Introduction to the Standard Model of particle physics

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III. The Standard Model of particle physics (2nd round)

The general procedure

- **Introduce Fields & Symmetries**

The general procedure

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- **Construct a local Lagrangian density**

The general procedure

- Introduce Fields & Symmetries
- Construct a local Lagrangian density
- **Describe Observables**
  - How to measure them?
  - How to calculate them?

The general procedure

- Introduce Fields & Symmetries
- Construct a local Lagrangian density
- Describe Observables
  - How to measure them?
  - How to calculate them?
- **Falsify: Compare theory with data**

Fields & Symmetries

Matter content of the Standard Model (including the antiparticles)

Matter				Higgs		Gauge	
$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$(\mathbf{3},\mathbf{2})_{1/3}$	$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$(\mathbf{1},\mathbf{2})_{-1}$	$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$	$(\mathbf{1},\mathbf{2})_1$	A	$(\mathbf{1},\mathbf{1})_0$
u_R^c	$(\overline{\mathbf{3}},\mathbf{1})_{-4/3}$	e_R^c	$(\mathbf{1},\mathbf{1})_{2}$			W	$(\mathbf{1},\mathbf{3})_0$
d_R^c	$(\overline{\mathbf{3}},\mathbf{1})_{2/3}$	ν_R^c	$(\mathbf{1},\mathbf{1})_{0}$			G	$(\mathbf{8},\mathbf{1})_0$

$Q^c = \begin{pmatrix} u_L^c \\ d_L^c \end{pmatrix}$	$(\overline{\mathbf{3}},\overline{\mathbf{2}})_{-1/3}$	$L^c = \begin{pmatrix} \nu_L^c \\ e_L^c \end{pmatrix}$	$(\mathbf{1},\overline{\mathbf{2}})_{1}$	$H = \begin{pmatrix} h^- \\ h^0 \end{pmatrix}$	$(\mathbf{1},\overline{\mathbf{2}})_{-1}$	A	$(\mathbf{1},\mathbf{1})_0$
u_R	$(\mathbf{3},\mathbf{1})_{4/3}$	e_R	$(\mathbf{1},\mathbf{1})_{-2}$			W	$(\mathbf{1},\mathbf{3})_0$
d_R	$(\mathbf{3},\mathbf{1})_{-2/3}$	ν_R	$(\mathbf{1},\mathbf{1})_{0}$			G	$(\mathbf{8},\mathbf{1})_0$

Matter content of the Standard Model

- Left-handed up quark $\mathbf{u_L}$:
  - LH Weyl fermion: $\mathbf{u_{L\alpha} \sim (1/2,0)}$ of so(1,3)
  - a color triplet: $\mathbf{u_{Li} \sim 3}$ of $SU(3)_c$
  - Indices: $\mathbf{(u_L)_{i\alpha}}$ with i=1,2,3 and α=1,2
- Similarly, left-handed down quark $\mathbf{d_L}$
- $\mathbf{u_L}$ and $\mathbf{d_L}$ components of a $SU(2)_L$ doublet: $\mathbf{Q_\beta = (u_L , d_L) \sim 2}$
  - **Q** carries a hypercharge 1/3: $\mathbf{Q \sim (3,2)_{1/3}}$ of $SU(3)_c \times SU(2)_L \times U(1)_Y$
  - Indices: $\mathbf{Q_{\beta i \alpha}}$ with β=1,2 ; i=1,2,3 and α=1,2

Matter content of the Standard Model

- There are three generations: Q_k , k =1,2,3
- Lot's of indices: $Q_{k\beta i\alpha}(x)$
- We know how the indices β, i, α transform under symmetry operations (i.e., which representations we have to use for the generators)

Matter content of the Standard Model

- Right-handed up quark u_R:
  - RH Weyl fermion: $u_{R\dot{\alpha}} \sim (0,1/2)$ of so(1,3)
  - a color triplet: $u_{Ri} \sim 3$ of $SU(3)_c$
  - a singlet of $SU(2)_L$: $u_R \sim 1$ (no index needed)
  - u_R carries hypercharge 4/3: $u_R \sim (3,1)_{4/3}$
  - Indices: $(u_R)_{i\dot{\alpha}}$ with i=1,2,3 and $\dot{\alpha}$=1,2 (Note the dot)
  - Note that $u_R^c \sim (3^*,1)_{-4/3}$

Matter content of the Standard Model

- Again there are three generations: u_{Rk} , k =1,2,3
- Lot's of indices: $u_{Rki\alpha.}(x)$
- And so on for the other fields ...

Terms for the Lagrangian

How to build Lorentz scalars? Scalar field (like the Higgs)

Real field ϕ

$$\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$$

Complex field $\phi = \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2)$

$$\partial_\mu\phi^*\partial^\mu\phi - m^2\phi^*\phi$$

Note: The mass dimension of each term in the Lagrangian has to be 4!

How to build Lorentz scalars? Fermions (spin 1/2)

Left-handed Weyl spinor

$$i\psi_L^\dagger \overline{\sigma}^\mu \partial_\mu \psi_L$$

Right-handed Weyl spinor

$$i\psi_R^\dagger \sigma^\mu \partial_\mu \psi_R$$

Mass term mixes left and right

$$i\psi_L^\dagger \overline{\sigma}^\mu \partial_\mu \psi_L + i\psi_R^\dagger \sigma^\mu \partial_\mu \psi_R - m(\psi_L^\dagger \psi_R + \psi_R^\dagger \psi_L)$$

$$\sigma^\mu = (1, \sigma^i)$$
$$\bar{\sigma}^\mu = (1, -\sigma^i)$$

Dirac spinor in chiral basis

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \qquad i\overline{\Psi}\gamma^\mu \partial_\mu \Psi - m\overline{\Psi}\Psi \quad \text{with} \quad \overline{\Psi} = \Psi^\dagger \gamma^0 \quad \text{and} \quad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}$$

How to build Lorentz scalars?
Vector boson (spin 1)

U(1) gauge boson ("Photon")

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu \quad \text{where} \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Mass term allowed by Lorentz invariance;
forbidden by gauge invariance

In principle, there is a second invariant

$$-\frac{1}{4}F_{\mu\nu}\widetilde{F}^{\mu\nu} \quad \text{with} \quad \widetilde{F}_{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

$$F\tilde{F} \propto \vec{E}\cdot\vec{B}$$

Violates Parity, Time reversal, and CP symmetry; prop. to a total divergence → doesn't contribute in QED

BUT strong CP problem in QCD

Gauge symmetry

- Idea: Generate interactions from free Lagrangian by imposing **local (i.e. $\alpha = \alpha(x)$) symmetries**
- Does not fall from heavens; generalization of 'minimal coupling' in electrodynamics
- Final judge is experiment: It works!

Local gauge invariance for a complex scalar field

$$\partial_\mu\phi^*\partial^\mu\phi - m^2\phi^*\phi \quad \text{is invariant under } \phi \to e^{i\alpha}\phi.$$

What if now α = α(x) depends on the space-time?

$$\begin{aligned}
&\partial_\mu(e^{i\alpha(x)}\phi)^*\partial^\mu(e^{i\alpha(x)}\phi) - m^2(e^{i\alpha(x)}\phi)^*(e^{i\alpha(x)}\phi)\\
&= [\partial_\mu e^{i\alpha(x)}\cdot\phi + e^{i\alpha(x)}\cdot\partial_\mu\phi]^*[\partial^\mu e^{i\alpha(x)}\cdot\phi + e^{i\alpha(x)}\cdot\partial^\mu\phi] - m^2\phi^*\phi\\
&= [ie^{i\alpha(x)}\partial_\mu\alpha(x)\cdot\phi + e^{i\alpha(x)}\cdot\partial_\mu\phi]^*[ie^{i\alpha(x)}\partial^\mu\alpha(x)\cdot\phi + e^{i\alpha(x)}\cdot\partial^\mu\phi] - m^2\phi^*\phi\\
&= [-ie^{-i\alpha(x)}\partial_\mu\alpha(x)\cdot\phi^* + e^{-i\alpha(x)}\cdot\partial_\mu\phi^*][ie^{i\alpha(x)}\partial^\mu\alpha(x)\cdot\phi + e^{i\alpha(x)}\cdot\partial^\mu\phi] - m^2\phi^*\phi\\
&= -ie^{-i\alpha(x)}\partial_\mu\alpha(x)\cdot\phi^*\cdot ie^{i\alpha(x)}\partial^\mu\alpha(x)\cdot\phi\\
&\quad - ie^{-i\alpha(x)}\partial_\mu\alpha(x)\cdot\phi^*\cdot e^{i\alpha(x)}\cdot\partial^\mu\phi\\
&\quad + e^{-i\alpha(x)}\cdot\partial_\mu\phi^*\cdot ie^{i\alpha(x)}\partial^\mu\alpha(x)\cdot\phi\\
&\quad + e^{-i\alpha(x)}\cdot\partial_\mu\phi^*\cdot e^{i\alpha(x)}\cdot\partial^\mu\phi\\
&\quad - m^2\phi^*\phi\\
&= \partial_\mu\phi\cdot\partial^\mu\phi - m^2\phi^*\phi + \text{non-zero terms}
\end{aligned}$$

Not invariant under U(1)!

Local gauge invariance for a complex scalar field

Can we find a derivative operator that commutes with the gauge transformation?

Define

$$D_\mu = \partial_\mu + iA_\mu,$$

where the *gauge field* A_μ transforms as

$$A_\mu \to A_\mu - \partial_\mu \alpha$$

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$$\begin{aligned}
D_\mu \phi \to & \, (\partial_\mu + i[A_\mu - \partial_\mu \alpha(x)])[e^{i\alpha(x)}\phi] \\
&= \partial_\mu[e^{i\alpha(x)}\phi] + i[A_\mu - \partial_\mu \alpha(x)][e^{i\alpha(x)}\phi] \\
&= ie^{i\alpha(x)}\partial_\mu \alpha(x) \cdot \phi + e^{i\alpha(x)}\partial_\mu \phi + iA_\mu e^{i\alpha(x)}\phi - i\partial_\mu \alpha(x) e^{i\alpha(x)}\phi \\
&= e^{i\alpha(x)}\partial_\mu \phi + iA_\mu e^{i\alpha(x)}\phi \\
&= e^{i\alpha(x)}[\partial_\mu \phi + iA_\mu]\phi \\
&= e^{i\alpha(x)} D_\mu \phi
\end{aligned}$$

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&= ie^{i\alpha(x)}\partial_\mu \alpha(x) \cdot \phi + e^{i\alpha(x)}\partial_\mu \phi + iA_\mu e^{i\alpha(x)}\phi - i\partial_\mu \alpha(x) e^{i\alpha(x)}\phi \\
&= e^{i\alpha(x)}\partial_\mu \phi + iA_\mu e^{i\alpha(x)}\phi \\
&= e^{i\alpha(x)}[\partial_\mu \phi + iA_\mu]\phi \\
&= e^{i\alpha(x)} D_\mu \phi
\end{aligned}$$

Nota bene:

- We call D_μ the *covariant derivative*, because it transforms just like ϕ itself:

$$\phi \to e^{i\alpha(x)}\phi \quad \text{and} \quad D_\mu \phi \to e^{i\alpha(x)} D_\mu \phi$$

Local gauge invariance for a complex scalar field

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Define

$$D_\mu = \partial_\mu + iA_\mu,$$

where the *gauge field* A_μ transforms as

$$A_\mu \to A_\mu - \partial_\mu \alpha$$

$$\begin{aligned} D_\mu\phi &\to (\partial_\mu + i[A_\mu - \partial_\mu\alpha(x)])[e^{i\alpha(x)}\phi] \\ &= \partial_\mu[e^{i\alpha(x)}\phi] + i[A_\mu - \partial_\mu\alpha(x)][e^{i\alpha(x)}\phi] \\ &= ie^{i\alpha(x)}\partial_\mu\alpha(x)\cdot\phi + e^{i\alpha(x)}\partial_\mu\phi + iA_\mu e^{i\alpha(x)}\phi - i\partial_\mu\alpha(x)e^{i\alpha(x)}\phi \\ &= e^{i\alpha(x)}\partial_\mu\phi + iA_\mu e^{i\alpha(x)}\phi \\ &= e^{i\alpha(x)}[\partial_\mu\phi + iA_\mu]\phi \\ &= e^{i\alpha(x)}D_\mu\phi \end{aligned}$$

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$$\phi \to e^{i\alpha(x)}\phi \quad \text{and} \quad D_\mu\phi \to e^{i\alpha(x)}D_\mu\phi$$

$$D_\mu\phi^* D^\mu\phi - m^2\phi^*\phi \to e^{-i\alpha(x)}D_\mu\phi^*\cdot e^{i\alpha(x)}D^\mu\phi - m^2 e^{-i\alpha(x)}\phi^*\cdot e^{i\alpha(x)}\phi = D_\mu\phi^* D^\mu\phi - m^2$$

Expanding the Lagrangian

$D_\mu\phi^* D^\mu\phi - m^2\phi^*\phi$ invariant under local U(1) transformations

$$D_\mu\phi^* D^\mu\phi - m^2\phi^*\phi = \partial_\mu\phi^*\partial^\mu\phi + iA^\mu(\phi\partial_\mu\phi^* - \phi^*\partial_\mu\phi) + \phi^*\phi A_\mu A^\mu - m^2\phi^*\phi$$

- Demand symmetry $\rightarrow$ Generate interactions
- Generated mass for gauge boson (after ϕ acquires a vacuum expectation value)
- Explicit mass term forbidden by gauge symmetry (although otherwise allowed):

$$m^2 A_\mu A^\mu \rightarrow m^2(A_\mu - \partial_\mu\alpha)(A_\mu - \partial_\mu\alpha) \neq m^2 A_\mu A^\mu$$

- Simplest form of Higgs mechanism
- Vector-scalar-scalar interaction

Non-Abelian gauge symmetry

Abelian	Non-Abelian: component notation	Non-Abelian: vector notation
$U = e^{i\alpha(x)}$	$U = e^{i\alpha^a(x)T_R^a}$	$U = e^{i\alpha^a(x)T_R^a}$
$\phi \to U\phi$	$\mathbf{\Phi}^i \to U^i{}_k \mathbf{\Phi}^k$	$\mathbf{\Phi} \to U\mathbf{\Phi}$
A_μ	$A_\mu^a T_R^a$	$\boldsymbol{A}_\mu$
$A_\mu \to A_\mu - \partial_\mu \alpha$	$A_\mu^a T^a \to U A_\mu^a T^a U^\dagger - \frac{i}{g}(\partial_\mu U)U^\dagger$	$\boldsymbol{A}_\mu \to U\boldsymbol{A}_\mu U^\dagger - \frac{i}{g}(\partial_\mu U)U^\dagger$
$F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu$	$F_{\mu\nu}^a := \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f^{abc} A_\mu^b A_\nu^c$	$\boldsymbol{F}_{\mu\nu} := \partial_\mu \boldsymbol{A}_\nu - \partial_\nu \boldsymbol{A}_\mu + ig[\boldsymbol{A}_\mu, \boldsymbol{A}_\nu]$
$F_{\mu\nu} \to F_{\mu\nu}$		$\boldsymbol{F}_{\mu\nu} \to U\boldsymbol{F}_{\mu\nu}U^\dagger$
$F_{\mu\nu}$ invariant	$F_{\mu\nu}^a F^{a\mu\nu}$ invariant	$\mathrm{Tr}(\boldsymbol{F}_{\mu\nu}\boldsymbol{F}^{\mu\nu})$ invariant

$$D_\mu = \partial_\mu + igA_\mu^a T_R^a$$

Conjecture

- All fundamental internal symmetries are gauge symmetries.
- Global symmetries are just "accidental" and not exact.

