Introduction to the Standard Model of particle physics

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III. The Standard Model of particle physics (2nd round)
The general procedure

• Introduce Fields & Symmetries
The general procedure

- Introduce Fields & Symmetries

- **Construct a local Lagrangian density**
The general procedure

- Introduce Fields & Symmetries
- Construct a local Lagrangian density
- **Describe Observables**
  - How to measure them?
  - How to calculate them?
The general procedure

- Introduce Fields & Symmetries
- Construct a local Lagrangian density
- Describe Observables
  
  - How to measure them?
  
  - How to calculate them?
- **Falsify: Compare theory with data**
Fields & Symmetries
### Matter content of the Standard Model (including the antiparticles)

<table>
<thead>
<tr>
<th>Matter</th>
<th>Higgs</th>
<th>Gauge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q = \begin{pmatrix} u_L \ d_L \end{pmatrix}$</td>
<td>$H = \begin{pmatrix} h^+ \ h^0 \end{pmatrix}$</td>
<td>$A \ (1, 1)_0$</td>
</tr>
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<td>$(3, 2)_{1/3}$</td>
<td>$(1, 2)_{-1}$</td>
<td>$(1, 3)_0$</td>
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<td>$u^c_R$</td>
<td>$L = \begin{pmatrix} \nu_L \ e_L \end{pmatrix}$</td>
<td>$W \ (1, 3)_0$</td>
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<tr>
<td>$(\overline{3}, 1)_{4/3}$</td>
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<td>$G \ (8, 1)_0$</td>
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<td>$d^c_R$</td>
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<td>$\nu_R$</td>
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<td></td>
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Matter content of the Standard Model

- Left-handed up quark $u_L$:
  - LH Weyl fermion: $u_{L\alpha} \sim (1/2,0)$ of $\text{so}(1,3)$
  - a color triplet: $u_{Li} \sim 3$ of $\text{SU}(3)_c$
  - Indices: $(u_L)_{i\alpha}$ with $i=1,2,3$ and $\alpha=1,2$

- Similarly, left-handed down quark $d_L$

- $u_L$ and $d_L$ components of a $\text{SU}(2)_L$ doublet: $Q_\beta = (u_L, d_L) \sim 2$
  - $Q$ carries a hypercharge $1/3$: $Q \sim (3,2)_{1/3}$ of $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$
  - Indices: $Q_{\beta i\alpha}$ with $\beta=1,2$; $i=1,2,3$ and $\alpha=1,2$
There are three generations: $Q_k$, $k = 1, 2, 3$

Lot's of indices: $Q_{k\beta i\alpha}(x)$

We know how the indices $\beta, i, \alpha$ transform under symmetry operations (i.e., which representations we have to use for the generators)
Matter content of the Standard Model

- Right-handed up quark $u_R$:
  - RH Weyl fermion: $u_{R\alpha} \sim (0, 1/2)$ of $so(1,3)$
  - a color triplet: $u_{Ri} \sim 3$ of $SU(3)_c$
  - a singlet of $SU(2)_L$: $u_R \sim 1$ (no index needed)
  - $u_R$ carries hypercharge $4/3$: $u_R \sim (3, 1)_{4/3}$
  - Indices: $(u_R)_{i\alpha}$ with $i=1,2,3$ and $\alpha.=1,2$ (Note the dot)
  - Note that $u_R^c \sim (3^*, 1)_{-4/3}$
Matter content of the Standard Model

- Again there are three generations: $U_{Rk}$, $k = 1, 2, 3$

- Lot’s of indices: $UR_{kia} (x)$

- And so on for the other fields ...
Terms for the Lagrangian
How to build Lorentz scalars?

Scalar field (like the Higgs)

Real field $\phi$

$$\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2$$

Complex field $\phi = \frac{1}{\sqrt{2}} (\varphi_1 + i \varphi_2)$

$$\partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi$$

Note: The mass dimension of each term in the Lagrangian has to be 4!
How to build Lorentz scalars?
Fermions (spin 1/2)

Left-handed Weyl spinor
\[ i \psi_L^\dagger \bar{\sigma}^\mu \partial_\mu \psi_L \]

Right-handed Weyl spinor
\[ i \psi_R^\dagger \sigma^\mu \partial_\mu \psi_R \]

Mass term mixes left and right
\[ i \psi_L^\dagger \bar{\sigma}^\mu \partial_\mu \psi_L + i \psi_R^\dagger \sigma^\mu \partial_\mu \psi_R - m (\psi_L^\dagger \psi_R + \psi_R^\dagger \psi_L) \]

Dirac spinor in chiral basis
\[ \Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \]
\[ i \bar{\Psi} \gamma^\mu \partial_\mu \Psi - m \bar{\Psi} \Psi \quad \text{with} \quad \bar{\Psi} = \Psi^\dagger \gamma^0 \quad \text{and} \quad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \]

Equations:
\[ \sigma^\mu = (1, \sigma^i) \]
\[ \bar{\sigma}^\mu = (1, -\sigma^i) \]
How to build Lorentz scalars?
Vector boson (spin 1)

