Université Joseph Fourier - Grenoble 1

**Rapport de Stage de Master 2**
Spécialité: Physique Subatomique et Astroparticules

présenté par
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# Poduction des slepton célibataires dans les collisions hadroniques polarisées

# Single Slepton Production at Polarized Hadron Collisions

18 juin 2008



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**Résumé**:

Le rapport présent commence avec la motivation pour sypersymétrie et donne en suite une introduction théorique de la construction des Lagrangiens supersymétriques. En passant, la R–parité est introduite et des termes brisants la R–parité sont considérés d'être une partie naturelle d'un Lagrangien sypersymétrique. Ce chapitre est complété par la dérivation des règles de Feynman correspondantes.
La deuxiéme partie constiste de l'exploration phénoménologique de la production resonante des sleptons célibataires dans les collisions hadroniques, divisée en résultats analytiques et numériques. Finallement, les modes de désintégration du slepton produit sont étudiés.
A la fin, le travail présenté est brèvement résumé et des conclusions courtes sont données.

**Abstract**:

The present report begins with the motivation for supersymmetry and gives in the following an introduction into the theoretical construction of a supersymmetric Lagrangian. Thereby, R-parity is introduced and R-parity violating terms are considered to be natural parts of a supersymmetric Lagrangian. The chapter is completed with the derivation of the corresponding Feynman rules.
The second large part consists of a phenomenological investigation of resonant single slepton production at both, polarized and unpolarized hadron colliders. It is divided into analytical and numerical results. Finally the possible decay modes of the produced slepton are studied.
At the end, the present work is briefly summarized and a short outlook is given.

**Acknowledgment**:

At this place, I'd like to thank first of all my family, especially my parents, who supported my decision to go to Grenoble.
Then I thank Ingo for his help, effort, advice and last but not least for his amicability.
Thanks to Joel Daniels for is correction of my English.
Finally, my thanks go to the whole theory group, where I want to mention Prof. M. Klasen for his taking care of me and Karol Kovarik for his given lectures in SUSY.

# Contents

# 1 Introduction

At present the starting up of the LHC (Large Hadron Collider) is highly awaited by nearly all the high energy physicists. The beam commissioning is planned to take place this summer. It will be the last collider of its design in regard of its size (27 km circumference of the pipe tunnel), reached energy (14 $TeV$) and costs ($>$ 3 billion Euros). It is expected to give answers on fundamental questions such as the search for a Higgs boson and physics beyond the SM (Standard Model), as well as new and more precise measurements of the SM parameters, for example the CKM matrix.

One of the favorite extension of the SM is considered for about 30 years: Supersymmetry. It predicts new particles, so called 'sparticles', which are supposed to be heavy, too heavy to have already been seen in experiments up to now, but light enough to be seen at the LHC , that's the hope. The report at hand follows this hope and investigates a possible extension of the MSSM (Minimal Supersymmetric SM): R-parity violation. Such research is important in order to take into account the possible phenomenology to be found at the LHC in the data analysis.

After an introduction into supersymmetry containing R-parity violation and their Feynman rules, single sleptons production and its implications are analyzed. This contains the cross section of resonant single sleptons production, taking into account the polarization of incoming particles and the parton distribution functions, and their decays. The results are grouped in an analytical and a numerical part being completed by our conclusions.

# 2 Supersymmetry

## 2.1 Motivation

The Standard Model (SM) of particle physics is a very powerful theory describing most of known phenomena in elementary particle physics. It is verified by precision tests up to a very high accuracy, see e.g. [1, 2, 3]. Nevertheless, the SM is not a complete theory, because neutrinos are not massless, as it assumes, and it doesn't explain gravity at all. Rather, the SM is considered to be an effective theory at low energies of the order of the electroweak energy scale $v = 246\,GeV$. Further questions beyond the SM are of the following kind: Why are left-handed fermions in SU(2) doublets and right-handed ones in singlets? Why are there three colors? Why is electric charge quantized? How many generations are there? etc. [4].

One possible extension of the SM is imposing supersymmetry (SUSY). It adds to each fermion in the SM supersymmetric boson partners and vice versa. Until now, none of these new particles have been discovered, but there are several theoretical arguments which make SUSY an interesting theory:

- SUSY provides a transformation $Q|fermion\rangle = |boson\rangle$ which connects bosons and fermions. This allows them to be seen as sharing a common origin rather than as fundamentally different particles.
- The three gauge couplings of the SM become very close at a possible grand unification scale (GUT scale). This is why supersymmetric models are favorite candidates for unified theories [5].
- If supersymmetry is realized as a local symmetry, then it includes a spin-2 field, the graviton, and a spin-$\frac{3}{2}$ field, the gravitino, and so it opens a way to explain gravity (SUGRA). This is also a possible scenario for the necessary supersymmetry breaking.
- The lightest supersymmetric particle (LSP) is a favorite dark matter candidate [6], which the SM doesn´t provide at all.
- There are possible neutrino mass generating mechanisms.
- String theory assumes supersymmetry in order to reduces dimensions and to make the theory more consistent.
- It provides a solution to the fine-tuning problem, which is strongly related to the hierarchy problem.