Spontaneous Symmetry Breaking

One page summary of the world

Gauge group

$\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

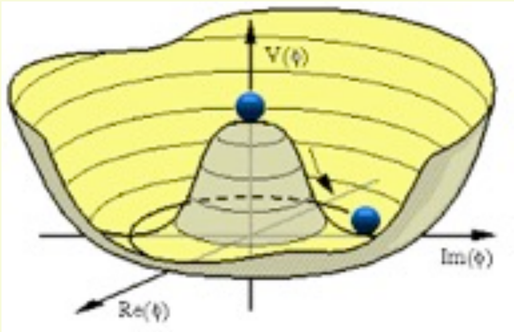
Particle content

MATTER				HIGGS		GAUGE	
$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$(\mathbf{3},\mathbf{2})_{1/3}$	$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$(\mathbf{1},\mathbf{2})_{-1}$	$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$	$(\mathbf{1},\mathbf{2})_1$	B	$(\mathbf{1},\mathbf{1})_0$
u_R^c	$(\overline{\mathbf{3}},\mathbf{1})_{-4/3}$	e_R^c	$(\mathbf{1},\mathbf{1})_2$			W	$(\mathbf{1},\mathbf{3})_0$
d_R^c	$(\overline{\mathbf{3}},\mathbf{1})_{2/3}$	ν_R^c	$(\mathbf{1},\mathbf{1})_0$			G	$(\mathbf{8},\mathbf{1})_0$

Lagrangian
(Lorentz + gauge + renormalizable)

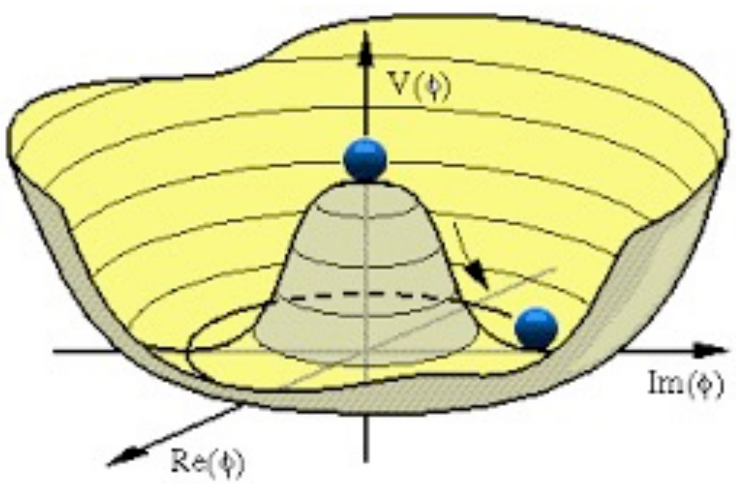
$$\mathcal{L} = -\frac{1}{4}G^{\alpha}_{\mu\nu}G^{\alpha\mu\nu} + \ldots \overline{Q}_k \not{D} Q_k + \ldots (D_\mu H)^\dagger (D^\mu H) - \mu^2 H^\dagger H - \frac{\lambda}{4!}(H^\dagger H)^2 + \ldots Y_{k\ell}\overline{Q}_k H (u_R)_\ell$$

SSB



- $H \to H' + \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$
- $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_Q$
- $B, W^3 \to \gamma, Z^0$ and $W^1_\mu, W^2_\mu \to W^+, W^-$
- Fermions acquire mass through Yukawa couplings to Higgs

The Higgs mechanism



- The Higgs potential: $V = \mu^2 \phi^\dagger\phi + \lambda (\phi^\dagger\phi)^2$
- Vacuum = Ground state = Minimum of V:
- If $\mu^2>0$ (massive particle): $\phi_{min} = 0$ (no symmetry breaking)
- If $\mu^2<0$: $\phi_{min} = \pm v = \pm(-\mu^2/\lambda)^{1/2}$
  These two minima in one dimension correspond to a continuum of minimum values in SU(2).
  The point $\phi = 0$ is now instable.
- Choosing the minimum (e.g. at +v) gives the vacuum a preferred direction in isospin space → spontaneous symmetry breaking
- Perform perturbation around the minimum

Higgs self-couplings

In the SM, the Higgs self-couplings are a consequence of the Higgs potential after expansion of the Higgs field $H \sim (1,2)_1$ around the vacuum expectation value which breaks the ew symmetry:

$$V_H = \mu^2 H^\dagger H + \eta (H^\dagger H)^2 \to \frac{1}{2} m_h^2 h^2 + \sqrt{\frac{\eta}{2}} m_h h^3 + \frac{\eta}{4} h^4$$

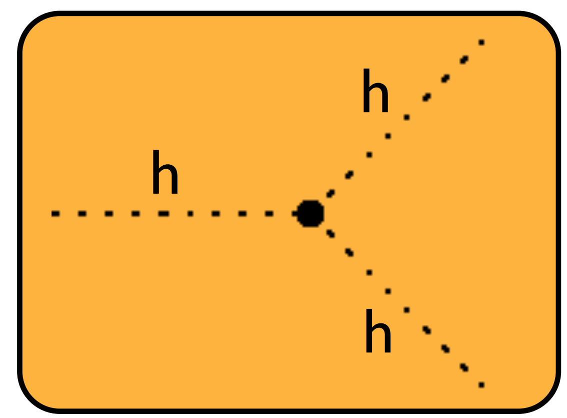
with: $$m_h^2 = 2\eta v^2 \, , \; v^2 = -\mu^2/\eta$$

Note: v=246 GeV is fixed by the precision measures of G_F

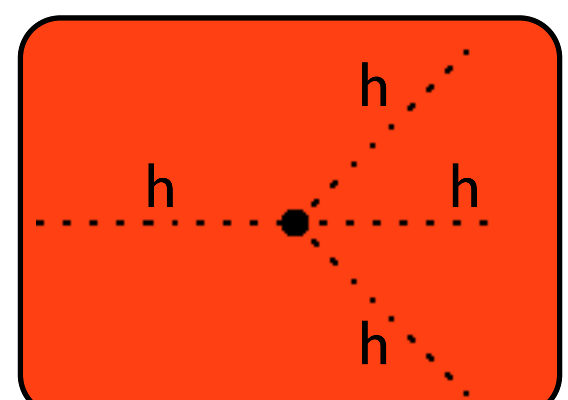
In order to completely reconstruct the Higgs potential, on has to:

- Measure the 3h-vertex:
  via a measurement of Higgs pair production

  $$\lambda_{3h}^{\mathrm{SM}} = \sqrt{\frac{\eta}{2}} m_h$$



- Measure the 4h-vertex:
  more difficult, not accessible at the LHC in the high-lumi phase



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with:

Note: v=246 GeV is fixed by the precision measures of G_F

Measuring the 3h-couplings: major goal for the high-lumi phase at the LHC

The Higgs particle is just the messenger!

Need to reconstruct the potential

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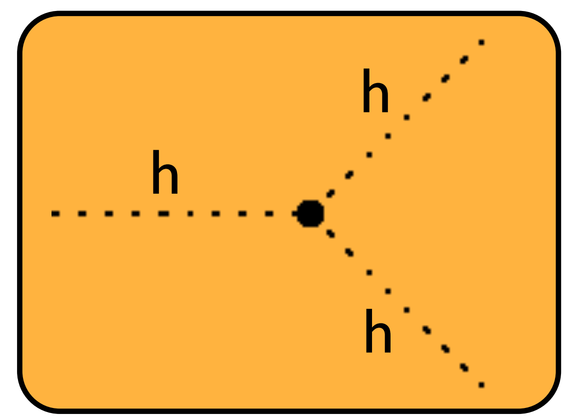

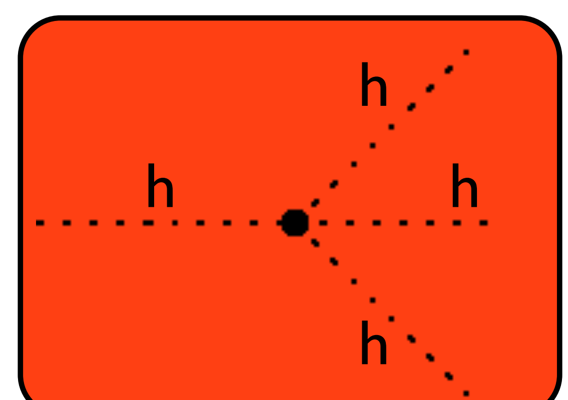

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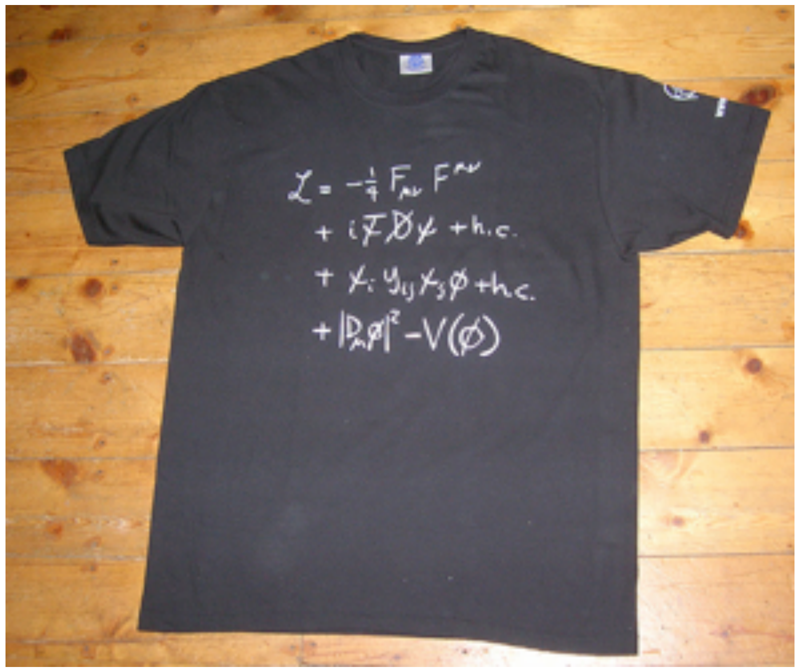
Not so compact anymore