**U(1) gauge boson ("Photon")**

\[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu \text{ where } F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\]

Mass term allowed by Lorentz invariance; forbidden by gauge invariance

In principle, there is a second invariant

\[-\frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} \text{ with } \tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}\]

**Violates Parity, Time reversal, and CP symmetry; prop. to a total divergence**

\[F \tilde{F} \propto \vec{E} \cdot \vec{B}\]

→ doesn’t contribute in QED

BUT strong CP problem in QCD
Gauge symmetry

- Idea: Generate interactions from free Lagrangian by imposing local (i.e. $\alpha = \alpha(x)$) symmetries

- Does not fall from heavens; generalization of ‘minimal coupling’ in electrodynamics

- Final judge is experiment: It works!
Local gauge invariance for a complex scalar field

\[ \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi \] is invariant under \( \phi \rightarrow e^{i\alpha} \phi \).

What if now \( \alpha = \alpha(x) \) depends on the space-time?

\[
\begin{align*}
\partial_\mu (e^{i\alpha(x)} \phi)^* \partial^\mu (e^{i\alpha(x)} \phi) &- m^2 (e^{i\alpha(x)} \phi)^* (e^{i\alpha(x)} \phi) \\
&= [\partial_\mu e^{i\alpha(x)} \cdot \phi + e^{i\alpha(x)} \cdot \partial_\mu \phi]^* [\partial^\mu e^{i\alpha(x)} \cdot \phi + e^{i\alpha(x)} \cdot \partial^\mu \phi] - m^2 \phi^* \phi \\
&= [ie^{i\alpha(x)} \partial_\mu \alpha(x) \cdot \phi + e^{i\alpha(x)} \cdot \partial_\mu \phi]^* [ie^{i\alpha(x)} \partial^\mu \alpha(x) \cdot \phi + e^{i\alpha(x)} \cdot \partial^\mu \phi] - m^2 \phi^* \phi \\
&= [-ie^{-i\alpha(x)} \partial_\mu \alpha(x) \cdot \phi^* + e^{-i\alpha(x)} \cdot \partial_\mu \phi^*][ie^{i\alpha(x)} \partial^\mu \alpha(x) \cdot \phi + e^{i\alpha(x)} \cdot \partial^\mu \phi] - m^2 \phi^* \phi \\
&= -ie^{-i\alpha(x)} \partial_\mu \alpha(x) \cdot \phi^* \cdot ie^{i\alpha(x)} \partial^\mu \alpha(x) \cdot \phi \\
&\quad - ie^{-i\alpha(x)} \partial_\mu \alpha(x) \cdot \phi^* \cdot e^{i\alpha(x)} \cdot \partial^\mu \phi \\
&\quad + e^{-i\alpha(x)} \cdot \partial_\mu \phi^* \cdot ie^{i\alpha(x)} \partial^\mu \alpha(x) \cdot \phi \\
&\quad + e^{-i\alpha(x)} \cdot \partial_\mu \phi^* \cdot e^{i\alpha(x)} \cdot \partial^\mu \phi \\
&\quad - m^2 \phi^* \phi \\
&= \partial_\mu \phi \cdot \partial^\mu \phi - m^2 \phi^* \phi + \text{non-zero terms}
\end{align*}
\]

Not invariant under U(1)!
Local gauge invariance for a complex scalar field

Can we find a derivative operator that commutes with the gauge transformation?

Define

$$D_\mu = \partial_\mu + iA_\mu,$$

where the gauge field $A_\mu$ transforms as

$$A_\mu \to A_\mu - \partial_\mu \alpha.$$
Local gauge invariance for a complex scalar field

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\[
\begin{align*}
D_\mu \phi &\rightarrow (\partial_\mu + i[A_\mu - \partial_\mu \alpha(x)]) [e^{i\alpha(x)} \phi] \\
&= \partial_\mu [e^{i\alpha(x)} \phi] + i[A_\mu - \partial_\mu \alpha(x)] [e^{i\alpha(x)} \phi] \\
&= ie^{i\alpha(x)} \partial_\mu \alpha(x) \cdot \phi + e^{i\alpha(x)} \partial_\mu \phi + iA_\mu e^{i\alpha(x)} \phi - i\partial_\mu \alpha(x) e^{i\alpha(x)} \phi \\
&= e^{i\alpha(x)} [\partial_\mu \phi + iA_\mu] \phi \\
&= e^{i\alpha(x)} D_\mu \phi
\end{align*}
\]
Local gauge invariance
for a complex scalar field

