

*D and B meson production at the LHC in the GM-VFN scheme at NLO*

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 Characterization of the Quark Gluon Plasma with Heavy Quarks  
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- ① HEAVY QUARKS IN PQCD
- ② HADROPRODUCTION OF HEAVY QUARKS: THEORY
- ③ NON-PERTURBATIVE INPUT
- ④ NUMERICAL RESULTS
- ⑤ SUMMARY

# Heavy quarks in pQCD

A quark  $h$  is heavy : $\Leftrightarrow m_h \gg \Lambda_{\text{QCD}} \sim 250 \text{ MeV}$

- $m_h \gg \Lambda_{\text{QCD}} \Rightarrow \alpha_s(m_h^2) \propto \ln^{-1}\left(\frac{m_h^2}{\Lambda_{\text{QCD}}^2}\right) \ll 1$  (asymptotic freedom)
- $m_h$  sets hard scale; acts as long distance cut-off
- Perturbation theory (pQCD) applicable

charm:	$m_c \sim 1.5 \text{ GeV}$	$\Lambda_{\text{QCD}}/m_c \sim 0.17$	$\alpha_s(m_c^2) \sim 0.34$
bottom:	$m_b \sim 5 \text{ GeV}$	$\Lambda_{\text{QCD}}/m_b \sim 0.05$	$\alpha_s(m_b^2) \sim 0.21$
top:	$m_t \sim 175 \text{ GeV}$	$\Lambda_{\text{QCD}}/m_t \sim 0.001$	$\alpha_s(m_t^2) \sim 0.11$

- The smaller the ratio  $\Lambda_{\text{QCD}}/m_h$ , the smaller effects of non-perturbative QCD (such as hadronization)
- Top quark decays before it could hadronize due to its large mass ( $\Gamma \propto m_t^3$ ):  

$$\Gamma \simeq \Gamma(t \rightarrow bW) \simeq \frac{G_F m_t^3}{8\pi\sqrt{2}} |V_{tb}|^2 \simeq 1.76 \text{ GeV} \left(\frac{m_t}{175 \text{ GeV}}\right)^3$$

## Requirements:

- (1)  $\mu \ll m$ : Decoupling of heavy degrees of freedom
- (2)  $\mu \gg m$ : IR-safety
- (3)  $\mu \sim m$ : Correct threshold behavior

## Problems:

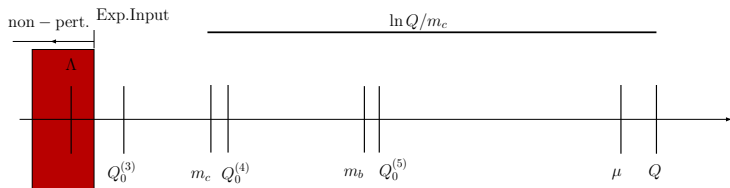
- Multiple hard scales:  $m_c, m_b, m_t, \mu$
- Mass-independent factorization/renormalization schemes like  $\overline{\text{MS}}$
- A single  $\overline{\text{MS}}$  scheme cannot meet requirements (1) and (3) (is unphysical).

Way out: Patchwork of  $\overline{\text{MS}}$  schemes  $S^{n_f, n_R}$ 

- Variable Flavor-Number Scheme (VFNS):  $S^{3,3} \xrightarrow{m_c} S^{4,4} \xrightarrow{m_b} S^{5,5} \xrightarrow{m_t} S^{6,6}$
- **Masses reintroduced** by backdoor: **matching**  $S^{3,3} \leftrightarrow S^{4,4}$  at  $m_c$ :  
 $f_i^{n_f=4}(x, m_c) = [A_{ij} \otimes f_j^{n_f=3}](x, m_c)$ ,  $\alpha_s^{n_f=4}(m_c) = B \alpha_s^{n_f=3}(m_c)$ , etc.
- Fixed Flavor-Number Scheme (3-FFNS):  $S^{3,3}$  or  $S^{3,3} \rightarrow S^{3,4} \rightarrow S^{3,5} \rightarrow S^{3,6}$

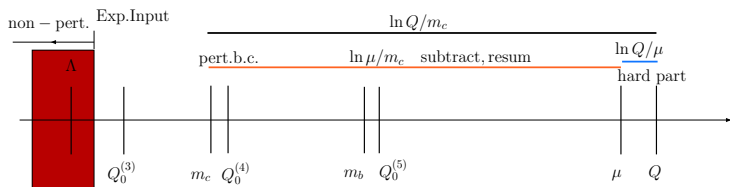
## 3-FFNS/Fixed Order:

- No charm PDF! Of course need **exp. Input** for  $u, d, s, g$  PDFs at scale  $Q_0^{(3)}$
- **finite** collinear logs  $\ln Q/m_c$  arise  $\rightarrow$  are kept in hard part (unresummed, in fixed order)
- Requirement (3) naturally satisfied
- Not IR-safe, does not meet requirement (2):
  - Not valid for  $Q \gg m_c$
  - Can we quantify? Valid for  $Q < m_c, 3m_c, 5m_c?$



Variable Flavour Number Scheme (VFNS):

- often large ratios of scales involved: **multi-scale problems**  
For  $Q \gg m_c$ : write  $\ln Q/m_c = \ln \mu/m_c + \ln Q/\mu$ , **subtract**  $\ln \mu/m_c$  and **resum**  $\ln \mu/m_c$  by introducing charm PDF at  $Q_0^{(4)} \simeq m_c$  using a **perturbative** boundary condition
- $Q < Q_0$ :  $n_f = 3$  no charm PDF,  $Q \geq Q_0$ :  $n_f \rightarrow n_f + 1$ , charm PDF **without fit parameters**
- IR-safe, satisfies requirement (2); resums collinear logarithms
- Problem: original ZM-VFNS (=massless parton model) only valid for  $Q \gg m$  (Quantify?)
- GM-VFNS: need extra work to satisfy requirement (3) but then **valid for all scales  $Q$ !** approaches FFNS for  $Q \sim m$ , approaches ZM-VFNS for  $Q \gg m$



- GM-VFNS essential for  $W, Z$  cross sections at the LHC
- Most of the most recent global analyses of proton PDFs use a version of a GM-VFNS [see Thorne, Tung, arXiv:0809.0714; Olness, IS, arXiv:08123371]
  - CTEQ6.6/CT10: S-ACOT $_{\chi}$
  - MSTW08: TR scheme
  - NNPDF2.1: FONLL
  - HERANPDF1.0: same as MSTW08
  - GJR08, JR09: as GRV in a FFNS
  - CTEQ5,CTEQ6.1,NNPDF2.0,... and older: ZM-VFNS
- The various GM-VFN schemes are 'tuned to' the DIS structure functions  $F_2^c, F_L^c$

IF THESE SCHEMES ARE NOT JUST COOKING RECIPES BUT PQCD FORMALISMS WITH HEAVY QUARKS, THEY SHOULD BE APPLICABLE TO OTHER PROCESSES AS WELL