## 2.2 Superalgebra

A crucial symmetry of quantum field theory is the space-time symmetry with its underlying Poincaré group satisfying the algebra

$$[P^\mu, P^\nu] = 0 \ , \tag{2.1}$$

$$\left[J^{\mu\nu}, P^\lambda\right] = i\left(g^{\nu\lambda}P^\mu - g^{\mu\lambda}P^\nu\right) \ , \tag{2.2}$$

$$\left[J^{\mu\nu}, J^{\lambda\rho}\right] = i\left(g^{\nu\lambda}J^{\mu\rho} - g^{\mu\lambda}J^{\nu\rho} + g^{\mu\rho}J^{\nu\lambda} - g^{\nu\rho}J^{\mu\lambda}\right) \ , \tag{2.3}$$

where $P^\mu$ is the generator of space-time translations and $J^{\mu\nu}$ is the generator of rotations and Lorentz boosts, rephrased space-time rotations or Lorentz transformations. $J^{\mu\nu}$ satisfies an $SO(3,1) \simeq SU(2) \times SU(2)$ Lie algebra (Lorentz algebra). This isomorphism allows the short notation for the representation of the Lorentz group $(j_1, j_2)$ with $j_1, j_2 \in \mathbb{N}/2$.

After Coleman and Mandula formulated their no-go theorem [7], which forbids any further external symmetry of the S-matrix, Haag, Lopuszanski and Sohnius [8] were yet able to find a way to extent this group by introducing *fermionic* generators $Q$, which transform under $(\frac{1}{2}, 0)$ and fulfill anticommutator relations. The minimal supersymmetric standard model (MSSM) has one such generator[1] $Q$ with the following algebra, called superalgebra:

$$[Q_\alpha, J^{\mu\nu}] = i\sigma^{\mu\nu}{}_\alpha{}^\beta Q_\beta \ , \tag{2.4}$$

$$[Q_\alpha, P^\mu] = 0 \ , \tag{2.5}$$

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0 \ , \tag{2.6}$$

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu \ . \tag{2.7}$$

The operator $\bar{Q}$ is defined as $\bar{Q}_{\dot\alpha} = Q_\alpha{}^\dagger$ and transforms as $(0, \frac{1}{2})$; $\alpha, \dot\beta = 1, 2$. The definitions and conventions of the several $\sigma$-matrices are listed in appendix A. It can be shown by using Eq. (2.7) that the energy of any non-vacuum state $\langle\Psi|P^0|\Psi\rangle$ is positive-definite and that the vanishing of the vacuum energy is a necessary and sufficient condition for the existence of a unique vacuum state $|0\rangle$ such that

$$\langle 0|P^0|0\rangle = 0 \ \Leftrightarrow \ Q_\alpha|0\rangle = 0 \ . \tag{2.8}$$

If $Q$ acts on a state, it changes the spin by an amount of $\frac{1}{2}$; $Q_1$ and $\bar{Q}_{\dot 2}$ increase it while $Q_2$ and $\bar{Q}_{\dot 1}$ decrease it. Another consequence of Eq. (2.7) is that all states that transform under a common irreducible representation of supersymmetry[2] are called supermultiplet and have the same mass, since $[Q, P^2] = 0$ and $P^2 = m^2$. This is obviously in contradiction to experiments, since no supersymmetric particle has yet been found, wherefore we know that if supersymmetry exists, it is a broken symmetry. We also remark that if supersymmetry is realized as a local symmetry, the energy-momentum operator on the right hand side of Eq. (2.7) becomes dependent on space-time as well, which opens the door to gravity (SUGRA).

[1]In extended SUSY models several generators $Q^1, Q^2, .., Q^N$ with $N = 1, 2, 4, 8$ are investigated. The higher the symmetry, the higher the restrictions. Therefore the framework of extended SUSY is often a higher dimensional space-time ($D > 4$).

[2]and if they are in the same multiplets of all other symmetry groups

## 2.3 Superfields and Superspace

### 2.3.1 Supersymmetry in Superspace

A consistent way of describing supersymmetry transformations and for constructing supersymmetric Lagrangians is provided by the formulation of superfields. The four ordinary space-time coordinates $x^\mu$ are extended by four anticommuting coordinates realized by the Grassmann variables $\theta_\alpha$ and $\bar{\theta}^{\dot{\beta}}$ with the following algebra

$$\forall\ \alpha, \beta = 1, 2: \quad \{\theta^\alpha, \theta^\beta\} = \{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\} = \{\theta^\alpha, \bar{\theta}_{\dot{\beta}}\} = 0\ . \tag{2.9}$$

Thus, generalized superfields $\widehat{\Phi}(x, \theta, \bar{\theta})$ that transform under irreducible representations of the supersymmetry algebra act on this superspace. The ordinary space-time translations can also be generalized to super-translations:

$$G(x, \theta, \bar{\theta}) = e^{i\left(xP + \theta Q + \bar{\theta}\bar{Q}\right)}\ , \tag{2.10}$$

$$\widehat{\Phi}(x, \theta, \bar{\theta}) = G(x, \theta, \bar{\theta})\ \widehat{\Phi}(0, 0, 0)\ G^{-1}(x, \theta, \bar{\theta})\ . \tag{2.11}$$