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where the gauge field \( A_\mu \) transforms as

\[ A_\mu \to A_\mu - \partial_\mu \alpha \]

\[ D_\mu \phi \to (\partial_\mu + i[A_\mu - \partial_\mu \alpha(x)])[e^{i\alpha(x)}\phi] \]

\[ = \partial_\mu [e^{i\alpha(x)}\phi] + i[A_\mu - \partial_\mu \alpha(x)][e^{i\alpha(x)}\phi] \]

\[ = ie^{i\alpha(x)}\partial_\mu \alpha(x) \cdot \phi + e^{i\alpha(x)}\partial_\mu \phi + iA_\mu e^{i\alpha(x)}\phi - i\partial_\mu \alpha(x)e^{i\alpha(x)}\phi \]

\[ = e^{i\alpha(x)}\partial_\mu \phi + iA_\mu e^{i\alpha(x)}\phi \]

\[ = e^{i\alpha(x)}[\partial_\mu \phi + iA_\mu]\phi \]

\[ = e^{i\alpha(x)}D_\mu \phi \]

Nota bene:

- We call \( D_\mu \) the **covariant derivative**, because it transforms just like \( \phi \) itself:

\[ \phi \to e^{i\alpha(x)}\phi \quad \text{and} \quad D_\mu \phi \to e^{i\alpha(x)}D_\mu \phi \]
Local gauge invariance for a complex scalar field

Can we find a derivative operator that commutes with the gauge transformation?

Define

\[ D_\mu = \partial_\mu + iA_\mu, \]

where the gauge field \( A_\mu \) transforms as

\[ A_\mu \rightarrow A_\mu - \partial_\mu \alpha \]

\[ D_\mu \phi \rightarrow (\partial_\mu + i[A_\mu - \partial_\mu \alpha(x)])[e^{i\alpha(x)}\phi] \]

\[ = \partial_\mu [e^{i\alpha(x)}\phi] + i[A_\mu - \partial_\mu \alpha(x)][e^{i\alpha(x)}\phi] \]

\[ = ie^{i\alpha(x)}\partial_\mu \alpha(x)\cdot \phi + e^{i\alpha(x)}\partial_\mu \phi + iA_\mu e^{i\alpha(x)}\phi - i\partial_\mu \alpha(x)e^{i\alpha(x)}\phi \]

\[ = e^{i\alpha(x)}\partial_\mu \phi + iA_\mu e^{i\alpha(x)}\phi \]

\[ = e^{i\alpha(x)}[\partial_\mu \phi + iA_\mu]\phi \]

\[ = e^{i\alpha(x)}D_\mu \phi \]

Notabene:

- We call \( D_\mu \) the covariant derivative, because it transforms just like \( \phi \) itself:

\[ \phi \rightarrow e^{i\alpha(x)}\phi \quad \text{and} \quad D_\mu \phi \rightarrow e^{i\alpha(x)}D_\mu \phi \]

\[ D_\mu \phi^* D^\mu \phi - m^2 \phi^* \phi \rightarrow e^{-i\alpha(x)} D_\mu \phi^* \cdot e^{i\alpha(x)} D^\mu \phi - m^2 e^{-i\alpha(x)} \phi^* \cdot e^{i\alpha(x)} \phi = D_\mu \phi^* D^\mu \phi - m^2 \]
Expanding the Lagrangian

\[ D_\mu \phi^* D^\mu \phi - m^2 \phi^* \phi \]  is invariant under local U(1) transformations

\[ D_\mu \phi^* D^\mu \phi - m^2 \phi^* \phi = \partial_\mu \phi^* \partial^\mu \phi + iA^\mu (\phi \partial_\mu \phi^* - \phi^* \partial_\mu \phi) + \phi^* \phi A_\mu A^\mu - m^2 \phi^* \phi \]

• Demand symmetry → Generate interactions
• Generated mass for gauge boson (after \(\phi\) acquires a vacuum expectation value)
• Explicit mass term forbidden by gauge symmetry (although otherwise allowed):
  \[ m^2 A_\mu A^\mu \rightarrow m^2 (A_\mu - \partial_\mu \alpha)(A_\mu - \partial_\mu \alpha) \neq m^2 A_\mu A^\mu \]
• Simplest form of Higgs mechanism
• Vector-scalar-scalar interaction
Non-Abelian gauge symmetry

<table>
<thead>
<tr>
<th>Abelian</th>
<th>Non-Abelian: component notation</th>
<th>Non-Abelian: vector notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U = e^{i\alpha(x)}$</td>
<td>$U = e^{i\alpha^a(x)T^a_R}$</td>
<td>$U = e^{i\alpha^a(x)T^a_R}$</td>
</tr>
<tr>
<td>$\phi \rightarrow U\phi$</td>
<td>$\Phi^i \rightarrow U^i_k \Phi^k$</td>
<td>$\Phi \rightarrow U\Phi$</td>
</tr>
<tr>
<td>$A_\mu$</td>
<td>$A^a_\mu T^a_R$</td>
<td>$A_\mu$</td>
</tr>
<tr>
<td>$A_\mu \rightarrow A_\mu - \partial_\mu \alpha$</td>
<td>$A^a_\mu T^a_R \rightarrow \frac{1}{g} (\partial_\mu U) U^\dagger$</td>
<td>$A_\mu \rightarrow U A_\mu U^\dagger - \frac{i}{g} (\partial_\mu U) U^\dagger$</td>
</tr>
<tr>
<td>$F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu$</td>
<td>$F^a_{\mu\nu} := \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$</td>
<td>$F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$</td>
</tr>
<tr>
<td>$F_{\mu\nu} \rightarrow F_{\mu\nu}$</td>
<td>$F^a_{\mu\nu}F^{a\mu\nu} \text{ invariant}$</td>
<td>$F_{\mu\nu} \rightarrow U F_{\mu\nu} U^\dagger$</td>
</tr>
<tr>
<td>$F_{\mu\nu}$ invariant</td>
<td></td>
<td>$\text{Tr}(F_{\mu\nu}F^{\mu\nu}) \text{ invariant}$</td>
</tr>
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</table>