Lagrangien du Modèle Standard

$$
\begin{aligned}
\mathcal{L}_{SM} = & \sum_{\ell=e,\mu,\tau} i\bar{\psi}_\ell \gamma^\mu \partial_\mu \psi_\ell + \sum_{\ell'=\nu_e,\nu_\mu,\nu_\tau} i\bar{\psi}_{\ell'} \gamma^\mu \partial_\mu \psi_{\ell'} + \sum_i^3 \sum_{q=u,c,t} i\bar{\psi}_{q_i} \gamma^\mu \partial_\mu \psi_{q_i} + \sum_i^3 \sum_{q'=d,s,b} i\bar{\psi}_{q'_i} \gamma^\mu \partial_\mu \psi_{q'_i} \\
& -\frac{1}{2}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)(\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}) - \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) \\
& -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{1}{4}\sum_{a=1}^8 (\partial_\mu G^a_\nu - \partial_\nu G^a_\mu)(\partial^\mu G^{a\nu} - \partial^\nu G^{a\mu}) + \frac{1}{2}\partial_\mu h \partial^\mu h \\
& - \sum_{\ell=e,\mu,\tau} \frac{\lambda_\ell v}{\sqrt{2}} \bar{\psi}_\ell \psi_\ell - \sum_i^3 \sum_{q=u,c,t} \frac{\lambda_q v}{\sqrt{2}} \bar{\psi}_{q_i} \psi_{q_i} - \sum_i^3 \sum_{q'=d,s,b} \frac{\lambda_{q'} v}{\sqrt{2}} \bar{\psi}_{q'_i} \psi_{q'_i} \\
& - \left(\frac{gv}{2}\right)^2 W^+_\mu W^{-\mu} - \frac{1}{2}\left(\frac{gv}{2\cos\theta_W}\right)^2 Z_\mu Z^\mu - \frac{1}{2}\left(-2m^2\right) h^2 \\
& + \frac{g}{4\cos\theta_W}\left(\sum_{\ell=e,\mu,\tau} \bar{\psi}_\ell \gamma^\mu (4\sin^2\theta_W - 1 + \gamma^5) \psi_\ell Z_\mu + \sum_{\ell'=\nu_e,\nu_\mu,\nu_\tau} \bar{\psi}_{\ell'} \gamma^\mu (1-\gamma^5) \psi_{\ell'} Z_\mu \right) \\
& + \frac{g}{4\cos\theta_W}\left(\sum_i^3 \sum_{q=u,c,t} \bar{\psi}_{q_i} \gamma^\mu (1 - \frac{8}{3}\sin^2\theta_W - \gamma^5) \psi_{q_i} Z_\mu + \sum_i^3 \sum_{q'=b,s,b} \bar{\psi}_{q'_i} \gamma^\mu (\frac{4}{3}\sin^2\theta_W - 1 + \gamma^5) \psi_{q'_i} Z_\mu \right) \\
& + \frac{g}{2\sqrt{2}}\left(\sum_{\ell=e,\mu,\tau} \bar{\psi}_{\nu_\ell} \gamma^\mu (1-\gamma^5) \psi_\ell W^+_\mu + \sum_{\ell=e,\mu,\tau} \bar{\psi}_\ell \gamma^\mu (1-\gamma^5) \psi_{\nu_\ell} W^-_\mu \right) \\
& + \frac{g}{2\sqrt{2}}\left(\sum_i^3 \sum_{\substack{q=u,c,t \\ q'=d,s,b}} V_{qq'} \bar{\psi}_q \gamma^\mu (1-\gamma^5) \psi_{q'_i} W^+_\mu + \sum_i^3 \sum_{\substack{q=u,c,t \\ q'=d,s,b}} V^*_{qq'} \bar{\psi}_{q'_i} \gamma^\mu (1-\gamma^5) \psi_{q_i} W^-_\mu \right) \\
& + g_{em}\left(- \sum_{\ell=e,\mu,\tau} \bar{\psi}_\ell \gamma^\mu \psi_\ell A_\mu + \frac{2}{3} \sum_{q=u,c,t} \bar{\psi}_{q_i} \gamma^\mu \psi_{q_i} A_\mu - \frac{1}{3} \sum_{q'=d,s,b} \bar{\psi}_{q'_i} \gamma^\mu \psi_{q'_i} A_\mu \right) \\
& + g_s\left(\sum_{i,j}^3 \sum_a^8 \sum_{q=u,c,t} \bar{\psi}_{q_j} \gamma^\mu \psi_{q_i} G^a_\mu T^a_{ij} + \sum_{i,j}^3 \sum_a^8 \sum_{q'=d,s,b} \bar{\psi}_{q'_j} \gamma^\mu \psi_{q'_i} G^a_\mu T^a_{ij} \right) \\
& - \sum_{\ell=e,\mu,\tau} \frac{\lambda_\ell}{\sqrt{2}} \bar{\psi}_\ell \psi_\ell h - \sum_i^3 \sum_{q=u,c,t} \frac{\lambda_q}{\sqrt{2}} \bar{\psi}_{q_i} \psi_{q_i} h - \sum_i^3 \sum_{q'=d,s,b} \frac{\lambda_{q'}}{\sqrt{2}} \bar{\psi}_{q'_i} \psi_{q'_i} h \\
& + ig_{em}\left[\partial_\mu A_\nu W^{-\mu} W^{+\nu} + \partial_\mu W^+_\nu W^{-\nu} A^\mu + \partial_\mu W^-_\nu W^{+\mu} A^\nu - \partial_\mu A_\nu W^{-\nu} W^{+\mu} \right. \\
& \qquad \left. - \partial_\mu W^+_\nu W^{-\mu} A^\nu - \partial_\mu W^-_\nu W^{+\nu} A^\mu \right] \\
& + ig\cos\theta_W\left[\partial_\mu Z_\nu W^{-\mu} W^{+\nu} + \partial_\mu W^+_\nu W^{-\nu} Z^\mu + \partial_\mu W^-_\nu W^{+\mu} Z^\nu - \partial_\mu Z_\nu W^{-\nu} W^{+\mu} \right. \\
& \qquad \left. - \partial_\mu W^+_\nu W^{-\mu} Z^\nu - \partial_\mu W^-_\nu W^{+\nu} Z^\mu \right] + \frac{g^2 v}{2} W^+_\mu W^{-\mu} h + \frac{g^2 v}{4\cos^2\theta_W} Z_\mu Z^\mu h - \lambda v h^3 \\
& + g_{em}^2\left[W^+_\nu W^{-\mu} A_\nu A^\mu - W^+_\mu W^{-\mu} A_\nu A^\nu \right] + g^2\cos^2\theta_W\left[W^+_\nu W^{-\mu} Z_\nu Z^\mu - W^+_\mu W^{-\mu} Z_\nu Z^\nu \right] \\
& + g^2\cos\theta_W\sin\theta_W\left[2W^+_\mu W^{-\mu} Z_\nu A^\nu - W^+_\mu W^{-\nu} A_\nu Z^\mu - W^+_\mu W^{-\nu} A^\mu Z_\nu \right] \\
& + \frac{g^2}{2}\left[W^-_\mu W^{-\mu} W^+_\nu W^{+\nu} - W^-_\mu W^{+\mu} W^-_\nu W^{+\nu} \right] + \frac{g^2}{4} W^+_\mu W^{-\mu} h^2 + \frac{g^2}{8\cos^2\theta_W} Z_\mu Z^\mu h^2 - \frac{\lambda}{4} h^4 \\
& - \frac{g_s}{2} \sum_{a,b,c}^8 f^{abc} (\partial_\mu G^{a\nu} - \partial_\nu G^a_\mu) G^{\mu b} G^{\nu c} - \frac{g_s^2}{4} \sum_{\substack{a,b,c \\ d,e,f}}^8 f^{abc} f^{ade} G^b_\mu G^c_\nu G^{\mu d} G^{\nu e}
\end{aligned}
$$

$g_{em} = g\sin\theta_W,\ v^2 = \frac{-m^2}{\lambda}\ (m^2 < 0, \lambda > 0),\ m_\ell = \frac{\lambda_\ell v}{\sqrt{2}},\ m_q = \frac{3\lambda_q v}{\sqrt{2}},\ m_W = \frac{gv}{2},\ m_Z = \frac{gv}{2\cos\theta_W},\ m_h = \sqrt{-2m^2}$

IV. From the SM to predictions at the LHC

Scattering theory

✦Cross sections can be calculated as

$$\sigma = \frac{1}{F} \int \mathrm{dPS}^{(n)} \overline{|M_{fi}|^2}$$

✤ We integrate over all final state configurations (momenta, *etc.*).
  ★The phase space (dPS) only depend on the final state particle momenta and masses
  ★ Purely kinematical

✤ We average over all initial state configurations
  ★ This is accounted for by the flux factor F
  ★ Purely kinematical

✤ The matrix element squared contains the physics model
  ★ Can be calculated from **Feynman diagrams**
  ★ Feynman diagrams can be drawn from the Lagrangian
  ★ The Lagrangian contains all the model information (particles, interactions)

Cross section

The differential cross section: $d\sigma = \frac{1}{F}|M|^2 d\Phi_n$

The Lorentz-invariant phase space:

$$d\Phi_n = (2\pi)^4 \delta^{(4)}(p_a + p_b - \sum_{f=1}^{n} p_f) \prod_{f=1}^{n} \frac{d^3 p_f}{(2\pi)^3 2E_f}$$

The flux factor: $F = \sqrt{(p_a \cdot p_b)^2 - p_a^2 p_b^2}$

Decay width

The differential decay width: $$d\Gamma = \frac{1}{2E_a}|M|^2 d\Phi_n$$

The Lorentz-invariant phase space:

$$d\Phi_n = (2\pi)^4 \delta^{(4)}(p_a - \sum_{f=1}^{n} p_f) \prod_{f=1}^{n} \frac{d^3 p_f}{(2\pi)^3 2E_f}$$

Rest frame of decaying particle: $$E_a = M_a$$

Life time and branching ratio

Life time: $$\tau = 1/\Gamma$$

Branching ratio: $$\mathrm{BR}(i \to f) = \frac{\Gamma(i \to f)}{\Gamma(i \to \mathrm{all})}$$

The model

✦ All the model information is included in the Lagrangian

✤Before electroweak symmetry breaking: very compact

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W_i^{\mu\nu} - \frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu} \\ & + \sum_{f=1}^{3}\left[\bar{L}_f\left(i\gamma^\mu D_\mu\right)L^f + \bar{e}_{Rf}\left(i\gamma^\mu D_\mu\right)e^f_R\right] \\ & + \sum_{f=1}^{3}\left[\bar{Q}_f\left(i\gamma^\mu D_\mu\right)Q^f + \bar{u}_{Rf}\left(i\gamma^\mu D_\mu\right)u^f_R + \bar{d}_{Rf}\left(i\gamma^\mu D_\mu\right)d^f_R\right] \\ & + D_\mu\varphi^\dagger D^\mu\varphi - V(\varphi)\end{aligned}$$

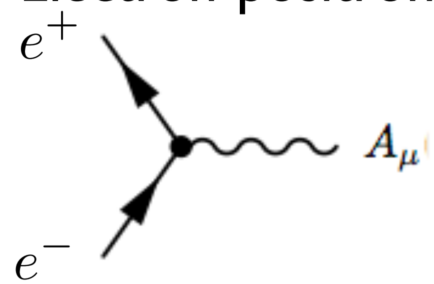
✤After electroweak symmetry breaking: quite large

Example: electroweak boson interactions with the Higgs boson:

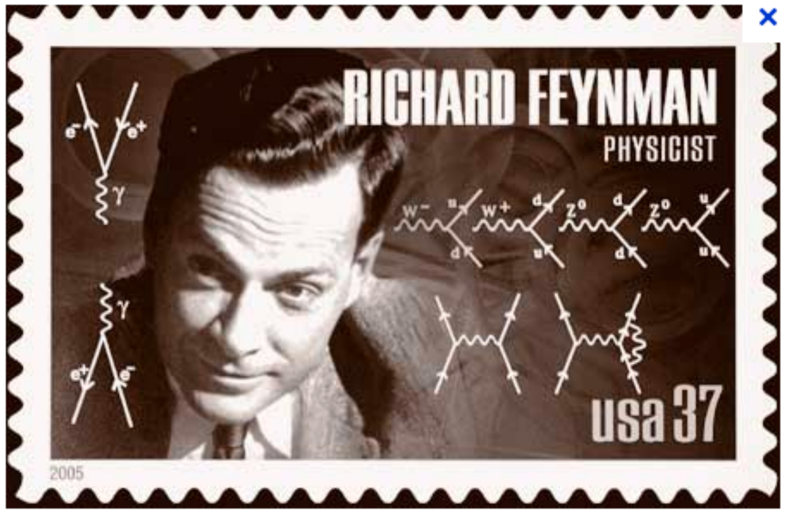
$$\begin{aligned}D_\mu\varphi^\dagger\ D^\mu\varphi = & \frac{1}{2}\partial_\mu h\partial^\mu h + \frac{e^2v^2}{4\sin^2\theta_w}W^+_\mu W^{-\mu} + \frac{e^2v^2}{8\sin^2\theta_w\cos^2\theta_w}Z_\mu Z^\mu \\ & + \frac{e^2v}{2\sin^2\theta_w}W^+_\mu W^{-\mu}h + \frac{e^2v}{4\sin^2\theta_w\cos^2\theta_w}Z_\mu Z^\mu h \\ & + \frac{e^2}{4\sin^2\theta_w}W^+_\mu W^{-\mu}hh + \frac{e^2}{8\sin^2\theta_w\cos^2\theta_w}Z_\mu Z^\mu hh\ .\end{aligned}$$

Feynman diagrams and Feynman rules I

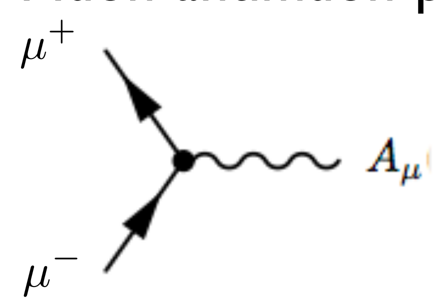
- Diagrammatic representation of the Lagrangian
  - Electron-positron-photon ($q = -1$)



$ieq\gamma^\mu$ → From the Lagrangian

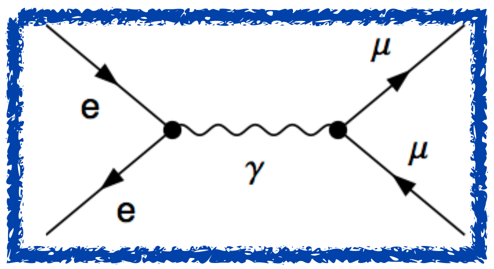


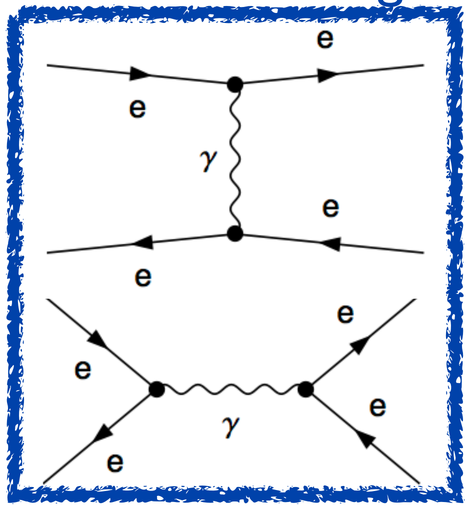
  - Muon-antimuon-photon ($q = -1$)

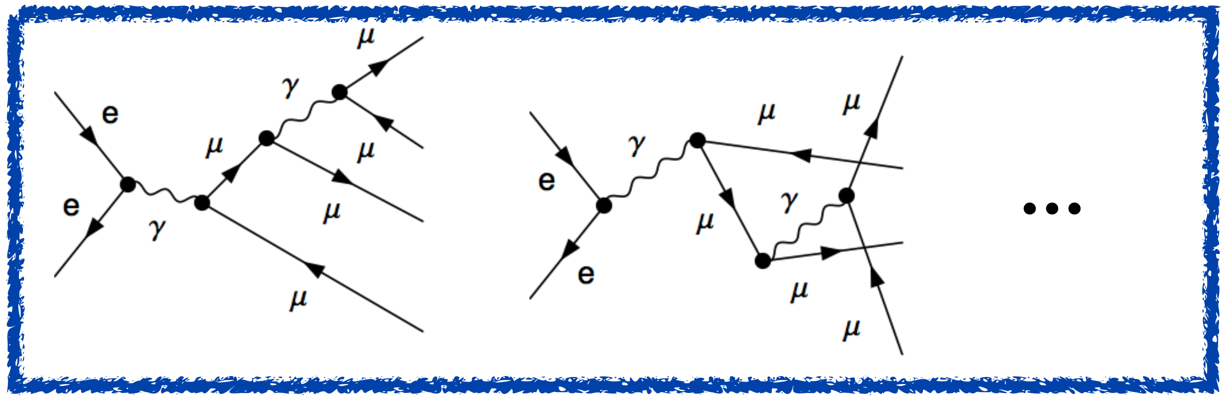


$ieq\gamma^\mu$

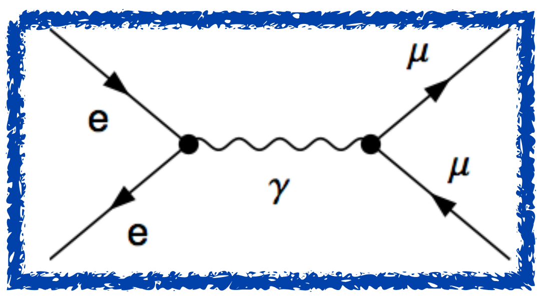
- The Feymman rules are the building blocks to construct Feynman diagrams