Using the identity

$$G(y, \xi, \bar{\xi})\ G(x, \theta, \bar{\theta}) = G(x + y + i(\xi\sigma\bar{\theta} - \theta\sigma\bar{\xi}), \xi + \theta, \bar{\xi} + \bar{\theta}) \tag{2.12}$$

the translation of a superfield $\widehat{\Phi}(x, \theta, \bar{\theta})$ reads as

$$\begin{aligned} G(y, \xi, \bar{\xi})\ &\widehat{\Phi}(x, \theta, \bar{\theta})\ G(x, \theta, \bar{\theta}) \\ &= G(y, \xi, \bar{\xi})\ G(x, \theta, \bar{\theta})\ \widehat{\Phi}(0, 0, 0)\ \left[G(y, \xi, \bar{\xi})\ G(x, \theta, \bar{\theta})\right]^{-1} \\ &= \widehat{\Phi}(x + y + i(\xi\sigma\bar{\theta} - \theta\sigma\bar{\xi}), \xi + \theta, \bar{\xi} + \bar{\theta}) \end{aligned} \tag{2.13}$$

Using the Taylor series of this equation, we obtain

$$[\widehat{\Phi}, P_\mu] = i\partial_\mu\ \widehat{\Phi}\ , \tag{2.14}$$

$$[\widehat{\Phi}, \xi^\alpha Q_\alpha] = i\xi^\alpha \left(\frac{\partial}{\partial\theta^\alpha} + i(\sigma^\nu\bar{\theta})_\alpha\partial_\nu\right)\ \widehat{\Phi}\ , \tag{2.15}$$

$$[\widehat{\Phi}, \bar{Q}_{\dot{\alpha}}\bar{\xi}^{\dot{\alpha}}] = -i\left(\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i(\theta\sigma^\nu)_{\dot{\alpha}}\partial_\nu\right)\bar{\xi}^{\dot{\alpha}}\ \widehat{\Phi}\ . \tag{2.16}$$

From these commutator relations we obtain explicit expressions, in terms of differential operators, for the supertranslation generators (cf. appendix A)

$$\hat{P}^\mu = i\partial^\mu\ , \tag{2.17}$$

$$\hat{Q}_\alpha = i\partial_\alpha - (\sigma^\mu\bar{\theta})_\alpha\partial_\mu\ , \tag{2.18}$$

$$\hat{\bar{Q}}_{\dot{\alpha}} = -i\bar{\partial}_{\dot{\alpha}} + (\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu\ . \tag{2.19}$$

The variation of space-time translations is defined by $\delta_a\Phi(x) := \Phi(x + a) - \Phi(x) = a^\mu\partial_\mu\ \Phi(x) = ia^\mu[P_\mu, \Phi(x)]$. Similarly, the supersymmetric variation is

$$\delta_\xi\widehat{\Phi} := i[\xi Q + \bar{Q}\bar{\xi}, \widehat{\Phi}] = -i(\xi\hat{Q} + \hat{\bar{Q}}\bar{\xi})\ \widehat{\Phi}\ . \tag{2.20}$$

In the following, our aim is to construct, by means of superfields, Lagrangians which are invariant under such supersymmetric transformations. We start with the general expression of a superfield $\widehat{\Phi}$, which is obtained by a Taylor series in the Grassmann variables $\theta$. It turns out to be finite and thus exact because of the nilpotence of the Grassmann variables following from Eq. (2.9):

$$\begin{aligned}\widehat{\Phi}(x,\theta,\bar{\theta}) =& f(x) + \theta\,\zeta(x) + \bar{\theta}\,\bar{\chi}(x) + \theta\theta\,m(x) + \bar{\theta}\bar{\theta}\,n(x) + \theta\sigma^\mu\bar{\theta}\,V_\mu(x) \\ &+ \theta\theta\bar{\theta}\,\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\theta\,\Psi(x) + \theta\theta\bar{\theta}\bar{\theta}\,d(x)\;.\end{aligned} \tag{2.21}$$

The fields $f$, $m$, $n$, $V_\mu$ and $d$ are bosonic, the others fermionic (8 bosonic and 8 fermionic degrees of freedom if $\widehat{\Phi}$ is real, and two times as many if it is complex). We are now searching for smallest possible, supersymmetry invariant terms, in other words we are searching for supermultiplets. Once found, we would like to use a minimal set of superfields with which we get the full particle content and interactions of the SM. In other words, we consider a minimal supersymmetric model containing the SM and we call it MSSM. We will need therefore two kinds of fields: *chiral* and *vector superfields*.

