4405 (1.40)	5085 (1.32)
$w = 1$	global2b	708	736 (1.04)	3134	4277 (1.36)	5014 (1.30)
$w = \infty$	nuanua1	-	-	3134	4192 (1.33)	4192 (1.33)

- $w = 0$: No. Problem: $R[F_2^{\nu Fe}]$
- $w = 1/7$: No. Problem: $R[F_2^{\nu Fe}]$
- $w = 1/4, 1/2$: No.
  - $Q^2 = 5$: Undershoots $R[F_2^{\ell Fe}]$ for $x < 0.2$. Overshoots $R[F_2^{\nu Fe}]$ for $x \in [0.1, 0.3]$
  - $Q^2 = 20$: $R[F_2^{\ell Fe}]$ still ok. Overshoots $R[F_2^{\nu Fe}]$.
- $w = 1$: No. Possibly there is a compromise if more strict Q^2 cut?
  - $Q^2 = 5$: Undershoots $R[F_2^{\ell Fe}]$ for $x < 0.2$. $R[F_2^{\nu Fe}]$ ok.
  - $Q^2 = 20$: $R[F_2^{\ell Fe}]$ still ok. $R[F_2^{\nu Fe}]$ ok.
- $w = \infty$: No. Problem: $R[F_2^{\ell Fe}]$

Discussion based on the comparison of the nuclear correction factors $R[F_2^{\ell A}]$ and $R[F_2^{\nu A}]$

- There is definitely a tension between the NuTeV and the charged lepton data
  - There is a clear dependence on the weight.
  - Theory curves for $R[F_2^{\ell A}]$ and $R[F_2^{\nu A}]$ are both shifted down with increasing weight of the neutrino data.
- Preliminary conclusion: At the level of the (high) precision there doesn't seem to be a good compromise fit of the combined ℓA, DY and νA data.
- However one has to be careful:
  - These are precision effects
  - For each weight, the curves have uncertainty bands not considered
  - The figures show the comparison to only few (representative) data

Consider next quantitative criterion based on χ^2

TOLERANCE CRITERION

Probability distribution for the χ^2 function

$$P_N(\chi^2) = \frac{(\chi^2)^{N/2-1} e^{-\chi^2/2}}{2^{N/2}\Gamma(N/2)}$$

Determine ξ^2_{50} and ξ^2_{90} (i.e. $p = 50$, $p = 90$):

$$\int_0^{\xi_p^2} d\chi^2 P_N(\chi^2) = p/100$$

Condition for compatibility of two fits:
The 2nd fit (χ^2_n) should be within the 90% C.L. region of the first fit ($\chi^2_{n,0}$)

$$\boxed{\chi^2_n/\chi^2_{n,0} < \xi^2_{90}/\xi^2_{50} \quad \Leftrightarrow \quad C_{90} \equiv \frac{\Delta\chi^2}{\frac{\chi^2_{n,0}}{\xi^2_{50}}(\xi^2_{90}-\xi^2_{50})} < 1}$$

see CTEQ'01, PRD65(2001)014012; MSTW'09, EPJC(2009)63,189-285

TOLERANCE CRITERION $C_{90} < 1$:

TOTAL χ^2 FOR A) ℓA+DY DATA AND B) NEUTRINO DATA

90% tolerance condition for the **charged lepton** χ^2 and the **neutrino** χ^2

- decut3: 638.9 ± 45.6 (best fit to only charged lepton and DY data)
- nuanua1: 4192 ± 138 (best fit to only neutrino data)

Is there a compromise fit compatible to both, decut3 **and** nuanua1?

Weight	Fit name	ℓ data	χ^2	ν data	χ^2	total χ^2 (/pt)
$w = 0$	decut3	708	639	-	nnnn **NO**	639 (0.90)
$w = 1/7$	glofac1a	708	645 **YES**	3134	4710 **NO**	5355 (1.39)
$w = 1/4$	glofac1c	708	654 **YES**	3134	4501 **NO**	5155 (1.34)
$w = 1/2$	**glofac1b**	708	680 **YES**	3134	4405 **NO*** **	5085 (1.32)
$w = 1$	global2b	708	736 **NO**	3134	4277 **YES**	5014 (1.30)
$w = \infty$	nuanua1	-	nnn **NO**	3134	4192	4192 (1.33)

Observations:

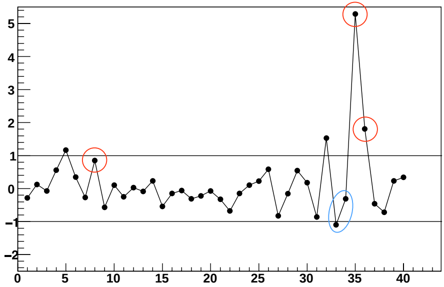
- There is no good compromise fit based on the 90% C.L. criterion.
- Our best candidate is **glofac1b** which is *marginally* compatible: $4405 - 4192 \simeq 1.5 \times 138$
- Observations in agreement with the previous conclusions based on $R[F_2^{\ell Fe}]$ and $R[F_2^{\nu Fe}]$.

Let's have a look at the tolerance criterion applied to the individual data sets!