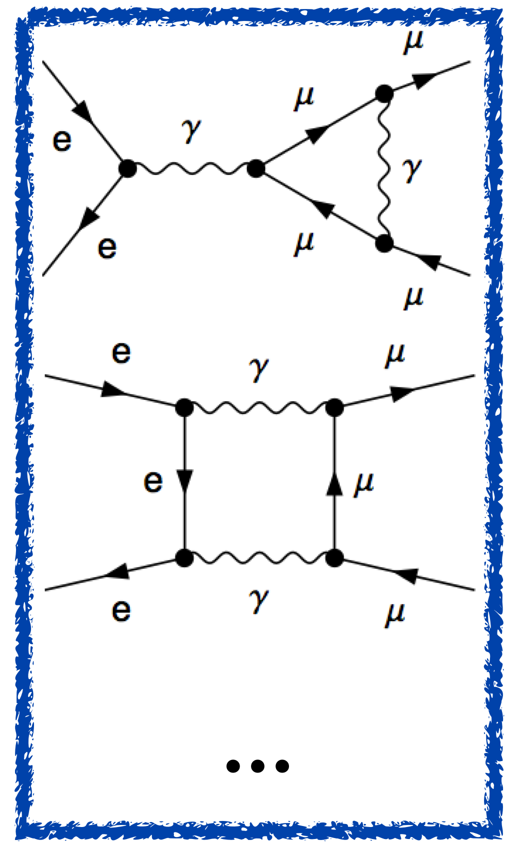




Loop diagrams



two interactions

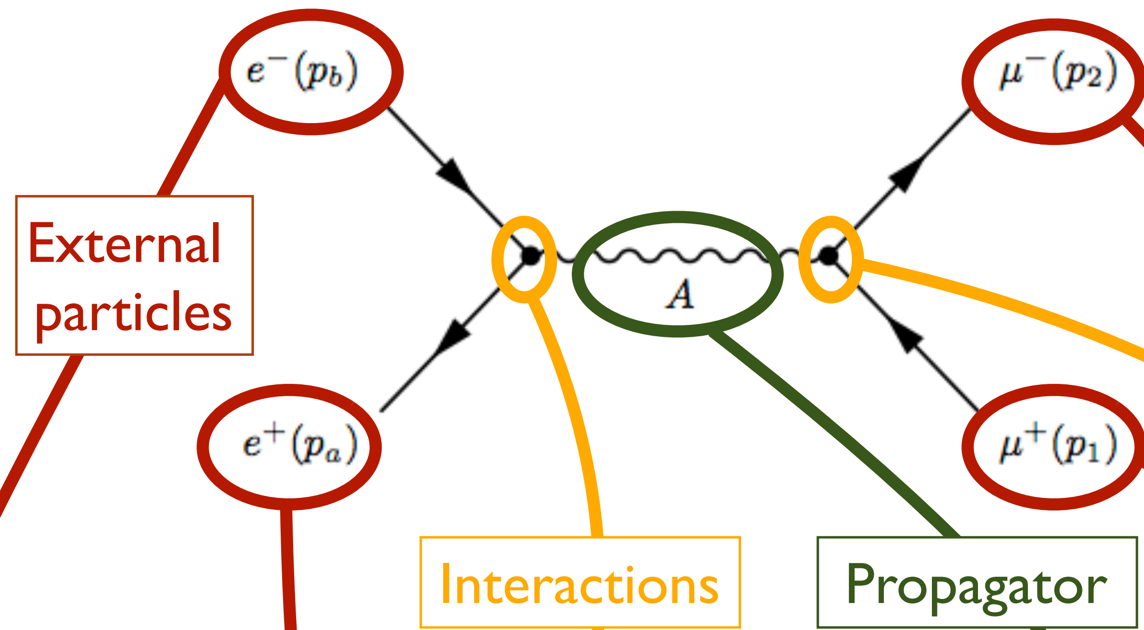


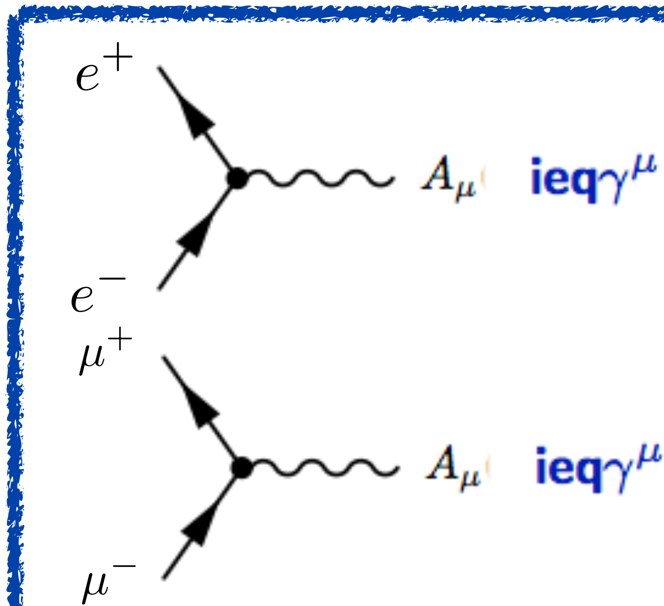
four interactions

Loops exist, but their contribution is often small

Feynman diagrams and Feynman rules II

✦ From Feynman diagrams to M_{fi} :



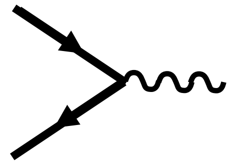
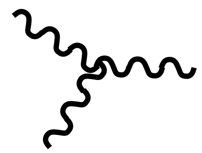
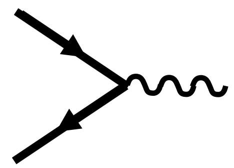
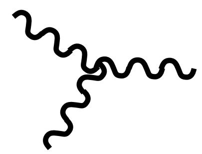
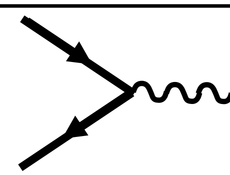
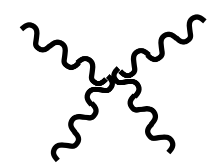
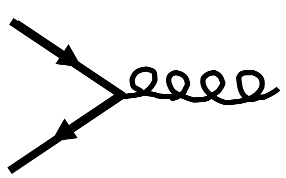
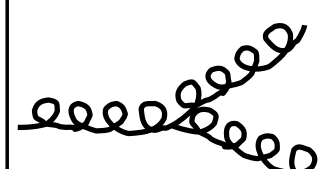
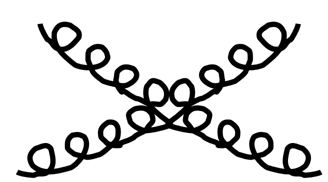
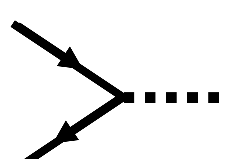
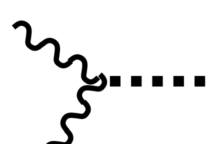
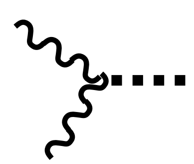


Feynman rules

$$iM_{fi} = \left[\bar{v}_{s_a}(p_a)\ (-ie\gamma^\mu)\ u_{s_b}(p_b)\right] \frac{-i\eta_{\mu\nu}}{(p_a+p_b)^2} \left[\bar{u}_{s_2}(p_2)\ (-ie\gamma^\nu)\ v_{s_1}(p_1)\right]$$

- ✤ We construct all possible diagrams with the set of rules at our disposal
- ✤ We can then calculate the squared matrix element and get the cross section

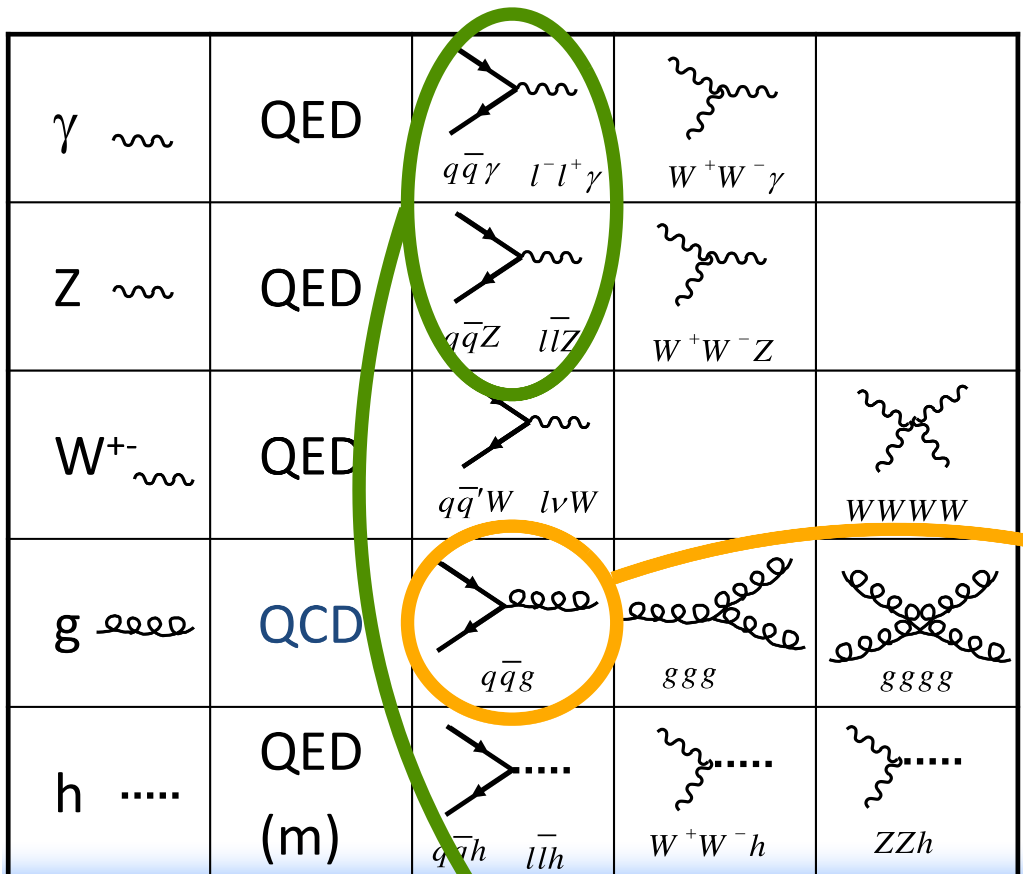
Feynman rules for the Standard Model

γ	QED	 $q\bar{q}\gamma$ $l^-l^+\gamma$	 $W^+W^-\gamma$	
Z	QED	 $q\bar{q}Z$ $l\bar{l}Z$	 W^+W^-Z	
W^{+-}	QED	 $q\bar{q}'W$ $l\nu W$		 $WWWW$
g	QCD	 $q\bar{q}g$	 ggg	 $gggg$
h	QED (m)	 $q\bar{q}h$ $l\bar{l}h$	 W^+W^-h	 ZZh

Almost all the building blocks necessary to draw any SM diagrams

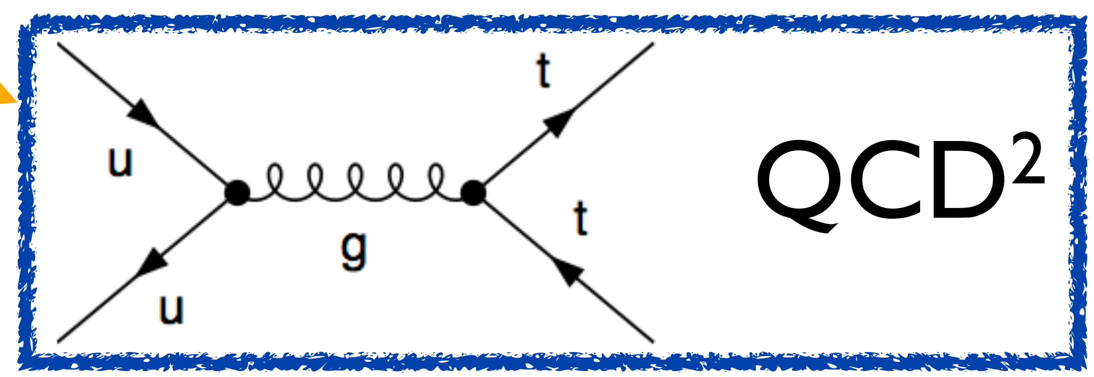
QCD coupling much stronger than QED coupling → dominant diagrams

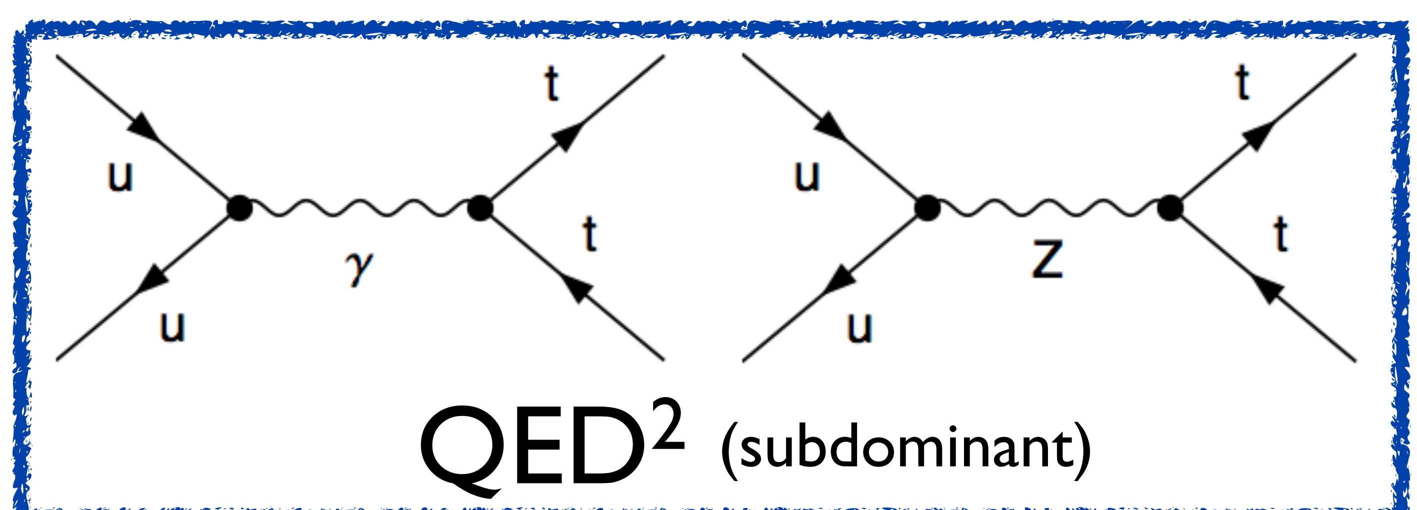
Drawing Feynman diagrams I



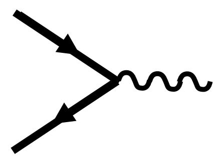
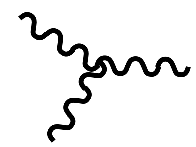
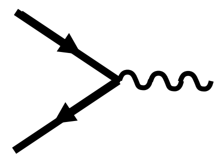
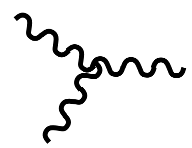
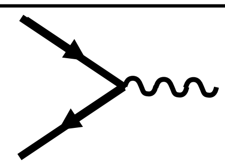
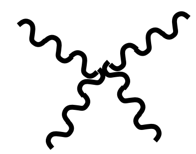
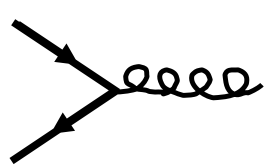
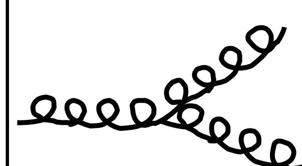
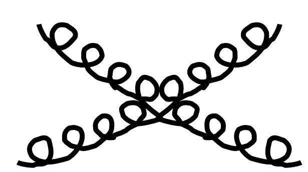
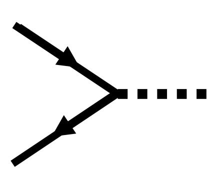
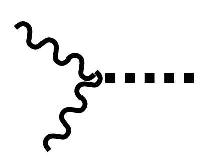
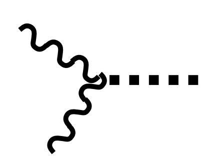
- We can now combine building blocks to draw diagrams
  - This ensures to focus only on the allowed diagrams
  - We must only consider the dominant diagrams