### 2.3.2 Chiral Superfields

Since $\partial_\alpha\widehat{\Phi}$ is not a superfield ($\partial_\alpha\delta_\xi\widehat{\Phi} \neq \delta_\xi\partial_\alpha\widehat{\Phi}$), we introduce the *fermionic covariant derivative* by

$$[D_\alpha, \delta_\xi] = 0 \quad \text{and} \quad [\bar{D}_{\dot{\alpha}}, \delta_\xi] = 0\;, \tag{2.22}$$

whose solutions are

$$D_\alpha = \partial_\alpha - i(\sigma^\mu\bar{\theta})_\alpha\partial_\mu\;, \tag{2.23}$$

$$\bar{D}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} + i(\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu\;. \tag{2.24}$$

They fulfill the same anticommutation relations between themselves as $Q$ and $\bar{Q}$. Then we define a chiral superfield as

$$\bar{D}_{\dot{\alpha}}\widehat{\Phi} := 0\;. \tag{2.25}$$

In order to simplify the solution for the chiral superfield we transform it into appropriate coordinates ("chiral representation"):

$$y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}\;, \quad \theta' = \theta\;, \quad \text{and} \quad \bar{\theta}' = \bar{\theta}\;, \tag{2.26}$$

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = \frac{\partial}{\partial y^\mu} = \partial_\mu{}'\;, \tag{2.27}$$

$$\partial_\alpha = \partial_\alpha{}' - i(\sigma^\mu\bar{\theta})_\alpha\partial_\mu\;, \tag{2.28}$$

$$\bar{\partial}_{\dot{\alpha}} = \bar{\partial}_{\dot{\alpha}}{}' + i(\theta\sigma^\mu)_{\dot{\alpha}}\partial_\mu\;. \tag{2.29}$$

Using this transformation Eq. (2.18)-(2.18) and (2.23) give (we let away the primes)

$$\hat{Q}_\alpha = i\partial_\alpha\;, \tag{2.30}$$

$$\hat{\bar{Q}}_{\dot{\alpha}} = -i\bar{\partial}_{\dot{\alpha}} + 2(\sigma^\mu\bar{\theta})_\alpha\partial_\mu\;, \tag{2.31}$$

$$D_\alpha = \partial_\alpha - 2i(\sigma^\mu\bar{\theta})_\alpha\partial_\mu\;, \tag{2.32}$$

$$\bar{D}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}}\;. \tag{2.33}$$

This allows us to enormously simplify the chiral superfield definition in (2.25) to

$$\bar{\partial}_{\dot{\alpha}}\widehat{\Phi} = 0 \; . \tag{2.34}$$

Thus $\widehat{\Phi} = \widehat{\Phi}(y,\theta)$ does not depend on $\bar{\theta}$ and the general superfield of Eq. (2.21) becomes

$$\widehat{\Phi}(y,\theta) = A(y) + \sqrt{2}\theta\Psi(y) - \theta\theta F(y). \tag{2.35}$$

The factor $\sqrt{2}$ and all the signs are just convention. The fields $A$, $\Psi$ and $F$ build an *irreducible chiral supermultiplet.* We can transform back from chiral coordinates to normal coordinates. But instead of developing the resulting $x^\mu - i\theta\sigma^\mu\bar{\theta}$ as a Taylor series in $\theta$, it is easier to use the equivalent supertranslation[3] $G = \exp(-i\theta\sigma^\mu\bar{\theta})$ leading to

$$\begin{aligned}\widehat{\Phi}(x,\theta,\bar{\theta}) =& A(x) + \sqrt{2}\theta\Psi(x) - \theta\theta F(x) - i\theta\sigma^\mu\bar{\theta}\partial_\mu A(x) \\ &+ \frac{i}{\sqrt{2}}\theta\theta\left(\partial_\mu\Psi(x)\sigma^\mu\bar{\theta}\right) - \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\Box A \; .\end{aligned} \tag{2.36}$$

Using the same arguments as for the chiral superfield $\widehat{\Phi}$, we can define an anti-chiral superfield $\widehat{\bar{\Phi}}$ as the solution of $\bar{D}_{\dot{\alpha}}\widehat{\bar{\Phi}} = 0$. We can again solve this equation using coordinate transformations, and the result is simply

$$\widehat{\bar{\Phi}}(x,\theta,\bar{\theta}) = \widehat{\Phi}^\dagger(x,\theta,\bar{\theta}) \; . \tag{2.37}$$

We are now interested in the explicit supersymmetry transformation (2.20), which, after a short calculation, yields for the component fields

$$\delta_\xi A = \sqrt{2}\xi^\alpha\Psi_\alpha \; , \tag{2.38}$$

$$\delta_\xi\Psi = -i\sqrt{2}\sigma^\mu_{\alpha\dot{\beta}}\bar{\xi}^{\dot{\beta}}\partial_\mu A - \sqrt{2}\xi_\alpha F \; , \tag{2.39}$$

$$\delta_\xi F = -i\sqrt{2}\partial_\mu\Psi^\alpha\sigma^\mu_{\alpha\dot{\beta}}\bar{\xi}^{\dot{\beta}} \; . \tag{2.40}$$

Most remarkable is the $F$-term, also written as $-\widehat{\Phi}\big|_{\theta\theta} = \widehat{\Phi}\big|_F$ because it transforms as a total divergence and hence it will serve as a tool to construct supersymmetry invariant Lagrangians. For this purpose we need an additional theorem.