# Hadroproduction of heavy quarks: Theory

$$A + B \rightarrow H + X: \quad d\sigma = \sum_{i,j,k} f_i^A(x_1) \otimes f_j^B(x_2) \otimes d\sigma(ij \rightarrow kX) \otimes D_k^H(z)$$

sum over all possible subprocesses  $i + j \rightarrow k + X$

Parton distribution functions:

$$f_i^A(x_1, \mu_F), f_j^B(x_2, \mu_F)$$

**non-perturbative** input

long distance

universal

Hard scattering

cross section:

$$d\sigma(\mu_F, \mu_F', \alpha_s(\mu_R), [\frac{m_h}{p_T}])$$

**perturbatively** computable

short distance

(coefficient functions)

Fragmentation functions:

$$D_k^H(z, [\mu_F'])$$

**non-perturbative** input

long distance

universal

Accuracy:

light hadrons:  $\mathcal{O}((\Lambda/p_T)^p)$  with  $p_T$  hard scale,  $\Lambda$  hadronic scale,  $p = 1, 2$

heavy hadrons: if  $m_h$  is neglected in  $d\sigma$ :  $\mathcal{O}((m_h/p_T)^p)$

Details (subprocesses, PDFs, FFs; mass terms) depend on the **Heavy Flavour Scheme**

# FFNS/Fixed Order

Start with FFNS = Fixed Order:

- NLO calculation more than 20 years old, very well tested
- Allows to predict the total cross section
- Allows to compute  $p_T$  spectrum if  $!(p_T \gg m)$   
(up to inclusion of a non-perturbative FF which is very hard)

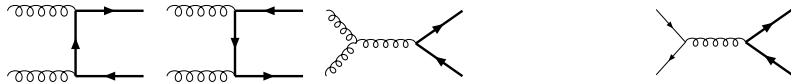
Compare data with best FFNS prediction!

- Find  $p_T$ -range where FFNS applicable. Guess:  $p_T = 5m$  still ok.
- When need for resummation of  $\ln m$  terms visible?  
(Apart from smaller uncertainty band in resummed theory)

AS DESCRIBED BEFORE: GM-VFNS  $\rightarrow$  FFNS FOR  $p_T \sim m$

## Leading order subprocesses:

1.  $gg \rightarrow Q\bar{Q}$
2.  $q\bar{q} \rightarrow Q\bar{Q}$  ( $q = u, d, s$ )



- The  $gg$ -channel is dominant at the LHC ( $\sim 85\%$  at  $\sqrt{S} = 14$  TeV).
- The total production cross section for **heavy quarks** is finite.  
The minimum virtuality of the t-channel propagator is  $m^2$ . Sets the scale in  $\alpha_s$ .  
Perturbation theory should be reliable.
- Note: For  $m^2 \rightarrow 0$  total cross section would diverge.

[See M. Mangano, hep-ph/9711337; Textbook by Ellis, Stirling and Webber]

Next-to-leading order (NLO) subprocesses:

1.  $gg \rightarrow Q\bar{Q}g$
2.  $q\bar{q} \rightarrow Q\bar{Q}g$  ( $q = u, d, s$ )
3.  $gg \rightarrow Q\bar{Q}q, g\bar{q} \rightarrow Q\bar{Q}\bar{q}$  [new at NLO]
4. Virtual corrections to  $gg \rightarrow Q\bar{Q}$  and  $q\bar{q} \rightarrow Q\bar{Q}$

NLO corrections for  $\sigma_{\text{tot}}$  and differential cross sections  $d\sigma/dp_T dy$  known since long:

- Nason, Dawson, Ellis, NPB303(1988)607; Beenakker, Kuif, van Neerven, Smith, PRD40(1989)54 [ $\sigma_{\text{tot}}$ ]
- NDE, NPB327(1989)49; (E)B335(1990)260; Beenakker *et al.*, NPB351(1991)507 [ $d\sigma/dp_T dy$ ]

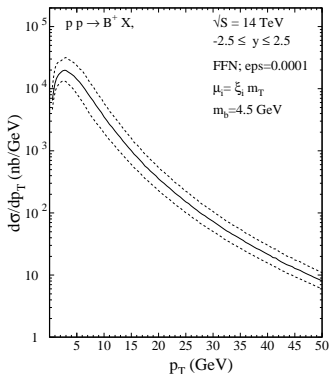
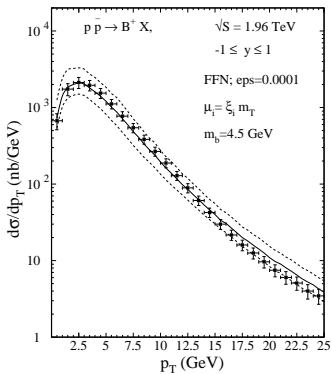
Well tested by recalculations and zero-mass limit:

- Bojak, Stratmann, PRD67(2003)034010 [ $d\sigma/dp_T dy$  (un)polarized]
- Kniehl, Kramer, Spiesberger, IS, PRD71(2005)014018 [ $m \rightarrow 0$  limit of diff. x-sec]
- Czakon, Mitov, NPB824(2010)111 [ $\sigma_{\text{tot}}$ , fully analytic]

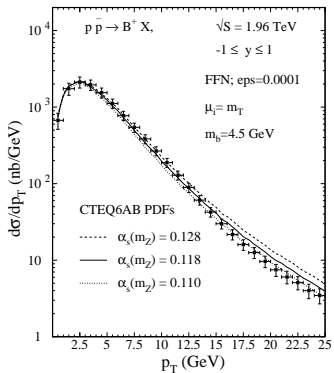
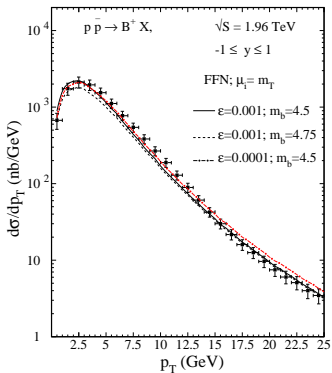
## ToDo:

- Calculate for  $q\bar{q} \rightarrow Q\bar{Q}$  and  $gg \rightarrow Q\bar{Q}$ 
  - $M^{(0)}(2 \rightarrow 4)$  (tree level)
  - $M^{(1)}(2 \rightarrow 3)$  (one-loop)
  - $M^{(2)}(2 \rightarrow 2)$  (two-loop)
  - NNLO subtraction of IR singularities!
- In addition  $gq$  channel
- Goal: total cross section and differential spectra
- Several groups are working on it (Bonciani, Gehrmann, Ferroglia, ...; Czakon, ...)
- $q\bar{q}$  complete (easier);  $M^{(2)}(2 \rightarrow 4)$  for  $gg$  channel diffi cult
- Complete calculation still a couple of years away
- See, e.g., [arXiv:1012.0258](https://arxiv.org/abs/1012.0258) for a recent talk

- $d\sigma/dp_T$  for the process  $pp \rightarrow B^+ X$ ; Fragmentation  $b \rightarrow B$  via Peterson-FF
- CTEQ6.1 PDFs
- Prediction in NLO perturbation theory