Tolerance criterion $C_{90} < 1$:

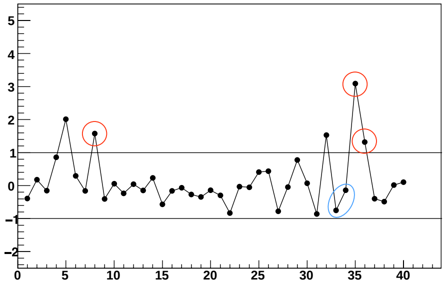
Individual data sets: $n = 1, \ldots, 32$ vs DECUT3; $n = 33, \ldots, 40$ vs NUANUA1

glofac1a ($w = 1/7$)



- **Y-axis**: $\mathbf{C_{90}}$; **X-axis**: Number of the data set ($n = 1, \ldots, 40$)
- Important data sets:
  - $n = 8$ (red circle): Fe/D charged lepton data
  - blue ellipse: CHORUS νPb, $\bar{\nu} Pb$ cross section data
  - $n = 35, 36$ (red ellipse): NuTeV νFe, $\bar{\nu} Fe$ cross section data

glofac1c ($w = 1/4$)

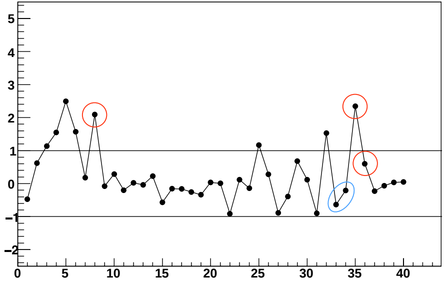


- **Y-axis**: C_{90}; **X-axis**: Number of the data set ($n = 1, \ldots, 40$)
- Important data sets:
  - $n = 8$ (red circle): Fe/D charged lepton data
  - blue ellipse: CHORUS νPb, $\bar{\nu} Pb$ cross section data
  - $n = 35, 36$ (red ellipse): NuTeV νFe, $\bar{\nu} Fe$ cross section data

TOLERANCE CRITERION $C_{90} < 1$:

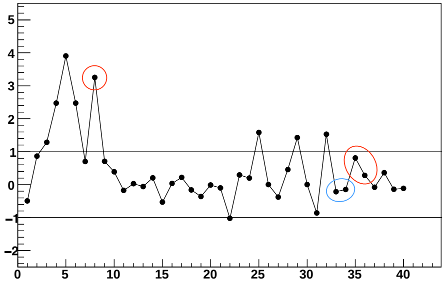
INDIVIDUAL DATA SETS: $n = 1, \ldots, 32$ VS DECUT3; $n = 33, \ldots, 40$ VS NUANUA1

glofac1b ($w = 1/2$)



- **Y-axis**: C_{90}; **X-axis**: Number of the data set ($n = 1, \ldots, 40$)
- Important data sets:
  - $n = 8$ (red circle): Fe/D charged lepton data
  - blue ellipse: CHORUS νPb, $\bar{\nu} Pb$ cross section data
  - $n = 35, 36$ (red ellipse): NuTeV νFe, $\bar{\nu} Fe$ cross section data

global2b ($w = 1$)



- **Y-axis**: $\mathbf{C_{90}}$; **X-axis**: Number of the data set ($n = 1, \ldots, 40$)
- Important data sets:
  - $n = 8$ (red circle): Fe/D charged lepton data
  - blue ellipse: CHORUS νPb, $\bar{\nu} Pb$ cross section data
  - $n = 35, 36$ (red ellipse): NuTeV νFe, $\bar{\nu} Fe$ cross section data

Tolerance criterion $C_{90} < 1$:

Individual data sets

Observations:

- $w = 1/7$: $C_{90} > 5$ for NuTeV νFe; $C_{90} \simeq 1.8$ for NuTeV $\bar{\nu} Fe$
- CHORUS data (blue ellipse) always compatible; little dependence on weight w
- increasing weight: NuTeV cross section data improve; charged lepton Fe/D data get worse
- our best candidate ($w = 1/2$)
  - Fe/D ($n = 8$): $C_{90} \simeq 2$
  - NuTeV νFe ($n = 35$): $C_{90} \simeq 2.2$
  - NuTeV $\bar{\nu} Fe$ ($n = 36$): $C_{90} < 1$
  - some other data sets $n = 3, 4, 5, 6, 32$ with $C_{90} > 1$
- $w = 1$: Fe/D ($n = 8$): $C_{90} > 3$
- Confi rms and quantifi es observations based on R plots

CONCLUSIONS III

Based on nuclear corrections factors R and the tolerance criterion $C_{90} < 1$:

- There is no good compromise fi t to the ℓA DIS + DY + νA DIS data.
- Most problematic: tension between NuTeV νFe cross section data and Fe/D data in charged lepton DIS.
- The NuTeV $\bar{\nu} Fe$ data are less problematic. They have larger errors.
- The CHORUS νPb and $\bar{\nu} Pb$ data are compatible with both, the charged lepton+DY and the NuTeV data, as is well known. They also have larger errors.

- Relaxing the tolerance criterion to $C_{90} \lesssim 2$ the fi t with weight $w = 1/2$ would be *marginally* acceptable.
- This can also (qualitatively) be verifi ed with the R-plots.
- A larger Q^2-cut, say $Q^2 > 5\ \mathrm{GeV}^2$, could also help to reduce the tension. (In particular, this would remove some of the rather precise NuTeV cross section data at small x.)