**Theorem.** *If $\widehat{\Phi}$ is a chiral superfield, then so is $\widehat{\Phi}^n$* $\forall n \in \mathbb{N}$

The proof is by induction, using the formula $\bar{D}_{\dot{\alpha}}\widehat{\Phi}^n = n\widehat{\Phi}^{n-1}\bar{D}_{\dot{\alpha}}\widehat{\Phi}$. This theorem assures that any holomorphic function $W(\widehat{\Phi})$ is another chiral superfield whose $F$-term transforms as a total divergence. $W(\widehat{\Phi})$ is called *superpotential* and describes non-gauge interactions. The demand of renormalizability restricts it to be of, at most, third order in superfields.

It can be shown that $[\widehat{\Phi}^\dagger\widehat{\Phi}]_{\theta\theta\bar{\theta}\bar{\theta}}$ also transforms as a total divergence. Its value is written as

$$[\widehat{\Phi}^\dagger\widehat{\Phi}]_{\theta\theta\bar{\theta}\bar{\theta}} = (\partial_\mu A)^\dagger(\partial^\mu A) + i\bar{\Psi}\sigma^\mu\partial_\mu\Psi + F^\dagger F + \text{total divergence.} \tag{2.41}$$

[3] As usual, fields - as long as not yet quantized - transform as $G\widehat{\Phi}$ and operators as $GOG^{-1}$.

We recognize the kinetic term of a scalar boson field $A$ and a (Weyl-) fermion field $\Psi$. By integrating the Grassmann variables we can now give a manifestly supersymmetric action, the Wess-Zumino action:

$$\begin{aligned} S &= \int \mathrm{d}^4x\, \mathrm{d}^4\theta\, \widehat{\Phi}^\dagger \widehat{\Phi} + \int \mathrm{d}^4x\, \mathrm{d}^2\theta\, W(\widehat{\Phi}) + \int \mathrm{d}^4x\, \mathrm{d}^2\bar{\theta}\, W(\widehat{\Phi}^\dagger) \\ &= \int \mathrm{d}^4x\, \mathcal{L} \end{aligned} \tag{2.42}$$

with

$$\begin{aligned} \mathcal{L} &= (\partial_\mu A)^\dagger (\partial^\mu A) + i\bar{\Psi}\sigma^\mu \partial_\mu \Psi + F^\dagger F \\ &- F\frac{\partial W}{\partial A} - F^\dagger \left(\frac{\partial W}{\partial A}\right)^\dagger - \frac{1}{2}\left[\frac{\partial^2 W}{\partial A^2}\Psi\Psi + \left(\frac{\partial^2 W}{\partial A^2}\right)^\dagger \bar{\Psi}\bar{\Psi}\right] + \text{total div. .} \end{aligned} \tag{2.43}$$

Possible indices for the fields are suppressed. The equation of motion for $F$ is $F = \left(\frac{\partial W}{\partial A}\right)^*$. It has no dynamic and furthermore cancels in the Lagrangian on-shell. It serves as an auxiliary field in order to guarantee supersymmetry for the Lagrangian, in especially for the fields $A, \Psi$ off-shell. The last term is the really interesting part describing Yukawa couplings between chiral supermultiplets. The derivative $\frac{\partial W}{\partial A_i}$ has to be understood as partially differentiating $W$ partially with respect to the superfield $\widehat{\Phi}_i$ and subsequently substituting all superfields by their scalar fields $A_i$.

### 2.3.3 Vector Fields

To rebuild the SM, we need vector bosons as gauge interaction particles[4]. Therefore, we define a real vector superfield as $\widehat{V}(x,\theta,\bar{\theta}) = \widehat{V}^\dagger(x,\theta,\bar{\theta})$ and expand it as a series

$$\begin{aligned} \widehat{V}(x,\theta,\bar{\theta}) &= C(x) + i\theta\chi(x) - i\bar{\theta}\bar{\chi}(x) + \frac{i}{2}\theta\theta[M(x) + iN(x)] \\ &- \frac{i}{2}\bar{\theta}\bar{\theta}[M(x) - iN(x)] + \theta\sigma^\mu\bar{\theta}V_\mu(x) + i\theta\theta\bar{\theta}[\bar{\lambda}(x) - \frac{i}{2}\bar{\sigma}^\mu\partial_\mu\xi(x)] \\ &- i\bar{\theta}\bar{\theta}\theta[\lambda(x) - \frac{i}{2}\sigma^\mu\partial_\mu\xi(x)] + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}[D(x) - \frac{1}{2}\Box C(x)] \;. \end{aligned} \tag{2.44}$$

There are 8 real bosonic $C$, $M$, $N$, $D$, $V_\mu$ and 8 fermionic degrees of freedom $\xi$, $\bar{\xi}$, $\lambda$, $\bar{\lambda}$. We define the (Abelian) field strength

$$F_{\mu\nu} := \partial_\mu V_\nu - \partial_\nu V_\mu \;. \tag{2.45}$$

In superspace the corresponding definition of the *field strength superfield* is

$$\widehat{W}_\alpha := -\frac{1}{4}\bar{D}^2 D_\alpha \widehat{V} \;. \tag{2.46}$$

Note that $\bar{D}_{\dot{\beta}}\widehat{W}_\alpha = 0$, so $\widehat{W}_\alpha$ is a spinor chiral superfield. The solution, worked out in the chiral representation, is

$$\widehat{W}_\alpha = -i\lambda_\alpha + \theta_\alpha D - \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu\theta)_\alpha F_{\mu\nu} - \theta\theta(\sigma^\mu\partial_\mu\bar{\lambda})_\alpha \;. \tag{2.47}$$

[4]Of course, if we introduce any boson particle, supersymmetry will automatically add fermion partners.