## Remarks:

- Fixed order theory in reasonable agreement with Tevatron data up to  $p_T \simeq 5m_b$
- At  $p_T \lesssim m_b$  factorization less obvious. Depends on definition of convolution variable  $z$ :  $p_B = zp_b$  or  $p_T^B = zp_T^b$  or  $p_B^+ = zp_b^+$  or  $\vec{p}_B = z\vec{p}_b$
- Less hadronization effects than originally believed:  
 $\epsilon$ -parameter small corresponding to a hard fragmentation function.  
Harder FF  $\rightarrow$  harder  $p_T$ -spectrum
- Larger  $\alpha_s(M_Z) \rightarrow$  harder  $p_T$ -spectrum
- Mass dependence important for  $p_T \lesssim m$  (peak)  $\rightarrow \sigma_{tot}$
- Only the 4th or 5th Mellin-moment of the FF is relevant for large  $p_T$  [M. Mangano]:  
 $d\sigma^b/dp_T(b) \simeq A/p_T(b)^n$  with  $n \simeq 4, \dots, 5$

$$d\sigma^B/dp_T(B) = \int dz/z D(z) d\sigma^b/dp_T(b)[p_T(b) = p_T(B)/z] = A/p_T(B)^n \times \int dz z^{n-1} D(z)$$

# ZM-VFNS/RS (RS: Resummed)

Next ZM-VFNS/RS which is the baseline for  $p_T \gg m$

- Again NLO calculation more than 20 years old, very well tested
- Allows to compute  $p_T$  spectrum if  $p_T \gg m$
- Needs scale-dependent FFs for quarks and gluons  $D_q^H(z, \mu'_F)$ ,  $D_g^H(z, \mu'_F)$
- Same theory used for the computation of inclusive  $\pi$  or  $K$  production.

Compare data with best ZM-VFNS prediction!

- Find smallest  $p_T$  where ZM-VFNS applicable.
- $m/p_T$  terms neglected.
- Is there an overlapping region where both, FFNS and ZM-VFNS are valid?

AS SAID BEFORE: GM-VFNS  $\rightarrow$  ZM-VFNS FOR  $p_T \gg m$

Massless NLO calculation: [Aversa,Chiappetta,Greco,Guillet,NPB327(1989)105]

1.  $gg \rightarrow qX$
2.  $gg \rightarrow gX$
3.  $qg \rightarrow gX$
4.  $qg \rightarrow qX$
5.  $q\bar{q} \rightarrow gX$
6.  $q\bar{q} \rightarrow qX$
7.  $qg \rightarrow \bar{q}X$
8.  $qg \rightarrow \bar{q}'X$
9.  $qg \rightarrow q'X$
10.  $q\bar{q} \rightarrow gX$
11.  $q\bar{q} \rightarrow qX$
12.  $q\bar{q} \rightarrow q'X$
13.  $q\bar{q}' \rightarrow gX$
14.  $q\bar{q}' \rightarrow qX$
15.  $q\bar{q}' \rightarrow q'X$
16.  $q\bar{q}' \rightarrow qX$

⊕ charge conjugated processes

# GM-VFNS/FONLL

$$\text{FONLL} = \text{FO} + (\text{RS} - \text{FOM0})G(m, p_T)$$

FO: Fixed Order; FOM0: Massless limit of FO; RS: Resummed

$$G(m, p_T) = \frac{p_T^2}{p_T^2 + 25m^2} \simeq \begin{cases} 0.04 & : p_T = m \\ 0.25 & : p_T = 3m \\ 0.50 & : p_T = 5m \\ 0.66 & : p_T = 7m \\ 0.80 & : p_T = 10m \end{cases}$$

$$\Rightarrow \text{FONLL} = \begin{cases} \text{FO} & : p_T \lesssim 3m \\ \text{RS} & : p_T \gtrsim 10m \end{cases}$$

Fragmentation functions (FFs):

- $D_i^H(z, \mu'_F) = D_i^Q \otimes D_Q^H$  where:
  - $D_i^Q(z, \mu'_F)$ : perturbative FFs of  $i = q, g, Q$  into an on-shell heavy quark  $Q$
  - $D_Q^H(z)$ : scale-independent, non-perturbative FF describing transition of heavy quark to heavy hadron
- Non-perturbative FF fitted to  $e^+e^- \rightarrow DX, BX$  data



Applications available for:

- $\gamma^* + p \rightarrow D^{*,0,+} + X$   
 photoproduction [JHEP0103(2001)006]
- $p + \bar{p} \rightarrow (D^0, D^{*\pm}, D^\pm, D_s^\pm, \Lambda_c^\pm) + X$   
 good description of Tevatron data [JHEP05(1998)007]
- $p + \bar{p} \rightarrow B + X$   
 good description of Tevatron data [PRL89(2002)122003, JHEP07(2004)033]
- $p + p \rightarrow D, B + X$   
 good description of RHIC data [PRL95(2005)122001]

Factorization Formula:

[1]

$$d\sigma(p\bar{p} \rightarrow D^* X) = \sum_{i,j,k} \int dx_1 dx_2 dz f_i^p(x_1) f_j^{\bar{p}}(x_2) \times \\ d\hat{\sigma}(ij \rightarrow kX) D_k^{D^*}(z) + \mathcal{O}(\alpha_s^{n+1}, (\frac{\Lambda}{Q})^p)$$

Q: hard scale,  $p = 1, 2$ 

- 
- $d\hat{\sigma}(\mu_F, \mu'_F, \alpha_s(\mu_R), \frac{m_h}{p_T})$ : hard scattering cross sections free of long-distance physics  $\rightarrow m_h$  kept
  - PDFs  $f_i^p(x_1, \mu_F), f_j^{\bar{p}}(x_2, \mu_F)$ :  $i, j = g, q, c$  [ $q = u, d, s$ ]
  - FFs  $D_k^{D^*}(z, \mu'_F)$ :  $k = g, q, c$

$\Rightarrow$  need short distance coefficients including heavy quark masses

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[1] J. Collins, 'Hard-scattering factorization with heavy quarks: A general treatment',  
PRD58(1998)094002

# LIST OF SUBPROCESSES: GM-VFNS

Only light lines

Heavy quark initiated ( $m_Q = 0$ )

Mass effects:  $m_Q \neq 0$

- 1  $gg \rightarrow qX$
- 2  $gg \rightarrow gX$
- 3  $qg \rightarrow gX$
- 4  $qg \rightarrow qX$
- 5  $q\bar{q} \rightarrow gX$
- 6  $q\bar{q} \rightarrow qX$
- 7  $qg \rightarrow \bar{q}X$
- 8  $qg \rightarrow \bar{q}'X$
- 9  $qg \rightarrow q'X$
- 10  $qq \rightarrow gX$
- 11  $qq \rightarrow qX$
- 12  $q\bar{q} \rightarrow q'X$
- 13  $q\bar{q}' \rightarrow gX$
- 14  $q\bar{q}' \rightarrow qX$
- 15  $qq' \rightarrow gX$
- 16  $qq' \rightarrow qX$

- 1 -
- 2 -
- 3  $Qg \rightarrow gX$
- 4  $Qg \rightarrow QX$
- 5  $Q\bar{Q} \rightarrow gX$
- 6  $Q\bar{Q} \rightarrow QX$
- 7  $Qg \rightarrow \bar{Q}X$
- 8  $Qg \rightarrow \bar{q}X$
- 9  $Qg \rightarrow qX$
- 10  $QQ \rightarrow gX$
- 11  $QQ \rightarrow QX$
- 12  $Q\bar{Q} \rightarrow qX$
- 13  $Q\bar{q} \rightarrow gX, q\bar{Q} \rightarrow gX$
- 14  $Q\bar{q} \rightarrow QX, q\bar{Q} \rightarrow qX$
- 15  $Qq \rightarrow gX, qQ \rightarrow gX$
- 16  $Qq \rightarrow QX, qQ \rightarrow qX$