- Process 0. $u\bar{u} \to t\bar{t}$





Drawing Feynman diagrams II

γ 	QED	 $q\bar{q}\gamma$ $l^-l^+\gamma$	 $W^+W^-\gamma$	
Z 	QED	 $q\bar{q}Z$ $l\bar{l}Z$	 W^+W^-Z	
W+- 	QED	 $q\bar{q}'W$ $l\nu W$		 $WWWW$
g 	QCD	 $q\bar{q}g$	 ggg	 $gggg$
h ·····	QED (m)	 $q\bar{q}h$ $l\bar{l}h$	 W^+W^-h	 ZZh

- Find out the dominant diagrams for
  - Process 1. $gg \to t\bar{t}$
  - Process 2. $gg \to t\bar{t}h$
  - Process 3. $u\bar{u} \to t\bar{t}\ b\bar{b}$
- What is the QCD/QED order?
  (keep only the dominant diagrams)

MadGraph5_aMC@NLO

- Check your answer online:

  MadGraph5_aMC@NLOwebpage

- Requires registration

Web process syntax

Initial state

```
u u~ > b b~ t t~
```

Final state

```
u u~ > b b~ t t~ QED=2
```

Minimal coupling order

```
u u~ > h > b b~ t t~
```

Required intermediate particles

Excluded particles

```
u u~ > b b~ t t~ / z a
```

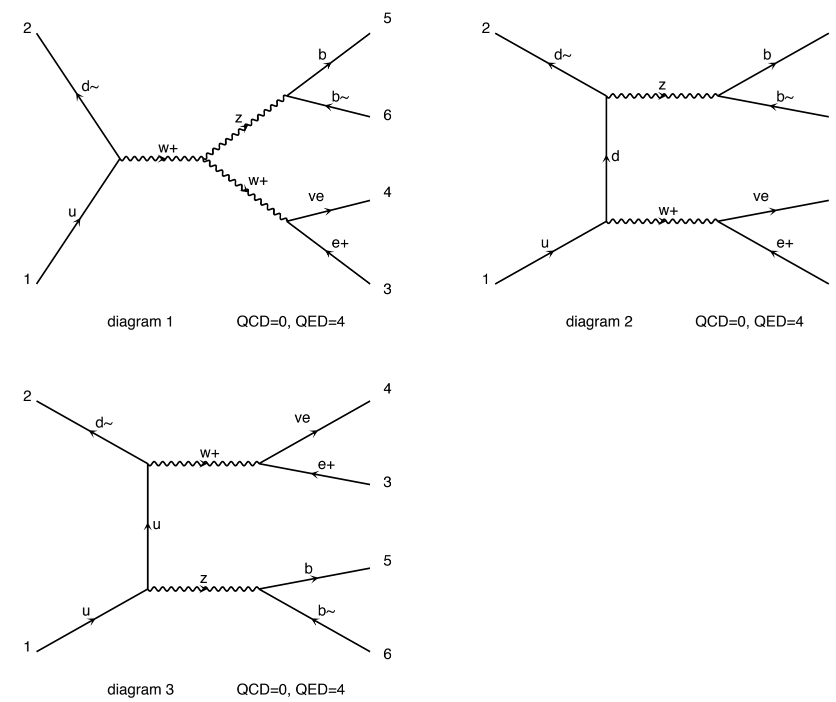
```
u u~ > b b~ t t~, t~ > w- b~
```

Specific decay chain

MadGraph output

✦ User requests a process

- `g g > t t~ b b~`
- `u d~ > w+ z, w+ > e+ ve, z > b b~`
- etc.




```
 SUBROUTINE SMATRIX(P1,ANS)
C
C Generated by MadGraph II Version 3.83. Updated 06/13/05
C RETURNS AMPLITUDE SQUARED SUMMED/AVG OVER COLORS
C AND HELICITIES
C FOR THE POINT IN PHASE SPACE P(0:3,NEXTERNAL)
C
C FOR PROCESS : g g -> t t~ b b~
C
C Crossing   1 is g g -> t t~ b b~
      IMPLICIT NONE
C
C CONSTANTS
C
      Include "genps.inc"
      INTEGER             NCOMB,     NCROSS
      PARAMETER (         NCOMB=  64, NCROSS=  1)
      INTEGER    THEL
      PARAMETER (THEL=NCOMB*NCROSS)
C
C ARGUMENTS
C
      REAL*8 P1(0:3,NEXTERNAL),ANS(NCROSS)
C
```

✦ MADGRAPH returns:

- Feynman diagrams
- Self-contained Fortran code for $|M_{fi}|^2$

✦ Still needed:

- What to do with a Fortran code?
- How to deal with hadron colliders?

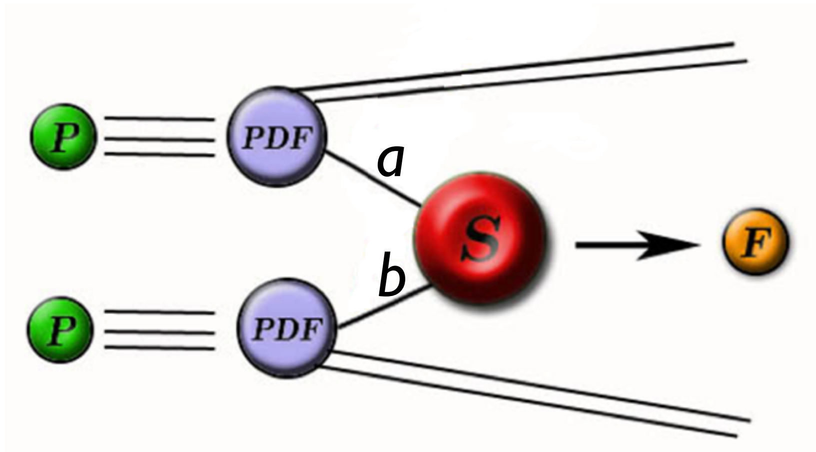
Proton-Proton collisions I

✦The master formula for hadron colliders

$$\sigma = \frac{1}{F}\sum_{ab}\int \mathrm{dPS}^{(n)}\mathrm{d}x_a\ \mathrm{d}x_b\ f_{a/p}(x_a)\ f_{b/p}(x_b)\overline{|M_{fi}|^2}$$

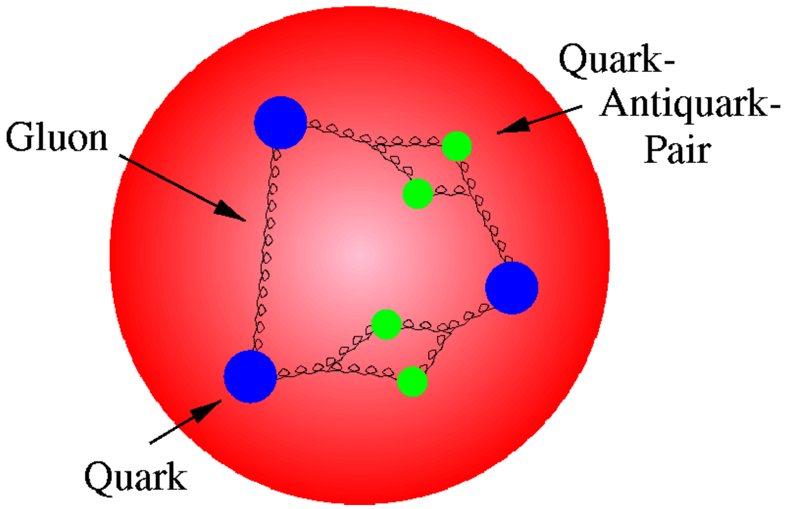
✤ We sum over all proton constituents (*a* and *b* here)

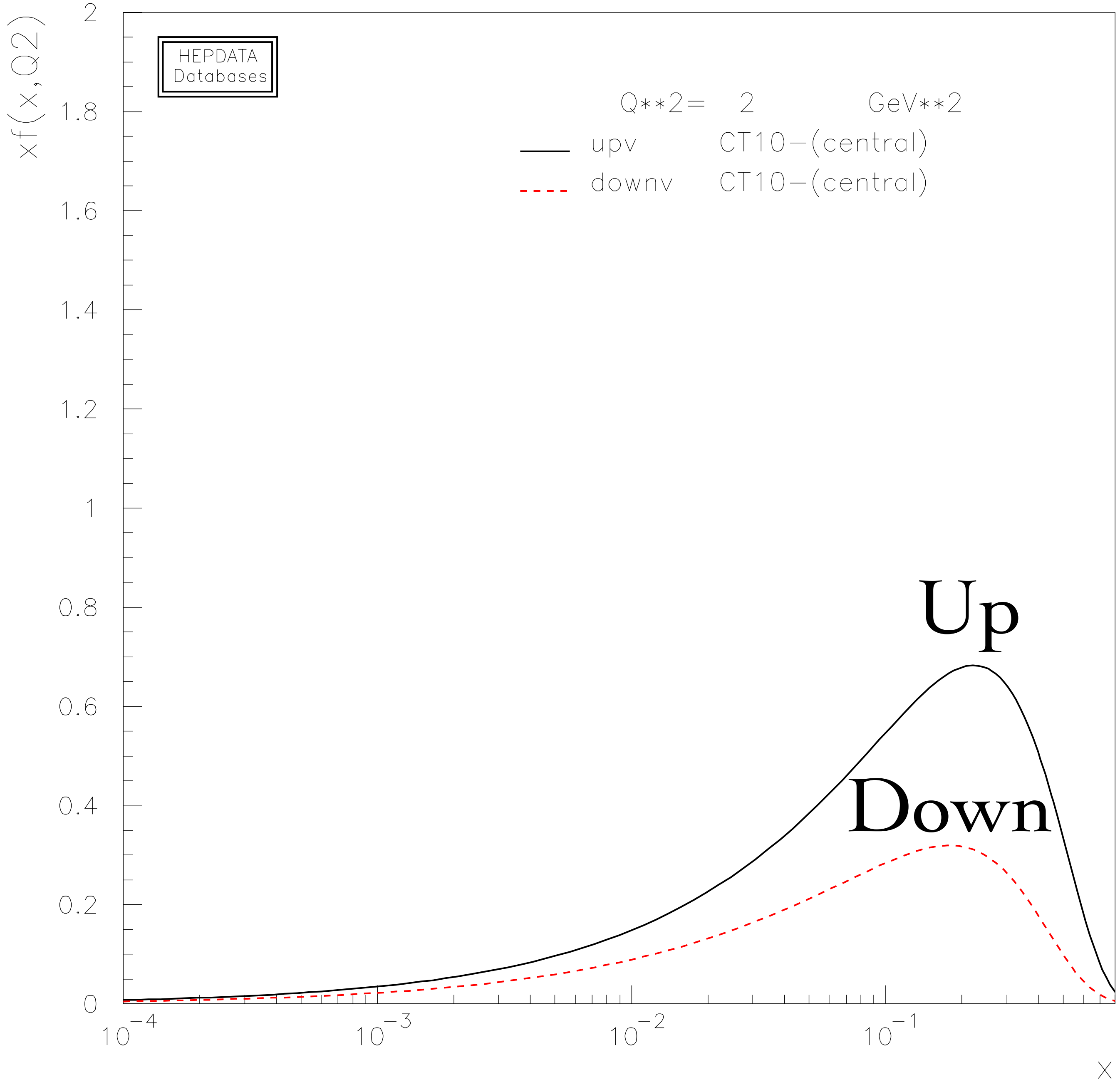
✤ We include the parton densities (the *f*-function)



They represent the probability of having a parton *a* inside the proton carrying a fraction x_a of the proton momentum