Under a supersymmetry transformation (2.20) the fields of $\widehat{W}_\alpha$ transform as

$$\delta_\xi \lambda = i\xi_\alpha D + \frac{1}{2}(\sigma^\mu \bar{\sigma}^\nu)_\alpha{}^\beta \xi_\beta F_{\mu\nu} , \tag{2.48}$$

$$\delta_\xi F_{\mu\nu} = i\partial_\mu(\xi\sigma_\nu\bar{\lambda} - \lambda\sigma_\nu\bar{\xi}) - i\partial_\nu(\xi\sigma_\mu\bar{\lambda} - \lambda\sigma_\mu\bar{\xi}) , \tag{2.49}$$

$$\delta_\xi D = \partial_\mu(\xi\sigma^\mu\bar{\lambda} - \lambda\sigma^\mu\bar{\xi}) . \tag{2.50}$$

It is once more important to note that the field $D$ transforms as a total divergence which provides a way to construct Lagrangians. The fields $\lambda$, $\bar{\lambda}$, $F_{\mu\nu}$ and $D$ form an *irreducible gauge supermultiplet*. Hence one possible term in the Lagrangian is

$$\begin{aligned}
\mathcal{L} &= \frac{1}{4}[\widehat{W}^\alpha \widehat{W}_\alpha]_{\theta\theta} + \frac{1}{4}[\widehat{\bar{W}}_{\dot{\alpha}} \widehat{\bar{W}}^{\dot{\alpha}}]_{\bar{\theta}\bar{\theta}} = \frac{1}{2}[\widehat{W}^\alpha \widehat{W}_\alpha]_{\theta\theta} + \text{total divergence} \\
&= i\bar{\lambda}\bar{\sigma}^\mu\partial_\mu\lambda + \frac{1}{2}D^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \text{total divergence} , \\
S &= \frac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^2\theta\; \widehat{W}^\alpha \widehat{W}_\alpha = \frac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^4\theta\; (D^\alpha \widehat{V})\widehat{W}_\alpha .
\end{aligned} \tag{2.51}$$

This Lagrangian is the kinetic energy for a vector field $V_\mu$ with field strength $F_{\mu\nu}$ and for its fermion partner $\lambda$. This provides a way to handle gauge interactions. Once realized we will call this fermion gaugino.

### 2.3.4 Gauge Interaction

First we restrict ourselves to a $U(1)$ gauge group. Then we use the fact that a field strength superfield $\widehat{W}_\alpha$ is invariant under the following transformation by a chiral superfield $\widehat{\Lambda}(x,\theta,\bar{\theta})$

$$\widehat{V} \to \widehat{V} + i(\widehat{\Lambda} - \widehat{\Lambda}^\dagger) \tag{2.52}$$

If we express $\widehat{\Lambda}$ as in (2.36), then the component fields of $\widehat{V}$ transform as

$$C \to C + i(A + A^\dagger) , \tag{2.53}$$

$$\xi \to \xi + \frac{1}{2}\Psi , \tag{2.54}$$

$$M + iN \to M + iN - 2F , \tag{2.55}$$

$$V_\mu \to V_\mu + \partial_\mu(A - A^\dagger) , \tag{2.56}$$

$$\lambda \to \lambda , \tag{2.57}$$

$$D \to D . \tag{2.58}$$

The good news is that $V_\mu$ transforms as a $U(1)$-gauge transformation and hence $F_{\mu\nu}$ is gauge invariant.

Let us now choose a fixed gauge to make things easier. We choose $A + A^\dagger$, $\Psi$ and $F$ such that $C = \xi = M = N = 0$. Note that $A - A^\dagger$ remains undetermined and the $U(1)$-gauge freedom of $V_\mu$ is thus still valid. This is the *Wess-Zumino gauge*, but it is not a supersymmetric gauge choice, i.e. supersymmetry transformations do not preserve these conditions. Supersymmetric invariant expressions, however, build up in this gauge choice remain invariant in whatever gauge. Having justified our choice, we get

$$\widehat{V}_{WZ} = \theta\sigma^\mu\bar\theta V_\mu(x) + i\theta\theta\bar\theta\bar\lambda(x) - \bar\theta\bar\theta\theta\lambda(x) + \frac{1}{2}\theta\theta\bar\theta\bar\theta D(x) \; . \tag{2.59}$$

A (complex) chiral superfield $\widehat{\Phi}$ can then be rotated by a gauge transformation

$$\widehat{\Phi} \to \exp(-2ig\widehat{\Lambda})\widehat{\Phi} \; . \tag{2.60}$$

As a consequence, the kinetic term $\mathcal{L}_{kin} = \int \mathrm{d}^4\theta \; \widehat{\Phi}^\dagger\widehat{\Phi}$ is not gauge invariant because $\widehat{\Lambda}$ is a complex field, or in other words not hermitian as a field operator[5]. But the modified term

$$\mathcal{L}_{kin} = \int \mathrm{d}^4\theta \; \widehat{\Phi}^\dagger \, e^{2g\widehat{V}} \, \widehat{\Phi} \tag{2.61}$$

is invariant under (2.60). The remaining part is a possible superpotential. The worked out supersymmetric and $U(1)$-gauge invariant Lagrangian in the Wess-Zumino gauge is given by