- 1  $gg \rightarrow QX$
- 2 -
- 3 -
- 4 -
- 5 -
- 6 -
- 7 -
- 8  $qg \rightarrow \bar{Q}X$
- 9  $qg \rightarrow QX$
- 10 -
- 11 -
- 12  $q\bar{q} \rightarrow QX$
- 13 -
- 14 -
- 15 -
- 16 -

⊕ charge conjugated processes

Mass terms contained in the hard scattering coefficients:

$$d\hat{\sigma}(\mu_F, \mu_{F'}, \alpha_s(\mu_R), \frac{m}{p_T})$$

Two ways to derive them:

- (1) Compare **massless limit** of a massive fixed-order calculation with a massless  $\overline{\text{MS}}$  calculation to determine subtraction terms

[Kniehl,Kramer,IS,Spiesberger,PRD71(2005)014018]

OR

- (2) Perform **mass factorization** using partonic PDFs and FFs

[Kniehl,Kramer,IS,Spiesberger,EPJC41(2005)199]

→ skip details

- Compare limit  $m \rightarrow 0$  of the massive calculation (Merebashvili et al., Ellis, Nason; Smith, van Neerven; Bojak, Stratmann; ...) with massless  $\overline{\text{MS}}$  calculation (Aurenche et al., Aversa et al., ...)

$$\lim_{m \rightarrow 0} d\tilde{\sigma}(m) = d\hat{\sigma}_{\overline{\text{MS}}} + \Delta d\sigma$$

⇒ Subtraction terms

$$d\sigma_{\text{sub}} \equiv \Delta d\sigma = \lim_{m \rightarrow 0} d\tilde{\sigma}(m) - d\hat{\sigma}_{\overline{\text{MS}}}$$

- Subtract  $d\sigma_{\text{sub}}$  from massive partonic cross section while keeping mass terms

$$d\hat{\sigma}(m) = d\tilde{\sigma}(m) - d\sigma_{\text{sub}}$$

→  $d\hat{\sigma}(m)$  short distance coefficient including  $m$  dependence

→ allows to use PDFs and FFs with  $\overline{\text{MS}}$  factorization  $\otimes$  massive short distance cross sections

- Treat contributions with charm in the initial state with  $m = 0$
- Massless limit: technically non-trivial, map from phase-space slicing to subtraction method

## Mass factorization

Subtraction terms are associated to mass singularities:  
can be described by

**partonic PDFs and FFs** for collinear splittings  $a \rightarrow b + X$

- initial state:
 
$$f_{g \rightarrow Q}^{(1)}(x, \mu^2) = \frac{\alpha_s(\mu)}{2\pi} P_{g \rightarrow q}^{(0)}(x) \ln \frac{\mu^2}{m^2}$$

$$f_{Q \rightarrow Q}^{(1)}(x, \mu^2) = \frac{\alpha_s(\mu)}{2\pi} C_F \left[ \frac{1+z^2}{1-z} \left( \ln \frac{\mu^2}{m^2} - 2 \ln(1-z) - 1 \right) \right]_+$$

$$f_{g \rightarrow g}^{(1)}(x, \mu^2) = -\frac{\alpha_s(\mu)}{2\pi} \frac{1}{3} \ln \frac{\mu^2}{m^2} \delta(1-x)$$
- final state:
 
$$d_{g \rightarrow Q}^{(1)}(z, \mu^2) = \frac{\alpha_s(\mu)}{2\pi} P_{g \rightarrow q}^{(0)}(z) \ln \frac{\mu^2}{m^2}$$

$$d_{Q \rightarrow Q}^{(1)}(z, \mu^2) = C_F \frac{\alpha_s(\mu)}{2\pi} \left[ \frac{1+z^2}{1-z} \left( \ln \frac{\mu^2}{m^2} - 2 \ln(1-z) - 1 \right) \right]_+$$
- Other partonic distribution functions are zero to order  $\alpha_s$

[Mele, Nason; Kretzer, Schienbein; Melnikov, Mitov]

(2) SUBTRACTION TERMS VIA  $\overline{\text{MS}}$  MASS FACTORIZATION:  $a(k_1)b(k_2) \rightarrow Q(p_1)X$  [1]

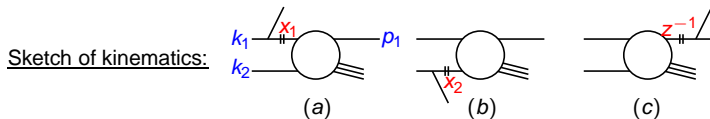


Fig. (a):

$$d\sigma^{\text{sub}}(ab \rightarrow QX) = \int_0^1 dx_1 f_{a \rightarrow i}^{(1)}(x_1, \mu_F^2) d\hat{\sigma}^{(0)}(ib \rightarrow QX)[x_1 k_1, k_2, p_1]$$

$$\equiv f_{a \rightarrow i}^{(1)}(x_1) \otimes d\hat{\sigma}^{(0)}(ib \rightarrow QX)$$

Fig. (b):

$$d\sigma^{\text{sub}}(ab \rightarrow QX) = \int_0^1 dx_2 f_{b \rightarrow j}^{(1)}(x_2, \mu_F^2) d\hat{\sigma}^{(0)}(aj \rightarrow QX)[k_1, x_2 k_2, p_1]$$

$$\equiv f_{b \rightarrow j}^{(1)}(x_2) \otimes d\hat{\sigma}^{(0)}(aj \rightarrow QX)$$

Fig. (c):

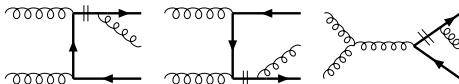
$$d\sigma^{\text{sub}}(ab \rightarrow QX) = \int_0^1 dz d\hat{\sigma}^{(0)}(ab \rightarrow kX)[k_1, k_2, z^{-1} p_1] d_{k \rightarrow Q}^{(1)}(z, \mu_F'^2)$$

$$\equiv d\hat{\sigma}^{(0)}(ab \rightarrow kX) \otimes d_{k \rightarrow Q}^{(1)}(z)$$

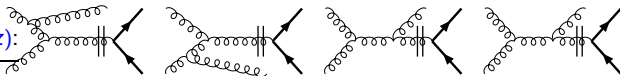
[1] Kniehl, Kramer, I.S., Spiesberger, EPJC41(2005)199

# GRAPHICAL REPRESENTATION OF SUBTRACTION TERMS FOR $gg \rightarrow Q\bar{Q}g$

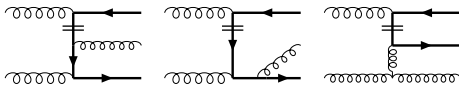
$$\underline{d\hat{\sigma}^{(0)}(gg \rightarrow Q\bar{Q}) \otimes d_{Q \rightarrow Q}^{(1)}(z):}$$