$$D_\mu = \partial_\mu + ig A^a_\mu T^a_R$$
Conjecture

- All fundamental internal symmetries are gauge symmetries.
- Global symmetries are just “accidental” and not exact.
Spontaneous Symmetry Breaking
**Gauge group**

\[ \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y \]

**Particle content**

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<td>( Q = \begin{pmatrix} u_L \ d_L \end{pmatrix} )</td>
<td>( (1, 2)_{-1} )</td>
<td>( B )</td>
</tr>
<tr>
<td>( L = \begin{pmatrix} \nu_L \ e_L \end{pmatrix} )</td>
<td>( (1, 2)_1 )</td>
<td>( W )</td>
</tr>
<tr>
<td>( u_R^c )</td>
<td>( (3, 1)_{-4/3} )</td>
<td>( \nu_R )</td>
</tr>
<tr>
<td>( d_R^c )</td>
<td>( (3, 1)_{2/3} )</td>
<td>( (1, 0) )</td>
</tr>
</tbody>
</table>

\[
\mathcal{L} = -\frac{1}{4} G^\alpha_{\mu\nu} G^{\alpha\mu\nu} + \cdots \overline{Q}_k Q_k + \cdots (D_\mu H)^\dagger (D^\mu H) - \mu^2 H^\dagger H - \frac{\lambda}{4!} (H^\dagger H)^2 + \cdots Y_{\ell k} \overline{Q}_k H (u_R)_\ell
\]

- \( H \to H' + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \)
- \( \text{SU}(2)_L \times \text{U}(1)_Y \to \text{U}(1)_Q \)
- \( B, W^3 \to \gamma, Z^0 \) and \( W^1_\mu, W^2_\mu \to W^+, W^- \)
- Fermions acquire mass through Yukawa couplings to Higgs
The Higgs mechanism

- The Higgs potential: \( V = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \)

- Vacuum = Ground state = Minimum of \( V \):

- If \( \mu^2 > 0 \) (massive particle): \( \phi_{\text{min}} = 0 \) (no symmetry breaking)

- If \( \mu^2 < 0 \): \( \phi_{\text{min}} = \pm v = \pm (-\mu^2/\lambda)^{1/2} \)
  These two minima in one dimension correspond to a continuum of minimum values in SU(2).
  The point \( \phi = 0 \) is now instable.

- Choosing the minimum (e.g. at +\( v \)) gives the vacuum a preferred direction in isospin space \( \rightarrow \) spontaneous symmetry breaking

- Perform perturbation around the minimum
In the SM, the Higgs self-couplings are a consequence of the Higgs potential after expansion of the Higgs field $H \sim (1,2)_{1}$ around the vacuum expectation value which breaks the ew symmetry:

$$V_H = \mu^2 H^\dagger H + \eta (H^\dagger H)^2 \rightarrow \frac{1}{2} m_h^2 h^2 + \sqrt{\frac{\eta}{2}} m_h h^3 + \frac{\eta}{4} h^4$$

with:

$$m_h^2 = 2\eta v^2, \quad v^2 = -\mu^2 / \eta$$

Note: $v=246$ GeV is fixed by the precision measures of $G_F$.

In order to completely reconstruct the Higgs potential, one has to:

- Measure the 3h-vertex:
  
  via a measurement of Higgs pair production

  $$\lambda_{3h}^{SM} = \sqrt{\frac{\eta}{2}} m_h$$

- Measure the 4h-vertex:
  
  more difficult, not accessible at the LHC in the high-lumi phase
In the SM, the Higgs self-couplings are a consequence of the Higgs potential after expansion of the Higgs field $H \sim (1,2)_1$ around the vacuum expectation value which breaks the ew symmetry:

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with:

Note: $v=246$ GeV is fixed by the precision measures of $G_F$

Measuring the 3h-couplings: major goal for the high-lumi phase at the LHC

The Higgs particle is just the messenger!