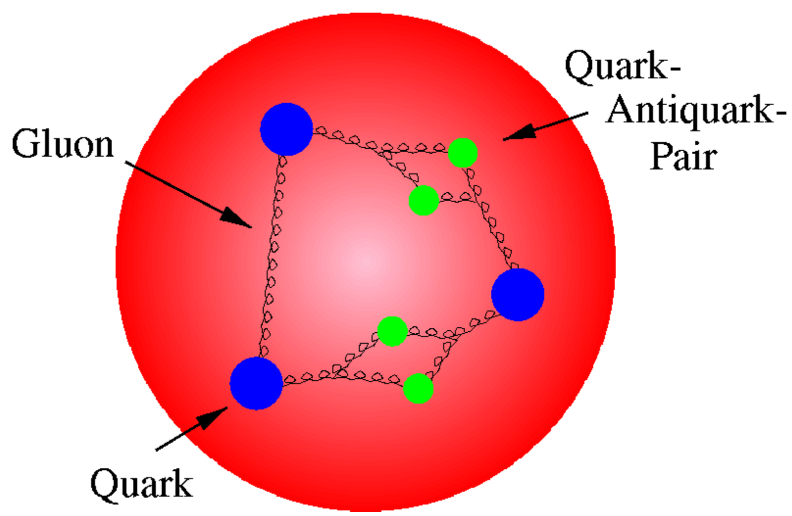
PDFs: x-dependence





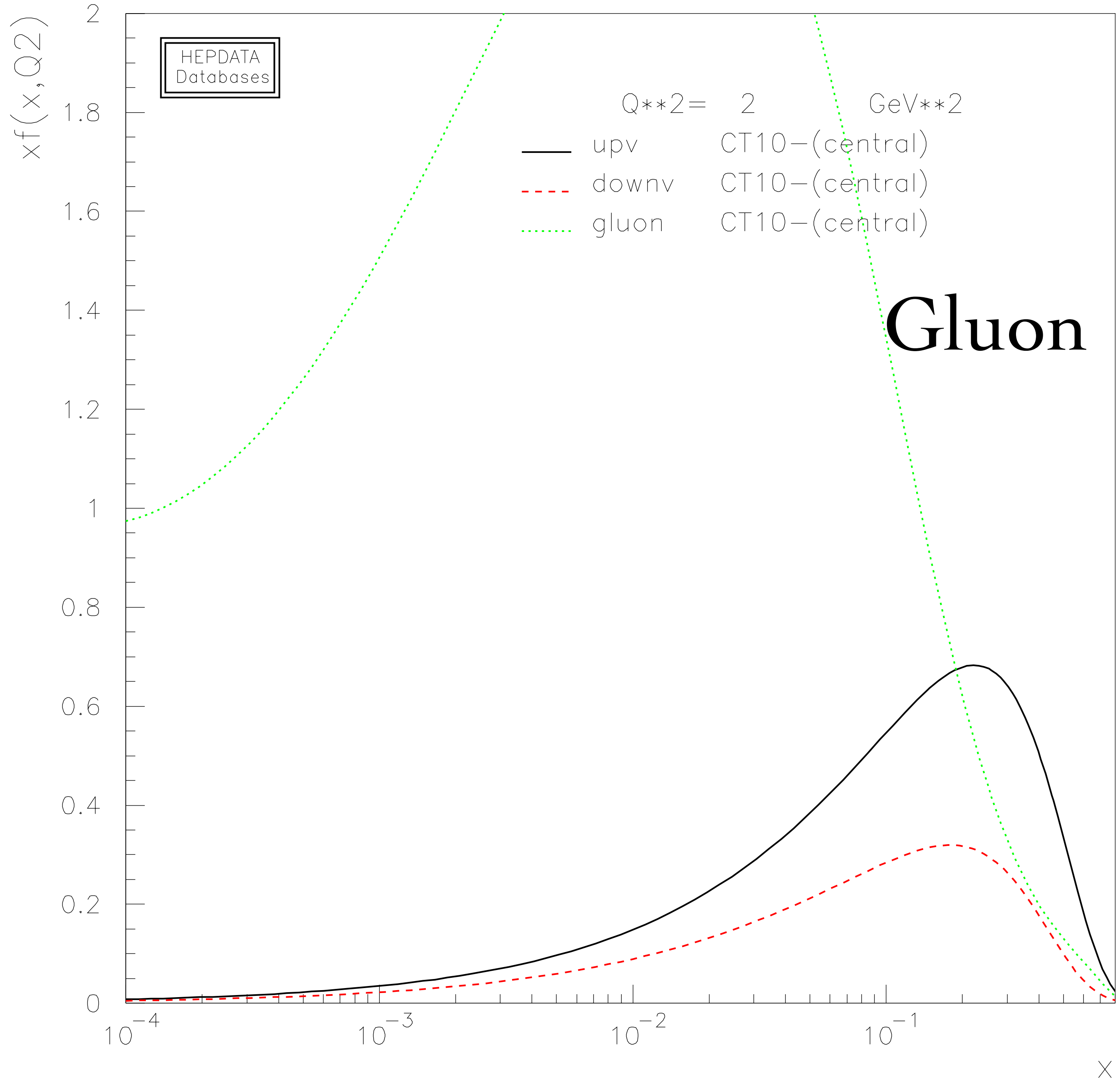
- Valence quarks
  p=|uud□

PDFs: x-dependence

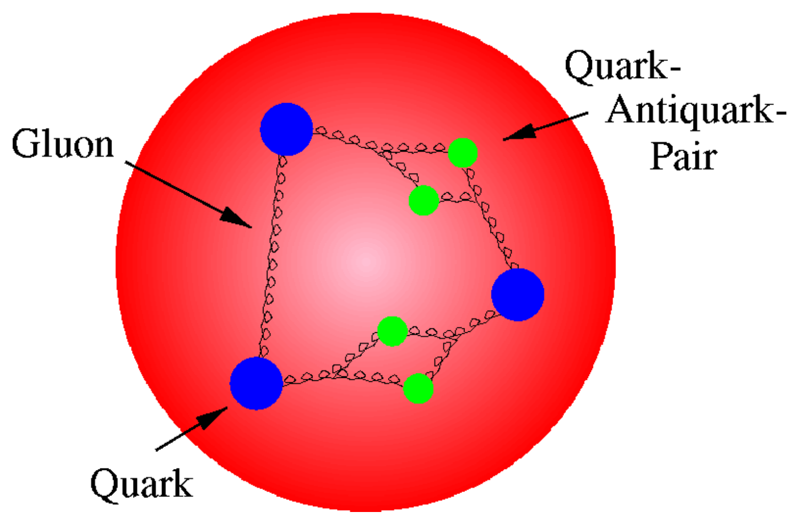


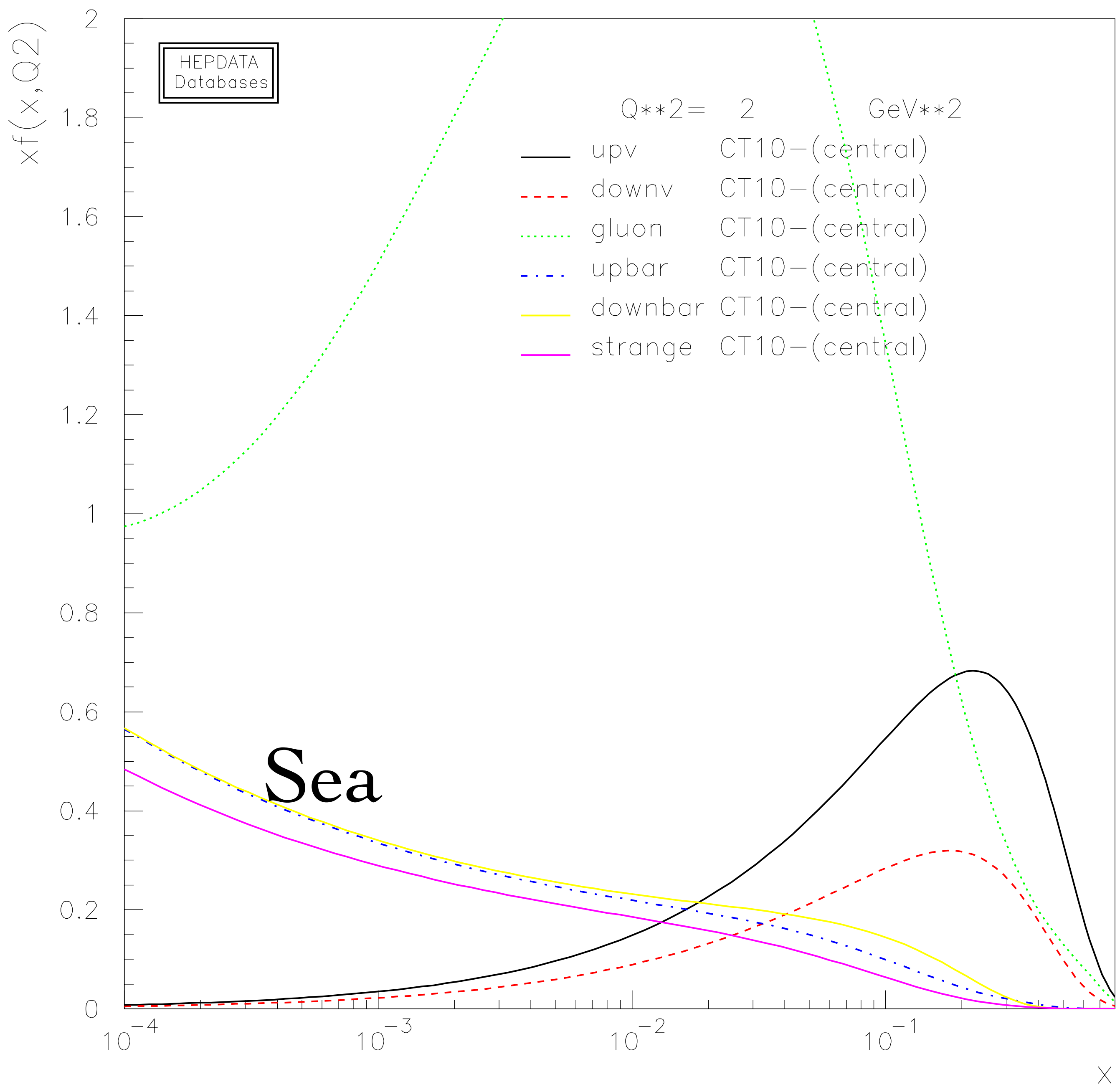
- Valence quarks
  p=|uud☐
- Gluons

  carry about 40% of momentum



PDFs: x-dependence





- Valence quarks
  p=|uud□
- Gluons
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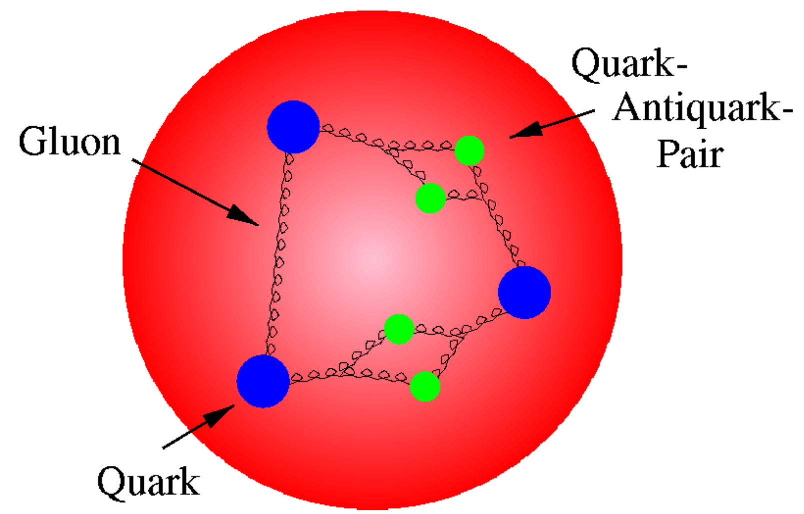
- Sea quarks
  light quark sea, strange sea

PDFs: Q-dependence

Altarelli-Parisi evolution equations



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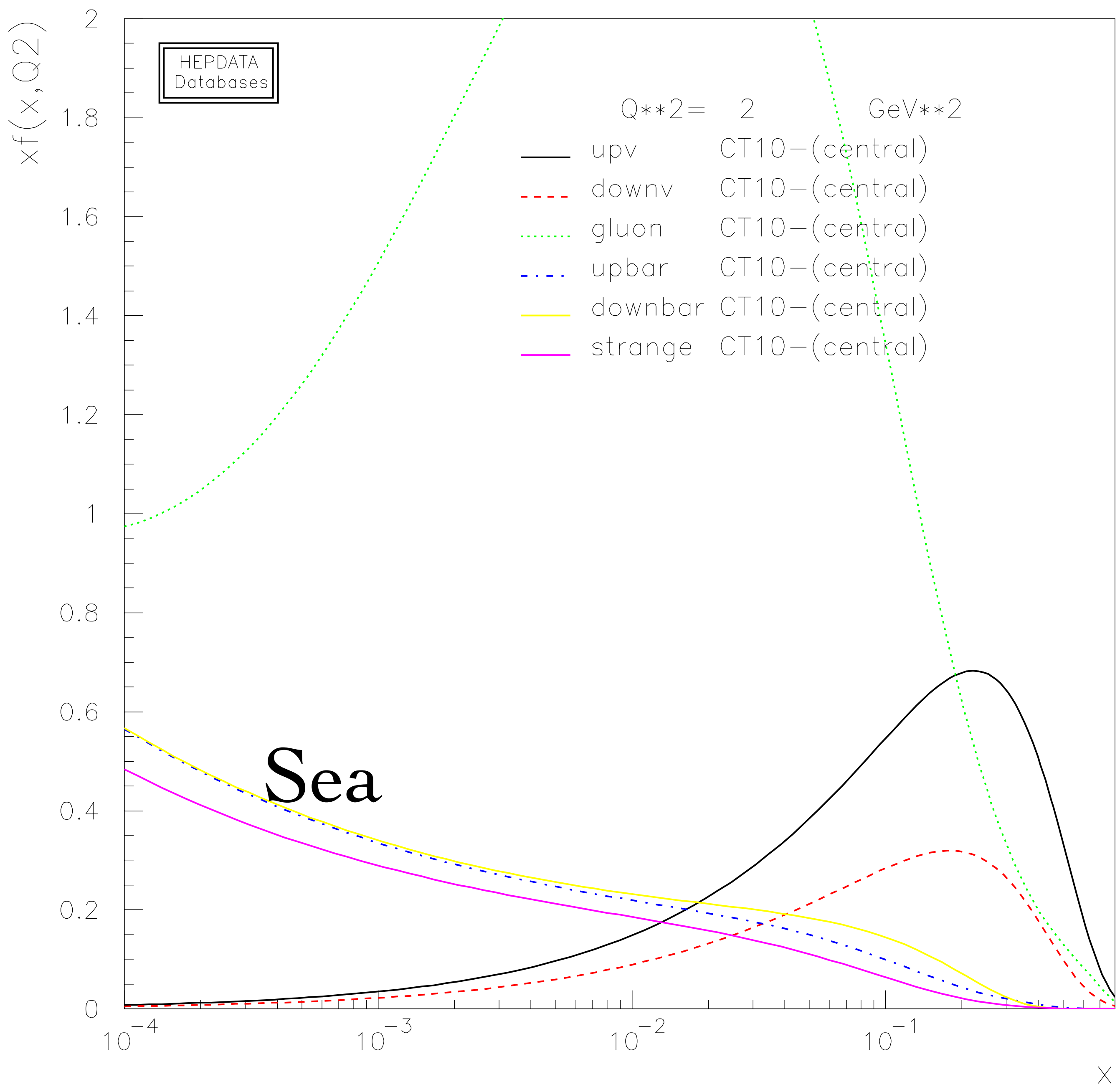

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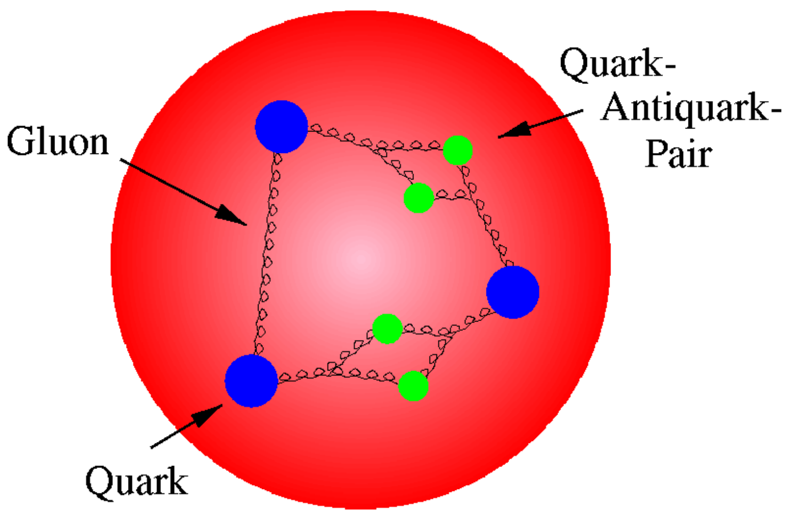