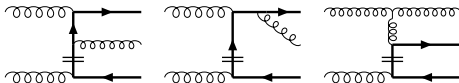
$$\underline{d\hat{\sigma}^{(0)}(gg \rightarrow gg) \otimes d_{g \rightarrow Q}^{(1)}(z):}$$



$$\underline{f_{g \rightarrow Q}^{(1)}(x_1) \otimes d\hat{\sigma}^{(0)}(Qg \rightarrow Qg):}$$

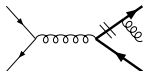


$$\underline{f_{g \rightarrow Q}^{(1)}(x_2) \otimes d\hat{\sigma}^{(0)}(gQ \rightarrow Qg):}$$

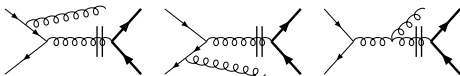




$d\hat{\sigma}^{(0)}(q\bar{q} \rightarrow Q\bar{Q}) \otimes d_{Q \rightarrow Q}^{(1)}(z):$



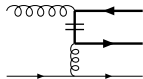
$d\hat{\sigma}^{(0)}(q\bar{q} \rightarrow gg) \otimes d_{g \rightarrow Q}^{(1)}(z):$



$d\hat{\sigma}^{(0)}(gq \rightarrow gq) \otimes d_{g \rightarrow Q}^{(1)}(z):$



$f_{g \rightarrow Q}^{(1)}(x_1) \otimes d\hat{\sigma}^{(0)}(Qq \rightarrow Qq):$



$$S - ACOT_{\chi} = FO + (RS - FOM0)H(m, p_T)$$

FO: Fixed Order; FOM0: Massless limit of FO; RS: Resummed

$$H(m, p_T) = \begin{cases} 1 & : \text{S-ACOT} \\ f(p_T/m) & : \text{S-ACOT}_{\chi} \end{cases}$$

Goal:  $H(m, p_T)$  not ad hoc! Calculated from a general principle.

$$\Rightarrow S - ACOT_{\chi} \rightarrow \begin{cases} \text{FO} & : p_T \sim m \\ \text{RS} & : p_T \gg m \end{cases}$$

Present implementation: S – ACOT (transition to FFNS at small  $p_T$  problematic)

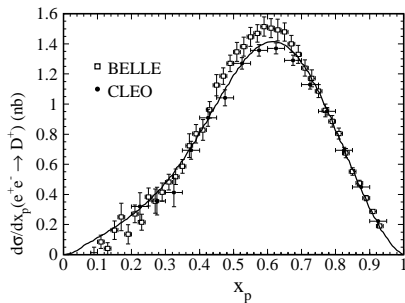
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[1] Cacciari, Greco, Nason, JHEP05(1998)007

## Applications available for

- $\gamma + \gamma \rightarrow D^{*\pm} + X$   
direct and resolved contributions EPJC22, EPJC28
- $\gamma^* + p \rightarrow D^{*\pm} + X$   
photoproduction EPJC38, EPJC62
- $p + \bar{p} \rightarrow (D^0, D^{*\pm}, D^\pm, D_s^\pm, \Lambda_c^\pm) + X$   
good description of Tevatron data PRD71, PRL96, PRD79
- $p + \bar{p} \rightarrow B + X$   
works for Tevatron data at large  $p_T$  PRD77
- work in progress for  $e + p \rightarrow D + X$

# Non-perturbative input



FF for  $c \rightarrow D^*$   
from fitting to  $e^+e^-$  data

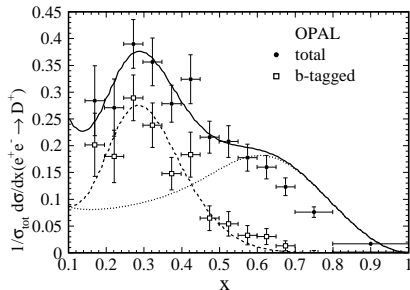
2008 analysis based on GM-VFNS

$\mu_0 = m$

global fit: data from  
ALEPH, OPAL, BELLE, CLEO

BELLE/CLEO fit

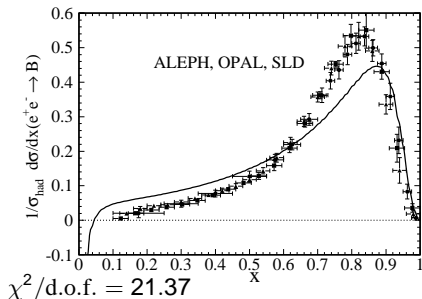
[KKKS: Kneesch, Kramer, Kniehl, IS  
NPB799 (2008)]



tension between low and high energy  
data sets  $\rightarrow$  speculations about non-  
perturbative (power-suppressed) terms

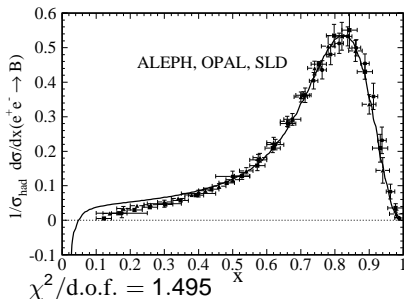
Petersen

$$D(x, \mu_0^2) = N \frac{x(1-x)^2}{[(1-x)^2 + \epsilon x]^2}$$



Kartvelishvili-Likhoded

$$D(x, \mu_0^2) = Nx^\alpha(1-x)^\beta$$



[1] Kniehl, Kramer, IS, Spiesberger, PRD77(2008)014011

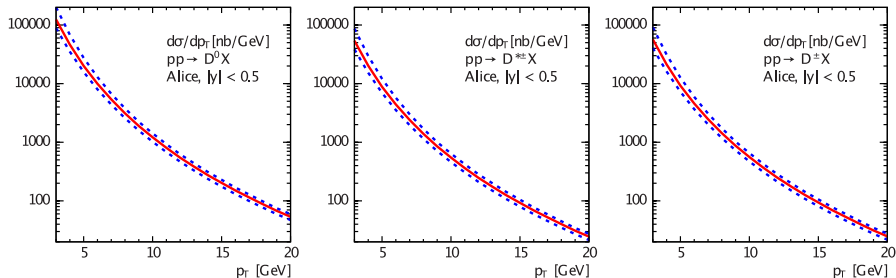
[2] ALEPH, PLB512(2001)30; OPAL, EPJC29(2003)463; SLD, PRL84(2000)4300;

PRD65(2002)092006

# Numerical Results

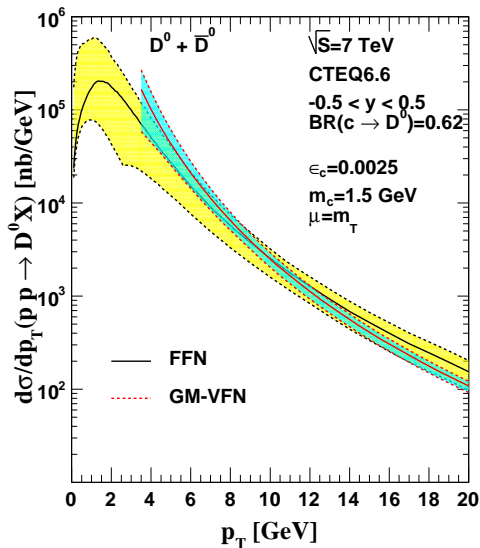
- Predictions for the LHC
  - ALICE:  $pp \rightarrow DX$  at 7 TeV
  - CMS:  $pp \rightarrow BX$  at 7 TeV
  - ATLAS:  $pp \rightarrow DX$  at 7 TeV
  - LHCb:  $pp \rightarrow DX$  at 7 TeV
- Predictions for the Tevatron
  - CDF:  $p\bar{p} \rightarrow DX$  at 1.96 TeV
  - CDF:  $p\bar{p} \rightarrow BX$  at 1.96 TeV
- Predictions for RHIC
  - RHIC:  $pp \rightarrow DX$  at 200 GeV and 500 GeV