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\int \mathrm{d}^4x \, \mathrm{d}^4\theta \; (D^\alpha\widehat{V})\widehat{W}_\alpha + \int \mathrm{d}^4\theta \; \widehat{\Phi}^\dagger \, e^{2g\widehat{V}} \, \widehat{\Phi} \\ &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar\lambda\bar\sigma^\mu\partial_\mu\lambda + \frac{1}{2}D^2 + F^\dagger F + gDA^\dagger A \\ &\quad + (\mathcal{D}_\mu A)^\dagger(\mathcal{D}^\mu A) + i\bar\Psi\bar\sigma^\mu\mathcal{D}_\mu\Psi + i\sqrt{2}g(A^\dagger\lambda\Psi - A\bar\lambda\bar\Psi) \; , \end{aligned} \tag{2.62}$$

where $\mathcal{D}_\mu = \partial_\mu + igV_\mu$ is the known gauge covariant derivative. The first two terms in the second line are just the normal gauge couplings between a vector boson $V_\mu$ and a scalar field $A$ or fermion $\Psi$ respectively. The last term describes the supersymmetry counterparts of the gauge interactions. It is the coupling of the fermionic partner of the gauge boson: The gaugino $\lambda$ couples to a scalar field $A$ and a fermion $\Psi$ of the same chiral supermultiplet with $\sqrt{2}$ times the coupling constant $g$. The equation of motion for $D$ is $D = -gA^\dagger A$ and thus it gives a mass term $-\frac{1}{2}gA^\dagger A$ on-shell. Is serves like $F$ as an auxiliary field to keep the whole expression even off-shell supersymmetry invariant.

We will briefly generalize to the non-Abelian case. The chiral superfield is thus a multiplet of the gauge group and transforms as

$$\widehat{\Phi}_i \to \exp(-2ig\widehat{\Lambda})_{ij}\widehat{\Phi}_j \; , \tag{2.63}$$

where $\widehat{\Lambda} = (\widehat{\Lambda}^a T^a)_{ij}$ and $T^a$ are the generators of the gauge group. Likewise $\widehat{V} := \widehat{V}^a T^a$. By this means, the kinetic term (2.61) is gauge invariant if the $\widehat{V}$ transforms as

$$\exp(2g\widehat{V}) \to \exp(-2ig\widehat{\Lambda}^\dagger)\exp(2g\widehat{V})\exp(2ig\widehat{\Lambda}) \tag{2.64}$$

The field strength superfield must be defined as

$$\widehat{W}_\alpha := -\frac{1}{8g}\bar D^2 \exp(-2g\widehat{V})D_\alpha\widehat{V}\exp(2g\widehat{V}) \; , \tag{2.65}$$

[5]The point is that $\exp(+2ig\widehat{\Lambda}^\dagger)\exp(-2ig\widehat{\Lambda}) \neq 1$.

such that it transforms as

$$\widehat{W}_\alpha \to \exp(-2g\widehat{V})\widehat{W}_\alpha \exp(2g\widehat{V}) \tag{2.66}$$

and $Tr(\widehat{W}^\alpha \widehat{W}_\alpha)$ is gauge invariant. Its explicit form in the Wess-Zumino gauge becomes

$$\widehat{W}^a_\alpha = -i\lambda^a_\alpha + \theta_\alpha D^a - \frac{i}{2}(\sigma^\mu \bar{\sigma}^\nu \theta)_\alpha F^a_{\mu\nu} - \theta\theta\sigma^\mu(\mathcal{D}^{ab}_\mu \bar{\lambda}^b)_\alpha \,. \tag{2.67}$$

Here $a$ is the adjoint group index and $f_{abc}$ as the structure constant of the gauge group. We recover the $U(1)$ field strength superfield of Eq. (2.47). The field strength and the covariant derivation are familiar

$$F^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu - g f_{abc} V^b_\mu V^c_\nu \,, \tag{2.68}$$
$$\mathcal{D}^{ac}_\mu = \delta^{ac}\partial_\mu - g f_{abc} V^b_\mu \,. \tag{2.69}$$

Finally, a superpotential must be a gauge singlet. This is the case when $W(e^{-2ig\widehat{\Lambda}}\widehat{\Phi}) = W(\widehat{\Phi})$.