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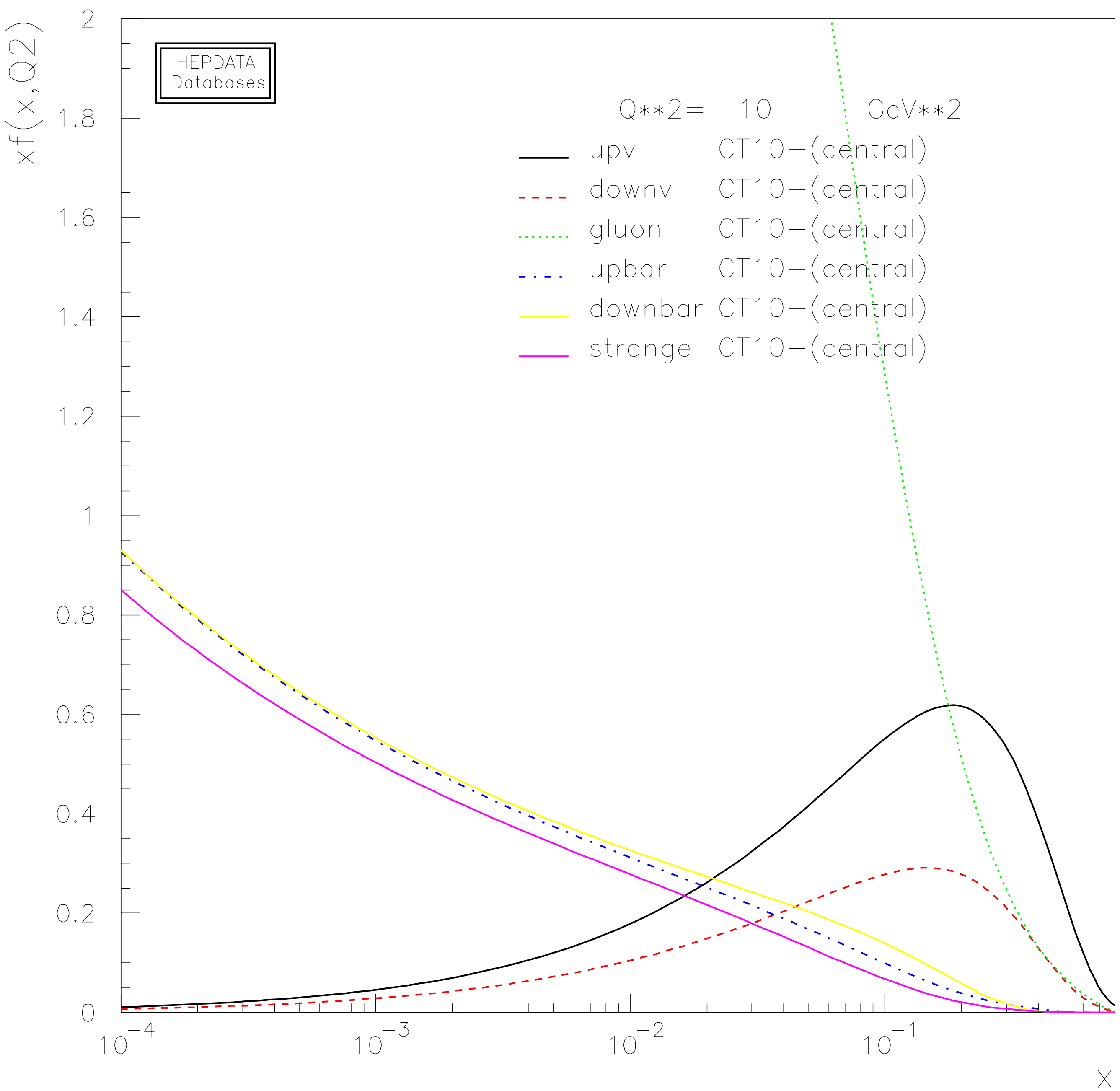

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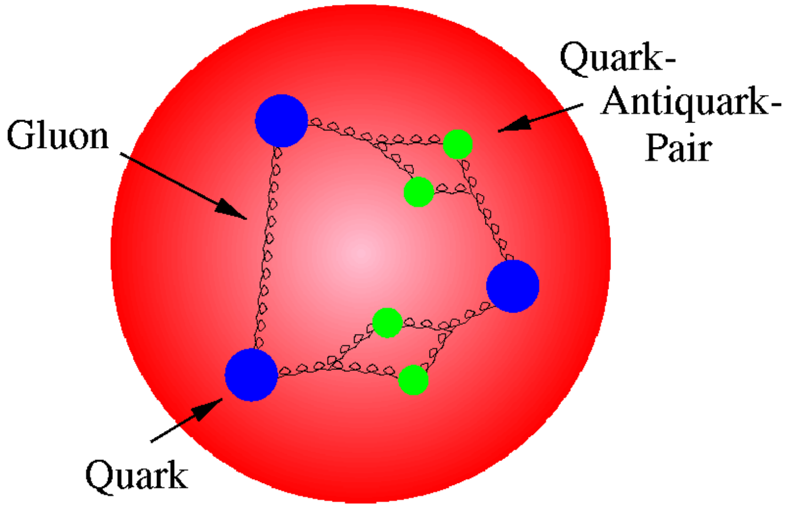

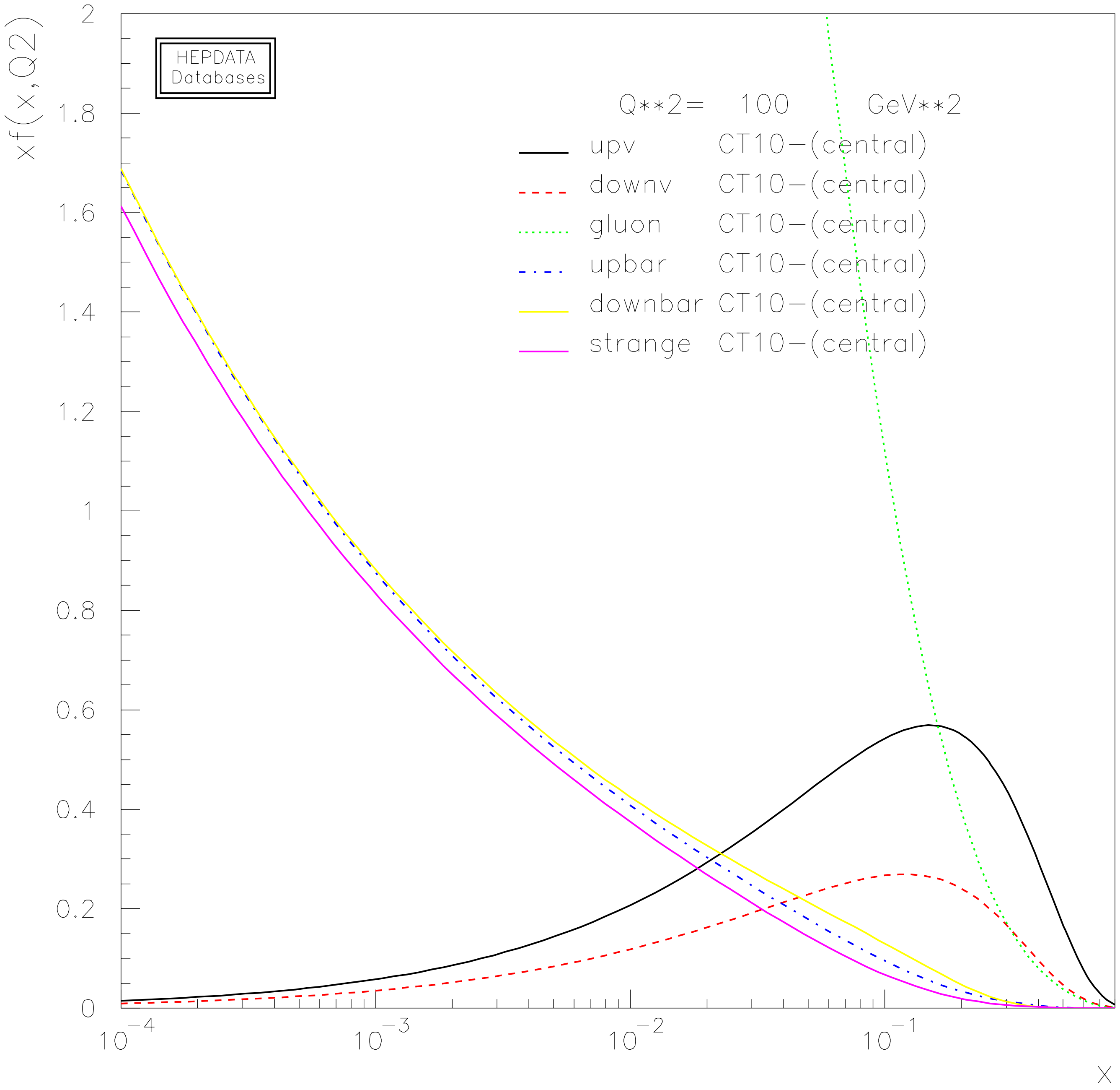

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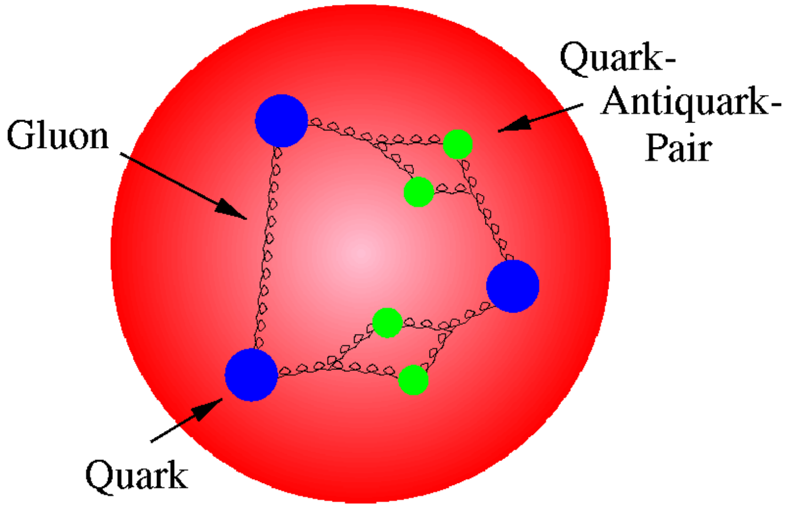