# GM-VFNS PREDICTIONS FOR $D^0$ , $D^{*\pm}$ , $D^\pm$ PRODUCTION AT ALICE

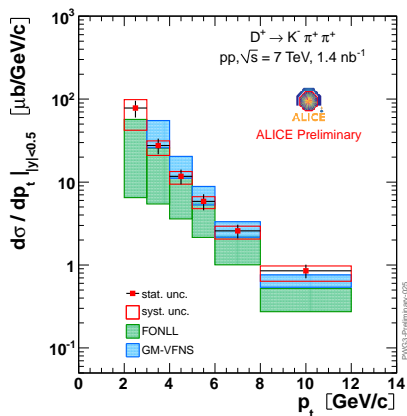
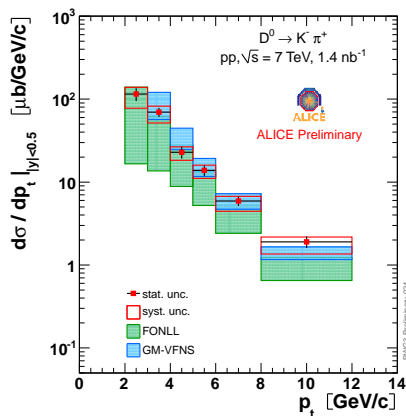


- $pp$  collisions,  $\sqrt{S} = 7$  TeV
- CTEQ6.5 PDF, KKKSc FF,  $m_c = 1.5$  GeV
- Results for  $(D^0 + \bar{D}^0)/2$ ,  $(D^{*+} + D^{*-})/2$ ,  $(D^+ + D^-)/2$  (check)
- Error bands: Varying  $\mu_R$  by factors 2 up and down  
(Except for very small  $p_T$  this gives maximal variation in the cross section)

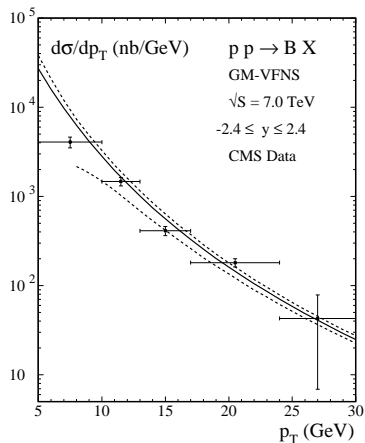




- Presented by A. Dainese at LHC Physics Day, 3. Dec. 2010

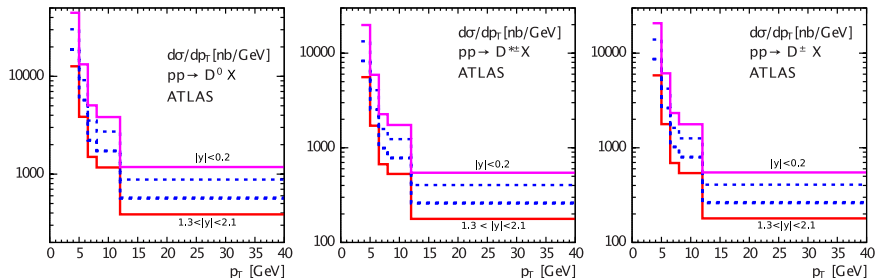


- pQCD predictions (FONLL, GM-VFNS) compatible with data



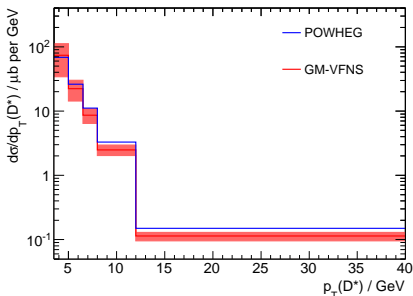
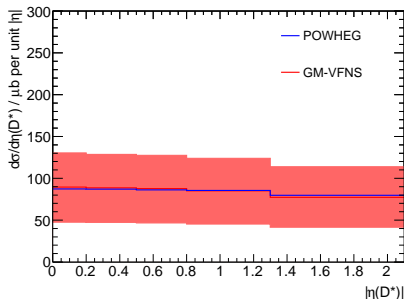
- CMS data: [PRL106\(2011\)112001](#)
- Error band: scale uncertainty

# GM-VFNS PREDICTIONS FOR $D^0$ , $D^{*\pm}$ , $D^\pm$ PRODUCTION AT ATLAS



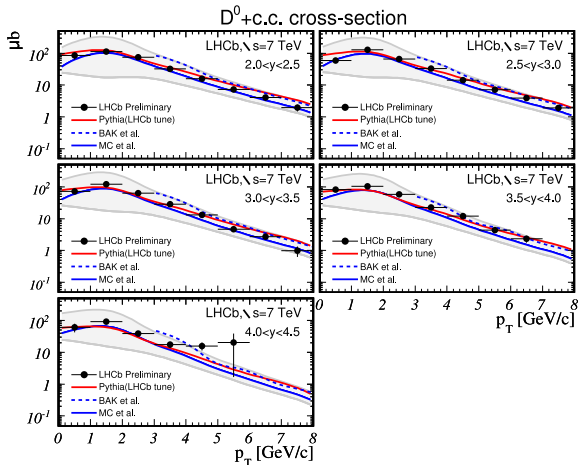
- $pp$  collisions,  $\sqrt{S} = 7$  TeV
- CTEQ6.6 PDF, KKKS06 FF,  $m_c = 1.5$  GeV
- Rapidity bins (top to bottom):  
 $|\eta| < 0.2$ ,  $0.2 < |\eta| < 0.5$ ,  $0.5 < |\eta| < 0.8$ ,  $0.8 < |\eta| < 1.3$ ,  $1.3 < |\eta| < 2.1$
- Results for **average**  $(D^0 + \bar{D}^0)/2$ ,  $(D^{*+} + D^{*-})/2$ ,  $(D^+ + D^-)/2$