Need to reconstruct the potential

• Measure the 4h-vertex: more difficult, not accessible at the LHC in the high-lumi phase
Not so compact anymore

\[ L_{SM} = \sum_{\ell, e, \mu, \tau} i \bar{\psi}_\ell \gamma^\mu \partial_\mu \psi_\ell + \sum_{\ell, e, \mu, \tau} i \bar{\psi}_e \gamma^\mu \partial_\mu \psi_e + \sum_{\ell, e, \mu, \tau} i \bar{\psi}_\mu \gamma^\tau \partial_\tau \psi_\mu + \sum_{\ell, e, \mu, \tau} i \bar{\psi}_\tau \gamma^\rho \partial_\rho \psi_\tau \]

\[ - \frac{1}{2} (\partial_\mu W^\mu_\nu - \partial_\nu W^\mu_\mu)(\partial^\nu W^{\rho \nu} - \partial^\rho W^\mu_\mu) - \frac{1}{4} (\partial_\mu Z^\mu - \partial_\nu Z^\nu)(\partial^\nu Z^\nu - \partial^\nu Z^\mu) \]

\[ - \frac{1}{4} (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) - \frac{8}{3} \sum_{a=1}^8 (\partial_\mu G^\mu_a - \partial_\nu G^\nu_a)(\partial^\nu G^{\rho \nu}_a - \partial^\rho G^{\mu \nu}_a) + \frac{1}{2} \partial_\mu h \partial^\mu h \]

\[ - \sum_{\ell, e, \mu, \tau} \lambda_\mu \frac{\gamma^\nu}{\sqrt{2}} \bar{\psi}_\mu \psi_\nu - 3 \sum_{i} \frac{\lambda_i \gamma^\nu}{\sqrt{2}} \bar{\psi}_\mu \psi_\nu - \sum_{\ell, e, \mu, \tau} \lambda_\mu \frac{\gamma^\nu}{\sqrt{2}} \bar{\psi}_\mu \psi_\nu \]

\[ - \frac{1}{2} \left( \frac{g}{2 \cos \theta_W} \right)^2 W^\mu_\nu W^{\rho \nu} - \frac{1}{2} \left( \frac{g}{2 \cos \theta_W} \right)^2 Z^\mu Z^\nu - \frac{1}{2} (2m^2) h^2 \]

\[ + \frac{g}{4 \cos \theta_W} \left( \sum_{\ell, e, \mu, \tau} \bar{\psi}_\mu \gamma^\mu \left( 4 \sin^2 \theta_W - 1 \right) \psi_\mu + \sum_{\ell, e, \mu, \tau} \bar{\psi}_\nu \gamma^\nu \left( 1 - \gamma^5 \right) \psi_\nu Z^\mu \right) \]

\[ + \frac{g}{4 \cos \theta_W} \left( \sum_\ell \sum_{e, \mu, \tau} \bar{\psi}_\mu \gamma^\mu \left( 1 - \gamma^5 \right) \psi_\mu W^\mu_\mu + \sum_\ell \sum_{e, \mu, \tau} \bar{\psi}_\nu \gamma^\nu \left( 1 - \gamma^5 \right) \psi_\nu W^\mu_\nu \right) \]

\[ + \frac{g}{\sqrt{2}} \left( \sum_\ell \sum_{e, \mu, \tau} \bar{\psi}_\mu \gamma^\mu \left( 1 - \gamma^5 \right) \psi_\mu W^\mu_\mu + \sum_\ell \sum_{e, \mu, \tau} \bar{\psi}_\nu \gamma^\nu \left( 1 - \gamma^5 \right) \psi_\nu W^\mu_\nu \right) \]

\[ + g_{em} \left( - \sum_{\ell, e, \mu, \tau} \bar{\psi}_\mu \gamma^\mu \psi_\mu A_\mu + \frac{2}{3} \sum_{\ell, e, \mu, \tau} \bar{\psi}_\mu \gamma^\nu \psi_\nu A_\mu - \frac{1}{3} \sum_{\ell, e, \mu, \tau} \bar{\psi}_\nu \gamma^\mu \psi_\mu A_\nu \right) \]

\[ + g_q \left( \sum_i \sum_{e, \mu, \tau} \bar{\psi}_\mu \gamma^\mu \psi_\mu G^\mu_\mu T^i_\mu + \sum_i \sum_{e, \mu, \tau} \bar{\psi}_\nu \gamma^\nu \psi_\nu G^\nu_\mu T^i_\mu \right) \]

\[ - \sum_{\ell, e, \mu, \tau} \lambda_\mu \frac{\gamma^\nu}{\sqrt{2}} \bar{\psi}_\mu \psi_\nu - 3 \sum_{\ell, e, \mu, \tau} \lambda_\mu \frac{\gamma^\nu}{\sqrt{2}} \bar{\psi}_\mu \psi_\nu - \sum_{\ell, e, \mu, \tau} \lambda_\mu \frac{\gamma^\nu}{\sqrt{2}} \bar{\psi}_\mu \psi_\nu \]

\[ + g_{em} \left( \partial_\mu A_\nu W^{\mu + \nu} + \partial_\nu A_\nu W^{\mu + \nu} - \partial_\nu A_\mu W^{\mu + \nu} - \partial_\nu A_\nu W^{\mu + \nu} \right) \]