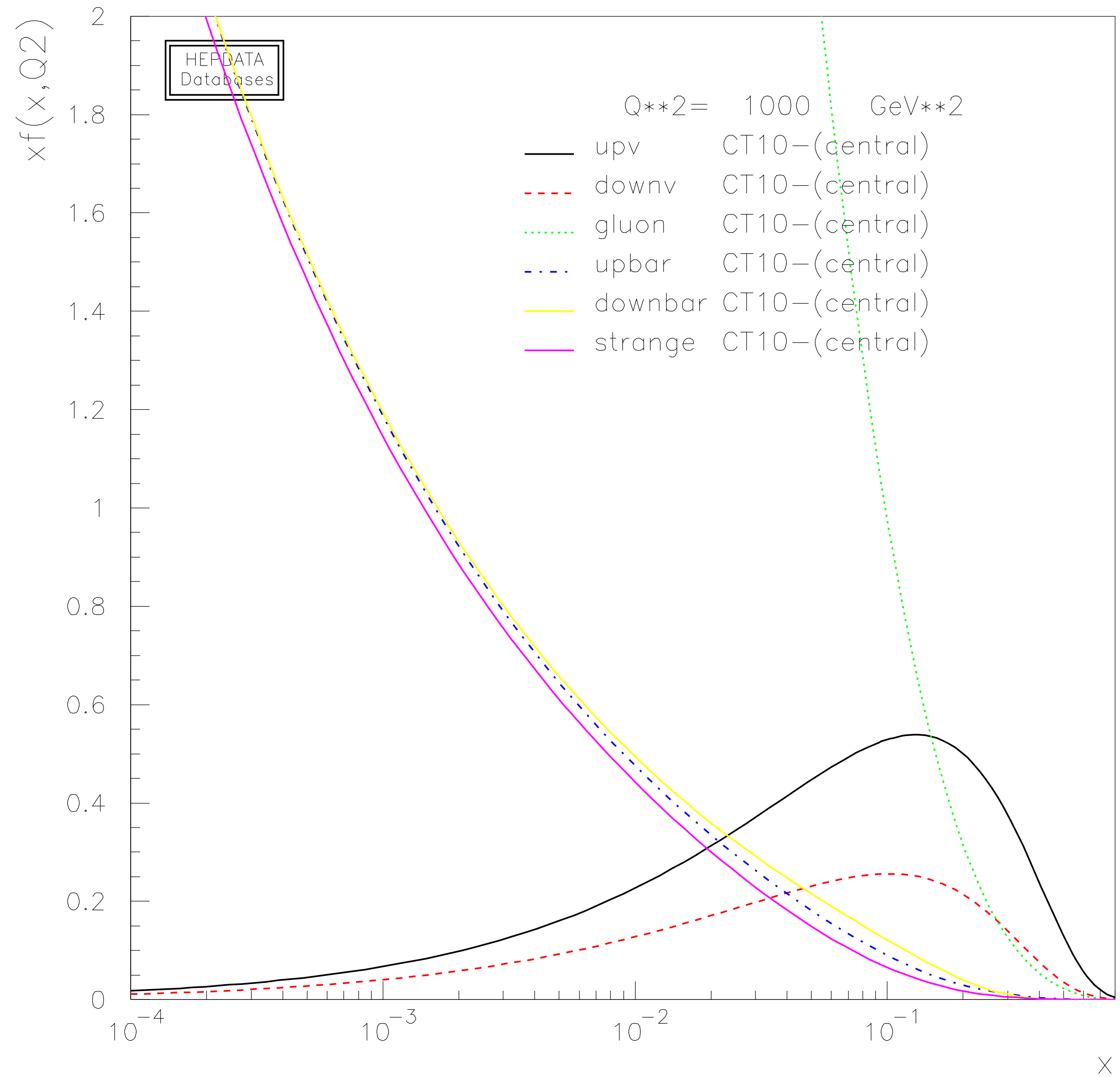

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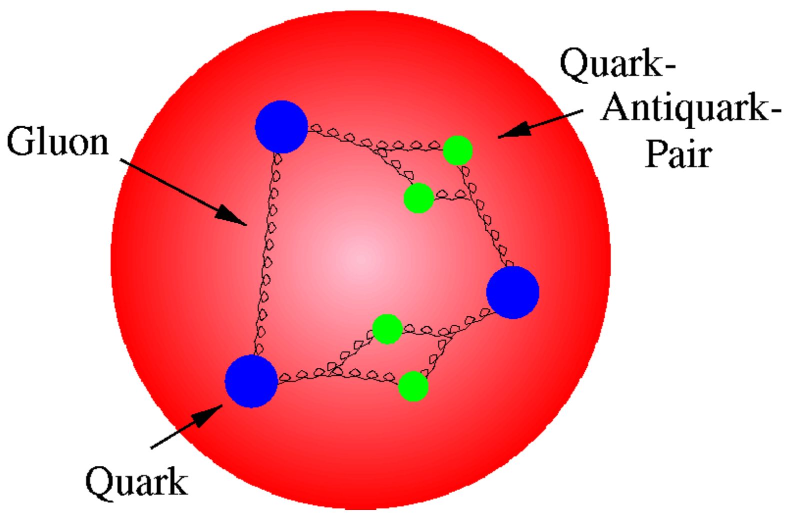
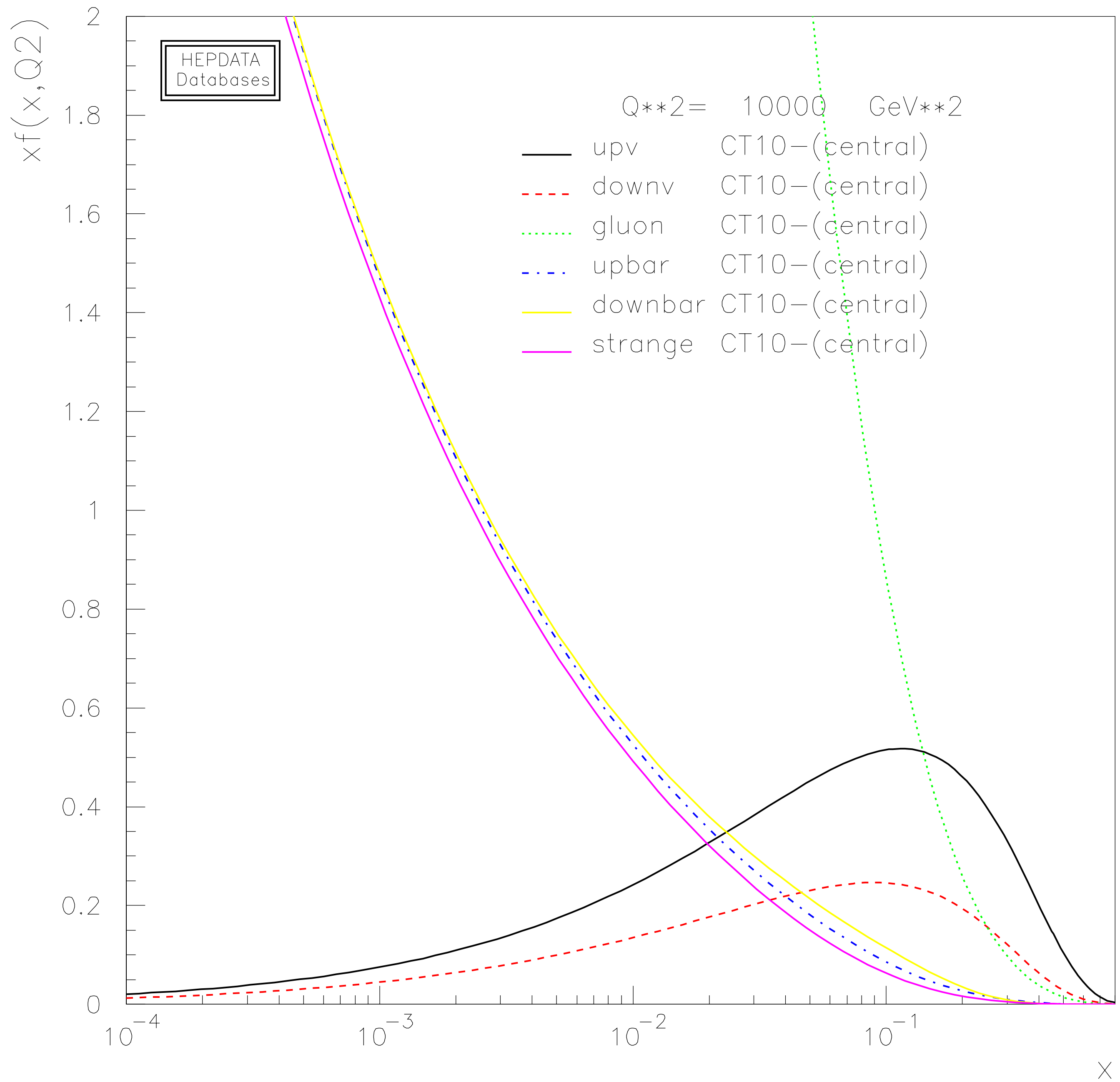


Altarelli-Parisi evolution equations



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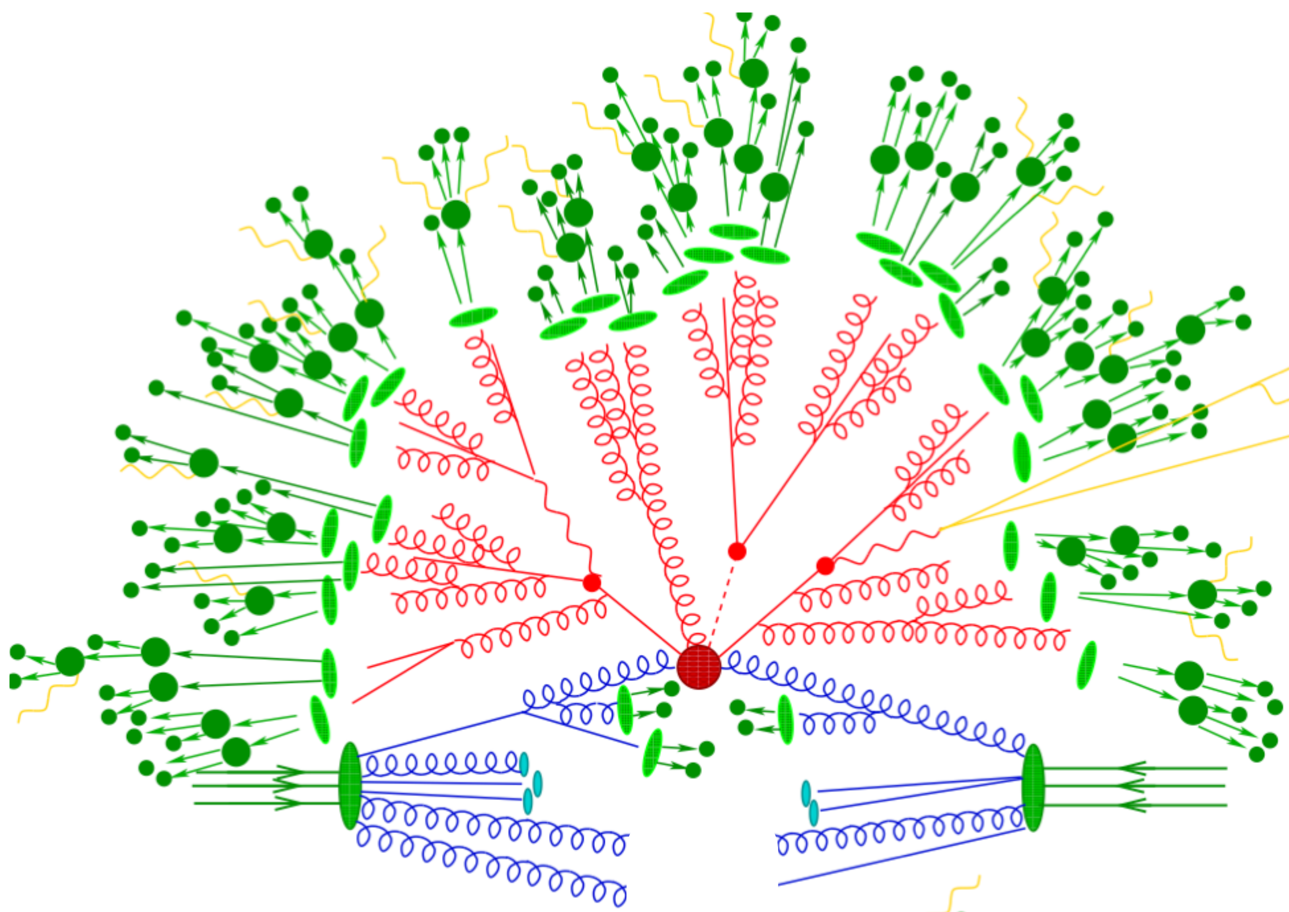
Proton-Proton collisions II

- This is not the end of the story...
  - At high energies, initial and final state quarks and gluons radiate other quark and gluons
  - The radiated partons radiate themselves
  - And so on...
  - Radiated partons hadronize
  - We observe hadrons in detectors



Input parameters

- The SM Lagrangian has **26 input parameters** (of course not all are equally important)
- They **need to be fixed** in order to make **predictions**
- The values and patterns of these parameters are quite **bizarre**!

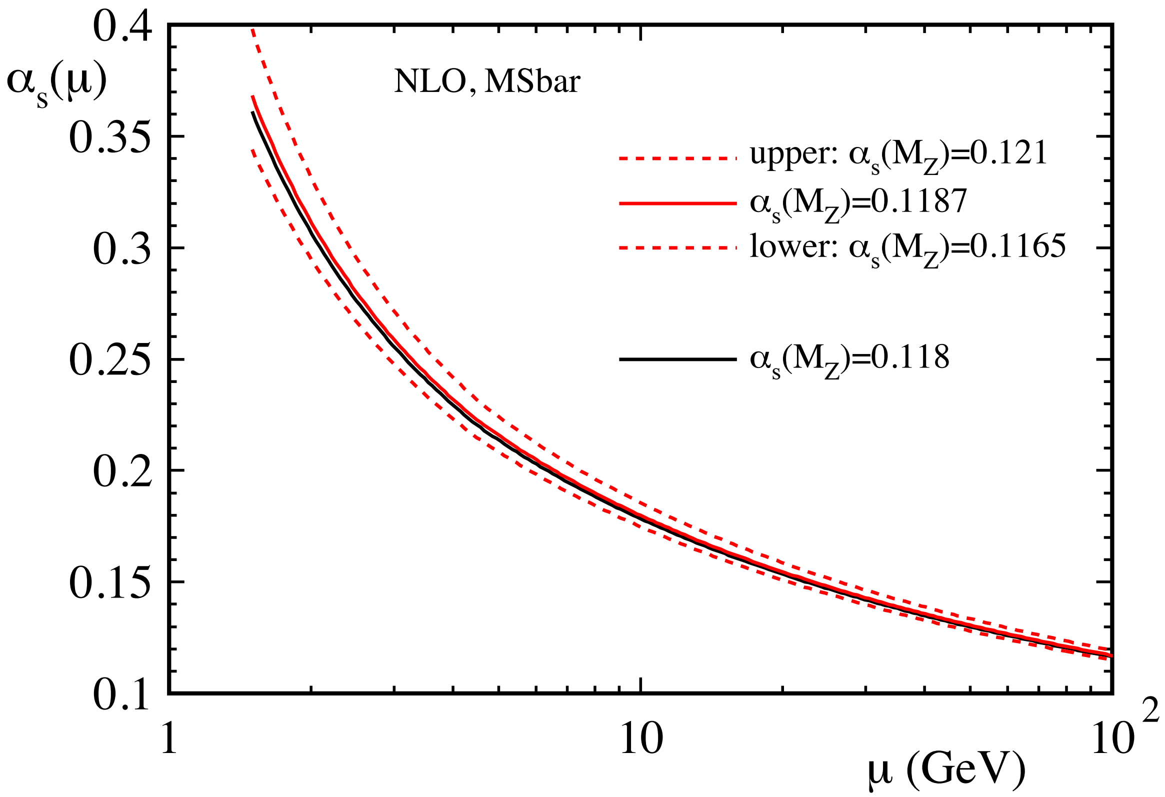
Quantum Corrections

- Quantum corrections have to be considered (otherwise some predictions very rough!)
- **UV** divergences appear
- **Renormalization** of Lagrangian parameters and fields
- This leads to **running parameters**
- Scale-dependence governed by **renormalization group equations** (RGEs)

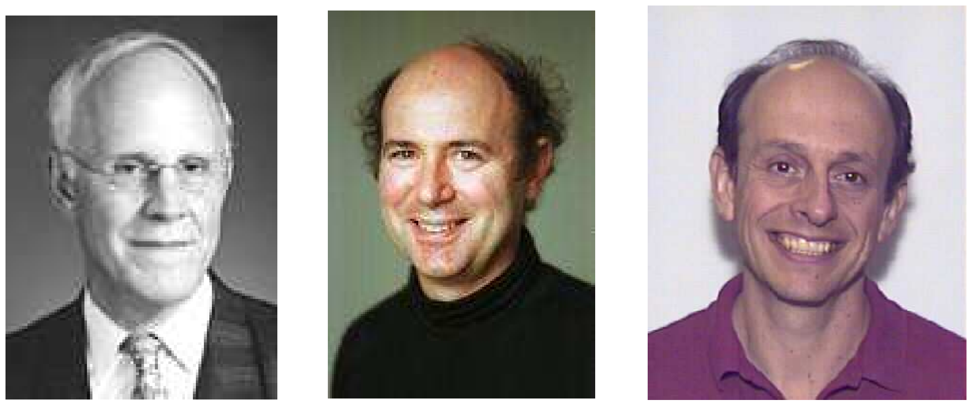
Asymptotic Freedom

Renormalization of UV-divergences:
Running coupling constant $a_s := \alpha_s/(4\pi)$

$$a_s(\mu) = \frac{1}{\beta_0 \ln(\mu^2/\Lambda^2)}$$



- Gross, Wilczek ('73); Politzer ('73)



Non-abelian gauge theories:
negative beta-functions

$$\frac{da_s}{d\ln\mu^2} = -\beta_0 a_s^2 + \ldots$$

where $\beta_0 = \frac{11}{3} C_A - \frac{2}{3} n_f$

$\Rightarrow$ asympt. freedom: $a_s \searrow$ for $\mu \nearrow$

- Nobel Prize 2004