Figures provided by S. Head



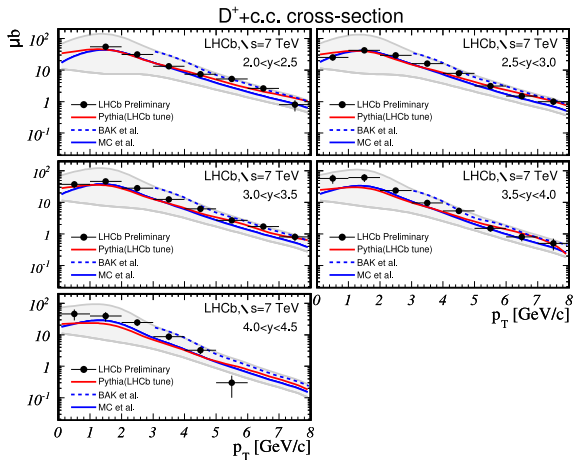
- $pp$  collisions,  $\sqrt{S} = 7$  TeV
- CTEQ6.6 PDF, KKKS06 FF,  $m_c = 1.5$  GeV
- Left figure:  $d\sigma/d\eta$  for  $3.5 < p_T < 40$
- Right figure:  $d\sigma/dp_T$  for  $0 < \eta < 2.1$
- Results for sum  $D^0 + \bar{D}^0$ ,  $D^{*+} + D^{*-}$ ,  $D^+ + D^-$
- Independent variation of  $\mu_R$  and  $\mu_F$  by a factor two up and down

- Prelim. results ( $\mathcal{L} = 1.8 \text{ nb}^{-1}$ ),  $D^0 \rightarrow K^- \pi^+$ , Data: 12 % correlated error not shown



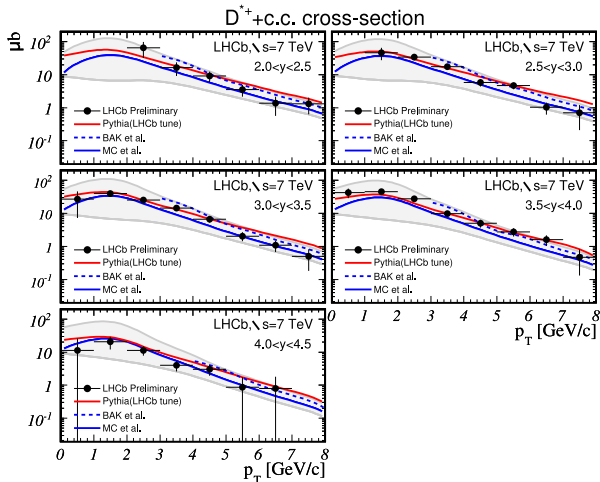
- BAK et al.= GM-VFNS: [B. Kniehl](#), [G. Kramer](#), [I. Schienbein](#), [H. Spiesberger](#)
- MC et al.= FONLL: [M. Cacciari](#), [S. Frixione](#), [M. Mangano](#), [P. Nason](#), [G. Ridolfi](#)

- Prelim. results ( $\mathcal{L} = 1.8 \text{ nb}^{-1}$ ),  $D^+ \rightarrow K^- \pi^+ \pi^+$ , Data: 14 % correlated error not shown



- BAK et al.= GM-VFNS: [B. Kniehl, G. Kramer, I. Schienbein, H. Spiesberger](#)
- MC et al.= FONLL: [M. Cacciari, S. Frixione, M. Mangano, P. Nason, G. Ridolfi](#)

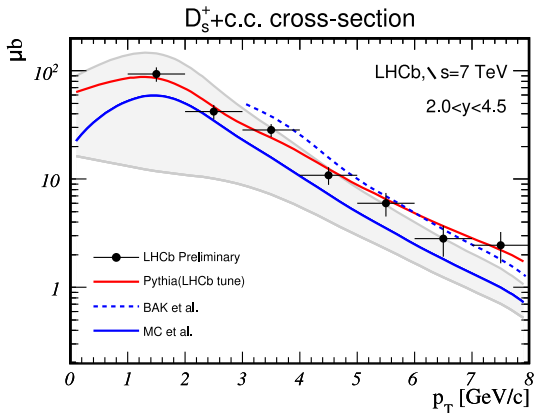
- Preliminary ( $\mathcal{L} = 1.8 \text{ nb}^{-1}$ ),  $D^{*+} \rightarrow (D^0 \rightarrow K^- \pi^+) \pi^+$ , Data: 14 % corr. error not shown



- BAK et al.= GM-VFNS: **B. Kniehl, G. Kramer, I. Schienbein, H. Spiesberger**
- MC et al.= FONLL: **M. Cacciari, S. Frixione, M. Mangano, P. Nason, G. Ridolfi**



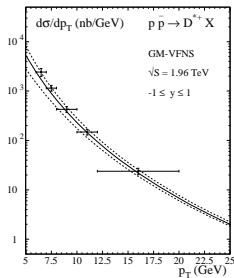
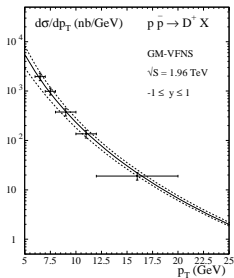
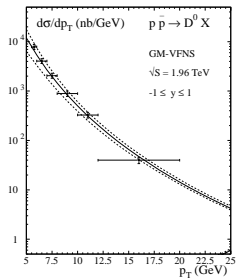
- Preliminary ( $\mathcal{L} = 1.8 \text{ nb}^{-1}$ ),  $D^s \rightarrow K^- K^+ \pi^+$ , Data: 16 % corr. error not shown



- BAK et al.= GM-VFNS: [B. Kniehl](#), [G. Kramer](#), [I. Schienbein](#), [H. Spiesberger](#)
- MC et al.= FONLL: [M. Cacciari](#), [S. Frixione](#), [M. Mangano](#), [P. Nason](#), [G. Ridolfi](#)

# HADROPRODUCTION OF $D^0, D^+, D^{*+}, D_s^+$

## GM-VFNS RESULTS W/ KKKSC FFs [1]

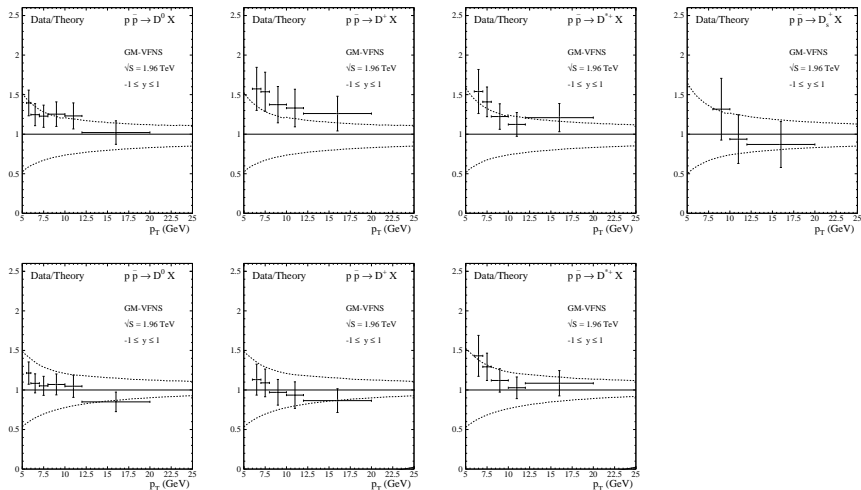


- $d\sigma/dp_T$  [nb/GeV]  $|y| \leq 1$  prompt charm
- Uncertainty band:  $1/2 \leq \mu_R/m_T, \mu_F/m_T \leq 2$  ( $m_T = \sqrt{p_T^2 + m_c^2}$ )
- CDF data from run II [2]
- **GM-VFNS describes data within errors**

[1] Kniehl, Kramer, IS, Spiesberger, arXiv:0901.4130[hep-ph], PRD(to appear)

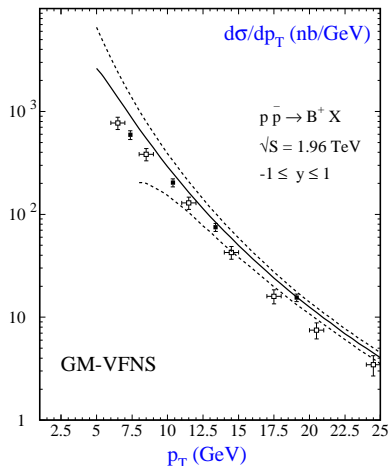
[2] Acosta et al., PRL91(2003)241804

# COMPARISON W/ PREVIOUS KK FFs [1]



- New KKKSc FFs improve agreement w/ CDF data.