\[ + g_{em} \left( \partial_\mu A_\nu W^{\mu + \nu} - \partial_\nu A_\mu W^{\mu + \nu} \right) \]

\[ - \frac{1}{2} \left( \frac{g}{2 \cos \theta_W} \right)^2 W^\mu_\nu W^{\rho \nu} + \frac{g}{4 \cos \theta_W} Z^\mu Z^\nu - \frac{1}{2} (2m^2) h^2 \]

\[ g_{em} = g \sin \theta_W, \quad v^2 = - \frac{m^2}{\lambda} \left( m^2 < 0, \lambda > 0 \right), \quad m_\ell = \frac{\lambda_v}{\sqrt{2}}, \quad m_q = \frac{3 \lambda_v}{\sqrt{2}}, \quad m_W = \frac{v}{\sqrt{2}}, \quad m_Z = \frac{v}{2 \cos \theta_W}, \quad m_h = \sqrt{-2m^2} \]
IV. From the SM to predictions at the LHC
Scattering theory

Cross sections can be calculated as

\[ \sigma = \frac{1}{F} \int \text{dPS}^{(n)} |M_{fi}|^2 \]

- We integrate over all final state configurations (momenta, etc.).
  - The phase space (dPS) only depend on the final state particle momenta and masses
  - Purely kinematical

- We average over all initial state configurations
  - This is accounted for by the flux factor F
  - Purely kinematical

- The matrix element squared contains the physics model
  - Can be calculated from Feynman diagrams
  - Feynman diagrams can be drawn from the Lagrangian
  - The Lagrangian contains all the model information (particles, interactions)
The differential cross section: \[ d\sigma = \frac{1}{F} |M|^2 d\Phi_n \]

The Lorentz-invariant phase space:

\[ d\Phi_n = (2\pi)^4 \delta^{(4)}(p_a + p_b - \sum_{f=1}^{n} p_f) \prod_{f=1}^{n} \frac{d^3 p_f}{(2\pi)^3 2E_f} \]

The flux factor:

\[ F = \sqrt{(p_a \cdot p_b)^2 - p_a^2 p_b^2} \]
Decay width

The differential decay width:

\[ d\Gamma = \frac{1}{2E_a} |M|^2 d\Phi_n \]

The Lorentz-invariant phase space:

\[ d\Phi_n = (2\pi)^4 \delta^{(4)}(p_a - \sum_{f=1}^{n} p_f) \prod_{f=1}^{n} \frac{d^3 p_f}{(2\pi)^3 2E_f} \]

Rest frame of decaying particle:

\[ E_a = M_a \]
Life time and branching ratio

Life time: \( \tau = 1/\Gamma \)

Branching ratio: \( \text{BR}(i \rightarrow f) = \frac{\Gamma(i \rightarrow f)}{\Gamma(i \rightarrow \text{all})} \)
The model

✧ All the model information is included in the Lagrangian

✧ Before electroweak symmetry breaking: very compact

\[ \mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W_i^{\mu\nu} W_i^{\mu\nu} - \frac{1}{4} G^{\mu\nu}_a G_a^{\mu\nu} \]

\[ + \sum_{f=1}^{3} \left[ \bar{L}_f \left( i \gamma^\mu D_\mu \right) L^f + \bar{e}_{R,f} \left( i \gamma^\mu D_\mu \right) e_R^f \right] \]

\[ + \sum_{f=1}^{3} \left[ \bar{Q}_f \left( i \gamma^\mu D_\mu \right) Q^f + \bar{u}_{R,f} \left( i \gamma^\mu D_\mu \right) u_R^f + \bar{d}_{R,f} \left( i \gamma^\mu D_\mu \right) d_R^f \right] \]

\[ + D_\mu \phi^\dagger D^\mu \phi - V(\phi) \]

✧ After electroweak symmetry breaking: quite large

Example: electroweak boson interactions with the Higgs boson:

\[ D_\mu \phi^\dagger D^\mu \phi = \frac{1}{2} \partial_\mu h \partial^\mu h + \frac{e^2 v^2}{4 \sin^2 \theta_w} W_\mu^+ W_-^\mu + \frac{e^2 v^2}{8 \sin^2 \theta_w \cos^2 \theta_w} Z_\mu Z^\mu \]

\[ + \frac{e^2 v}{2 \sin^2 \theta_w} W_\mu^+ W_-^\mu h + \frac{e^2 v}{4 \sin^2 \theta_w \cos^2 \theta_w} Z_\mu Z^\mu h \]

\[ + \frac{e^2}{4 \sin^2 \theta_w} W_\mu^+ W_-^\mu hh + \frac{e^2}{8 \sin^2 \theta_w \cos^2 \theta_w} Z_\mu Z^\mu hh . \]
Feynman diagrams and Feynman rules I

✦ Diagrammatic representation of the Lagrangian

❖ Electron-positron-photon \((q = -1)\)

\[
\begin{align*}
e^+ & \quad A_{\mu} \quad e^- \\
& \quad \text{From the Lagrangian}
\end{align*}
\]

❖ Muon-antimuon-photon \((q = -1)\)

\[
\begin{align*}
\mu^+ & \quad A_{\mu} \quad \mu^- \\
& \quad \text{From the Lagrangian}
\end{align*}
\]

✦ The Feynman rules are the building blocks to construct Feynman diagrams
Loop diagrams

Loops exist, but their contribution can usually be neglected.