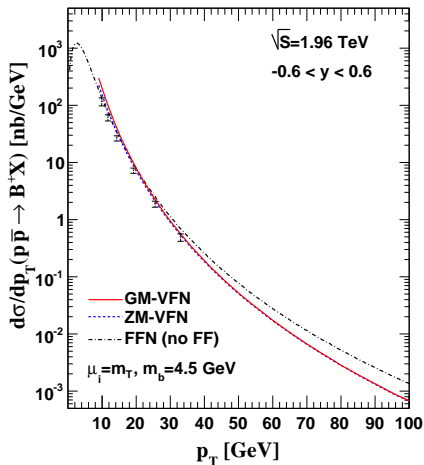
[1] Kniehl, Kramer, PRD74(2006)037502



- CDF (1.96 TeV):
  - open squares  $J/\psi X$  [1]
  - solid squares  $J/\psi K^+$  [2]
- CTEQ6.1M PDFs
- $m_b = 4.5$  GeV
- $\Lambda_{\overline{MS}}^{(5)} = 227$  MeV  $\rightsquigarrow \alpha_s^{(5)} = 0.1181$
- $1/2 \leq \mu_R/m_T, \mu_F/m_T, \mu_R/\mu_F \leq 2$   
 $(m_T = \sqrt{p_T^2 + m_b^2})$

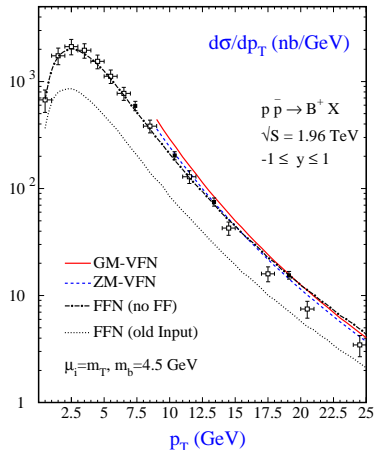
[1] CDF, PRD71(2005)032001

[2] CDF, PRD75(2007)012010



- CDF II (preliminary) [1]
- $\mu_R = \mu_F = m_T$
- for  $p_T \gg m_b$ :
  - GM-VFN merges w/ ZM-VFN
  - FFN breaks down
- data point in bin [29,40] favors GM-VFN

[1] Kraus, FERMILAB-THESIS-2006-47; Annovi, FERMILAB-CONF-07-509-E

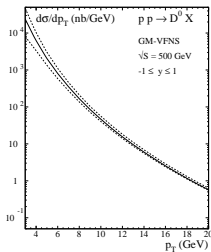
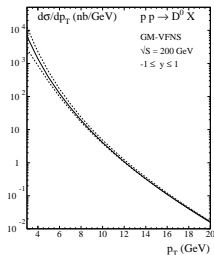


- obsolete FFN as above
- up-to-date FFN evaluated with
  - CTEQ6.1M PDFs
  - $m_b = 4.5 \text{ GeV}$
  - $\Lambda_{\overline{MS}}^{(5)} = 227 \text{ MeV} \rightsquigarrow \alpha_s^{(5)} = 0.1181$
  - $D(x) = B(b \rightarrow B)\delta(1-x)$  with  $B(b \rightarrow B) = 39.8\%$

[1] Kniehl, Kramer, IS, Spiesberger, PRD77(2008)014011

# INTRINSIC CHARM IN THE PROTON

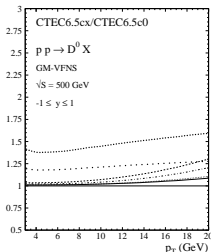
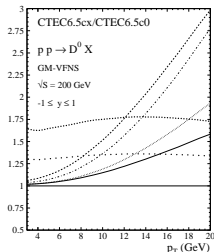
## D-MESONS AT RHIC



$$(\frac{d\sigma}{dp_T})(pp \rightarrow D^0 + X)$$

$$\sqrt{s} = 200, 500 \text{ GeV}$$

$$|y| < 1$$

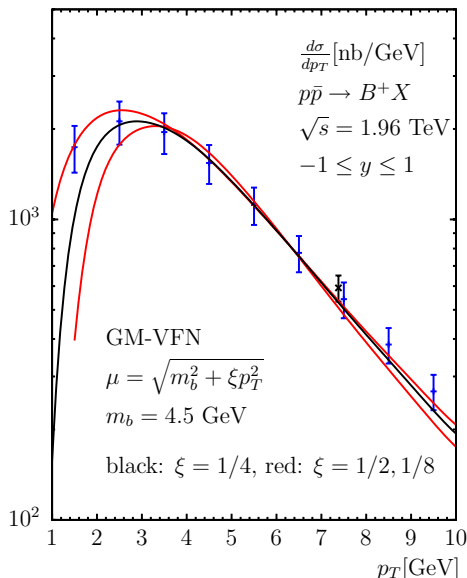


IC Model	moderate	marginal
BHPS	solid	dashed
Meson-cloud	densely dotted	dot-dashed
Sealike	scarsely dotted	dotted

- Presented an overview of theoretical approaches to hadroproduction of heavy quarks
- Main message: GM-VFNS predictions in good agreement with first LHC data
- Paper in preparation
  - More predictions (GM-VFNS,FFNS) for  $D$  and  $B$  mesons
  - Uncertainties
  - Matching to FFNS at small  $p_T$



# Backup



- evaluate  $d\hat{\sigma}_{\text{ZM}}^{(1)}(Q + g/q \rightarrow Q + X)$   
 @ LO to match  
 $f_{g \rightarrow Q}^{(1)} \otimes d\hat{\sigma}^{(0)}(Q + g/q \rightarrow Q + g/q)$
- evaluate  
 $d\hat{\sigma}^{(0)}(gg/q\bar{q} \rightarrow Q\bar{Q}) \otimes d_{Q \rightarrow Q}^{(1)}$   
 w/  $m_Q \neq 0$  to match  
 $d\hat{\sigma}_{\text{GM}}^{(1)}(gg/q\bar{q} \rightarrow Q/\bar{Q} + X)$
- impose  $\theta(\hat{s} - 4m_Q^2)$  on massless kinematics
- choose  $\mu_F^2 = m_Q^2 + \xi p_T^2$  so that  
 $\mu_F \xrightarrow{p_T \rightarrow 0} m_Q = \mu_0$
- $G(m, p_T) \equiv 1$  in contrast to FONLL