Loops exist, but their contribution is often small.
From Feynman diagrams to $M_f$:

We construct all possible diagrams with the set of rules at our disposal.

We can then calculate the squared matrix element and get the cross section.

$$i M_{fi} = \left[ \bar{u}_{s_a} (p_a) (-ie\gamma^\mu) u_{s_b} (p_b) \right] \frac{-\eta_{\mu\nu}}{(p_a + p_b)^2} \left[ \bar{u}_{s_2} (p_2) (-ie\gamma^\nu) u_{s_1} (p_1) \right]$$
## Feynman rules for the Standard Model

<table>
<thead>
<tr>
<th>Process</th>
<th>QED</th>
<th>QCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>$q\bar{q}\gamma$</td>
<td>$W^+W^-\gamma$</td>
</tr>
<tr>
<td>$Z$</td>
<td>$q\bar{q}Z$</td>
<td>$W^+W^-Z$</td>
</tr>
<tr>
<td>$W^+$</td>
<td>$q\bar{q}W$</td>
<td>$WWW$</td>
</tr>
<tr>
<td>$g$</td>
<td>$q\bar{g}$</td>
<td>$gg$</td>
</tr>
<tr>
<td>$h$</td>
<td>$q\bar{q}h$</td>
<td>$W^+W^-h$</td>
</tr>
</tbody>
</table>

### QED
- $\gamma$: Photons
- $Z$: Z bosons
- $W^+$: W bosons

### QCD
- $g$: Gluons

Almost all the building blocks necessary to draw any SM diagrams.

QCD coupling much stronger than QED coupling → dominant diagrams.
### Drawing Feynman diagrams I

<table>
<thead>
<tr>
<th>Electron (e)</th>
<th>QED</th>
<th>$\gamma\gamma$</th>
<th>$\gamma\gamma$</th>
<th>$\gamma\gamma$</th>
<th>$\gamma\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z</td>
<td>QED</td>
<td>$q\bar{q}Z$</td>
<td>$W^+W^-Z$</td>
<td>$W^+W^-Z$</td>
<td>$W^+W^-Z$</td>
</tr>
<tr>
<td>$W^+$</td>
<td>QED</td>
<td>$q\bar{q}W$</td>
<td>$W^+W^-W$</td>
<td>$W^+W^-W$</td>
<td>$W^+W^-W$</td>
</tr>
<tr>
<td>Gluon (g)</td>
<td>QCD</td>
<td>$q\bar{q}g$</td>
<td>$g$</td>
<td>$g$</td>
<td>$g$</td>
</tr>
<tr>
<td>Higgs (h)</td>
<td>QED</td>
<td>$q\bar{h}h$</td>
<td>$W^+W^-h$</td>
<td>$W^+W^-h$</td>
<td>$W^+W^-h$</td>
</tr>
</tbody>
</table>

**Note:**
- We can now combine building blocks to draw diagrams.
- This ensures to focus only on the **allowed** diagrams.
- We must only consider the **dominant** diagrams.

**Process 0.** $u\bar{u} \rightarrow t\bar{t}$

**QCD**

**QED**

**QED** (subdominant)

---

Monday 24 July 17
## Drawing Feynman diagrams II

<table>
<thead>
<tr>
<th></th>
<th>QED</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>QED</td>
<td>$\gamma\gamma l^-l^+$</td>
<td>$W^+W^-\gamma$</td>
<td></td>
</tr>
<tr>
<td>$Z$</td>
<td>QED</td>
<td>$q\bar{q}Z \ l\bar{l}Z$</td>
<td>$W^+W^-Z$</td>
<td></td>
</tr>
<tr>
<td>$W^{+-}$</td>
<td>QED</td>
<td>$q\bar{q}' W \ l\bar{l}W$</td>
<td>$WWWW$</td>
<td></td>
</tr>
<tr>
<td>$g$</td>
<td>QCD</td>
<td>$q\bar{q}g$</td>
<td>$gg$</td>
<td>$ggg$</td>
</tr>
<tr>
<td>$h$ (m)</td>
<td>QED</td>
<td>$\gamma h \ l\bar{l}h$</td>
<td>$W^+W^-h$</td>
<td>$ZZh$</td>
</tr>
</tbody>
</table>

**Find out the dominant diagrams for**

- **Process 1.** $gg \rightarrow t\bar{t}$
- **Process 2.** $gg \rightarrow t\bar{t}h$
- **Process 3.** $u\bar{u} \rightarrow t\bar{t} b\bar{b}$

**What is the QCD/QED order?**
(keep only the dominant diagrams)

**CERN Summer Program**

Tim Stelzer

Monday 24 July 17
MadGraph5_aMC@NLO

- Check your answer online:
  MadGraph5_aMC@NLOwebpage
- Requires registration
Web process syntax

Initial state

\[
\begin{align*}
&\quad u \; u^\sim > \quad b \; b^\sim \quad t \quad t^\sim \\
&\text{Final state} \\
&\quad u \; u^\sim > \quad b \; b^\sim \quad t \quad t^\sim \quad \text{QED=2} \quad \\
&\quad \text{Minimal coupling order} \\
&\quad u \; u^\sim > \quad h > \quad b \; b^\sim \quad t \quad t^\sim \\
&\quad \text{Required intermediate particles} \\
&\quad u \; u^\sim > \quad b \; b^\sim \quad t \quad t^\sim \quad / \quad z \quad a \\
&\quad \text{Excluded particles} \\
&\quad u \; u^\sim > \quad b \; b^\sim \quad t \quad t^\sim , \quad t^\sim > \quad w^- \quad b^- \\
&\quad \text{Specific decay chain}
\end{align*}
\]
User requests a process

- $g g \rightarrow t \bar{t} b \bar{b}$
- $u d \rightarrow w^+ z, w^+ \rightarrow e^+ \nu e, z \rightarrow b \bar{b}$
- etc.

MadGraph returns:

- Feynman diagrams
- Self-contained Fortran code for $|M_{fi}|^2$

Still needed:

- What to do with a Fortran code?
- How to deal with hadron colliders?
Proton-Proton collisions I

The master formula for hadron colliders

\[ \sigma = \frac{1}{F} \sum_{ab} \int dPS^{(n)} dx_a \, d x_b \, f_{a/p}(x_a) \, f_{b/p}(x_b) |M_{fi}|^2 \]

- We sum over all proton constituents \((a \text{ and } b \text{ here})\)
- We include the parton densities \((\text{the } f\text{-function})\)

They represent the probability of having a parton \(a\) inside the proton carrying a fraction \(x_a\) of the proton momentum
PDFs: $x$-dependence

- Valence quarks $p=|uud\rangle$
PDFs: $x$-dependence

- Valence quarks $p = |uud\rangle$
- Gluons carry about 40% of momentum
• Valence quarks  
  \( p = |uud| \)

• Gluons  
  carry about 40% of momentum

• Sea quarks  
  light quark sea, strange sea
PDFs: Q-dependence

- Valence quarks
  \( p=|uuud\) 
- Gluons
  carry about 40% of momentum
- Sea quarks
  light quark sea, strange sea
PDFs: $Q$-dependence

- **Valence quarks**
  $p = |uud\rangle$

- **Gluons**
  carry about 40% of momentum

- **Sea quarks**
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---

Altarelli-Parisi evolution equations

---

Monday 24 July 17
PDFs: Q-dependence

- **Valence quarks**
  \[ p = |uud| \]

- **Gluons**
  carry about 40% of momentum

- **Sea quarks**
  light quark sea, strange sea

**Altarelli-Parisi evolution equations**

\[ Q^2 = 100 \text{ GeV}^2 \]
PDFs: $Q$-dependence

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  \[ p = |uud| \]

- **Gluons**
  carry about 40% of momentum

- **Sea quarks**
  light quark sea, strange sea

**Altarelli-Parisi evolution equations**
PDFs: Q-dependence

- Valence quarks
  \( p = |uud\) □
- Gluons
  carry about 40% of momentum
- Sea quarks
  light quark sea, strange sea

Altarelli-Parisi evolution equations

\[ Q^2 = 10000 \text{ GeV}^2 \]
Proton-Proton collisions II

This is not the end of the story...

- At high energies, initial and final state quarks and gluons radiate other quark and gluons
- The radiated partons radiate themselves
- And so on...
- Radiated partons hadronize
- We observe hadrons in detectors
Input parameters

- The SM Lagrangian has **26 input parameters** (of course not all are equally important)
- They **need to be fixed** in order to make **predictions**
- The values and patterns of these parameters are quite **bizarre**!
Quantum Corrections

• Quantum corrections have to be considered (otherwise some predictions very rough!)

• **UV** divergences appear

• **Renormalization** of Lagrangian parameters and fields

• This leads to **running parameters**

• Scale-dependence governed by **renormalization group equations** (RGEs)
Asymptotic Freedom

Renormalization of UV-divergences:
Running coupling constant $a_s := \alpha_s / (4\pi)$

$$a_s(\mu) = \frac{1}{\beta_0 \ln(\mu^2 / \Lambda^2)}$$

- Gross, Wilczek (’73); Politzer (’73)

Non-abelian gauge theories:
Negative beta-functions

$$\frac{d a_s}{d \ln \mu^2} = -\beta_0 a_s^2 + \ldots$$

where $\beta_0 = \frac{11}{3} C_A - \frac{2}{3} n_f$

$\Rightarrow$ asympt. freedom: $a_s \downarrow$ for $\mu \uparrow$

- Nobel Prize 2004