## 2.4 Particle Content of the MSSM

We are now ready to list the particle contents of the MSSM in terms of supermultiplets using chiral and gauge/vector superfields. The chiral supermultiplets are listed in table 2.1. All the SM leptons and quarks can be found together with their superpartners, the scalar sleptons and squarks. One may ask why we have introduced two Higgs doublets

| Name | Superfield | spin 0 | spin 1/2 | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ |
|---|---|---|---|---|---|---|
| squarks, quarks | $\widehat{Q}$ | $(\widetilde{u}_L \;\; \widetilde{d}_L)$ | $(u_L \;\; d_L)$ | $\mathbf{3}$ | $\mathbf{2}$ | $+\frac{1}{6}$ |
| ($\times$3 families) | $\widehat{U}$ | $\widetilde{u}^*_R$ | $u^\dagger_R = u^c_L$ | $\bar{\mathbf{3}}$ | $\mathbf{1}$ | $-\frac{2}{3}$ |
| | $\widehat{D}$ | $\widetilde{d}^*_R$ | $d^\dagger_R = d^c_L$ | $\bar{\mathbf{3}}$ | $\mathbf{1}$ | $+\frac{1}{3}$ |
| sleptons, leptons | $\widehat{L}$ | $(\widetilde{\nu} \; \widetilde{e}_L)$ | $(\nu \;\; e_L)$ | $\mathbf{1}$ | $\mathbf{2}$ | $-\frac{1}{2}$ |
| ($\times$3 families) | $\widehat{E}$ | $\widetilde{e}^*_R$ | $e^\dagger_R = e^c_L$ | $\mathbf{1}$ | $\mathbf{1}$ | $+1$ |
| Higgs, higgsinos | $\widehat{H}_u$ | $(H^+_u \;\; H^0_u)$ | $(\widetilde{H}^+_u \;\; \widetilde{H}^0_u)$ | $\mathbf{1}$ | $\mathbf{2}$ | $+\frac{1}{2}$ |
| | $\widehat{H}_d$ | $(H^0_d \;\; H^-_d)$ | $(\widetilde{H}^0_d \;\; \widetilde{H}^-_d)$ | $\mathbf{1}$ | $\mathbf{2}$ | $-\frac{1}{2}$ |

Table 2.1: Chiral supermultiplets in the MSSM

in contrast to the SM which has only one? A short look at the MSSM superpotential will clarify. This is a general superpotential that is invariant under the Lorentz group, the local gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ and invariant under R-parity (see section 2.5):

$$W_{MSSM} = h^e_{ij}\widehat{L}_i\widehat{H}_d\widehat{E}_j + h^d_{ij}\widehat{Q}_i\widehat{H}_d\widehat{D}_j + h^u_{ij}\widehat{Q}_i\widehat{H}_u\widehat{U}_j + \mu\widehat{H}_u\widehat{H}_d \,. \tag{2.70}$$

The indices $i$, $j$ are family indices, color and weak isospin indices are suppressed, $\mu$ is the Higgs mass parameter and all $h$ are Yukawa coupling constants bestowing masses to the particles once the electroweak gauge symmetry is broken. The answer is that a second Higgs doublet is needed to create *all* the masses. The superpotential is a holomorphic function of the fields and hence the term $h^u_{ij}\widehat{Q}_i\widehat{H}_u\widehat{U}_j$ cannot be substituted by $h^u_{ij}\widehat{Q}_i\widehat{H}^*_d\widehat{U}_j$. The solution is a second Higgs doublet $\widehat{H}_u$ with an opposite $U_Y$ charge relative to $H_d$. A second reason is the avoidance of a gauge anomaly by this second Higgs doublet.

The vector supermultiplets, which are also called gauge supermultiplets because they mediate the gauge interactions, are listed in table 2.2.

| Name | Superfield | spin 1/2 | spin 1 | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ |
|---|---|---|---|---|---|---|
| gluino, gluon | $\widehat{g}$ | $\widetilde{g}$ | $g$ | **8** | **1** | 0 |
| winos, W bosons | $\widehat{W}$ | $\widetilde{W}^\pm$ $\widetilde{W}^0$ | $W^\pm$ $W0$ | **1** | **3** | 0 |
| bino, B boson | $\widehat{B}$ | $\widetilde{B}$ | $B$ | **1** | **1** | 0 |

Table 2.2: Gauge supermultiplets in the MSSM

As far as the particle contents has been discussed, supersymmetry breaking was not considered. There are several possible mechanisms to break supersymmetry spontaneously, among them (see, e.g., Chap. 12.5 of [9]):

1. Gravity mediated breaking
2. Gauge mediated F-type breaking [10]
3. Gauge mediated D-type breaking [11]
4. Anomaly-mediated breaking

Though these breaking mechanisms break supersymmetry spontaneously, it is difficult to obtain the right SM particle mass spectrum. Because of this difficulty, and the ambiguity in the model, it is a more phenomenological approach to write down the most general soft supersymmetry breaking superpotential $W_{\text{soft}}$. It breaks supersymmetry explicitly and introduces a lot of new parameters, but renders, for all cases in which supersymmetry is broken, the right phenomenology. To be "soft" means that it fulfills conditions of renormalization, more precisely it maintains still a solution of the hierarchy problem. Once supersymmetry breaking is introduced, mass mixings become possible between particles with same quantum numbers (cf. appendix D of this report and appendix C of [4]):

- Sleptons of each flavor:

$$\widetilde{l}_1 = \cos\theta\,\widetilde{l}_L + \sin\theta\,\widetilde{l}_R\,, \quad \widetilde{l}_2 = -\sin\theta\,\widetilde{l}_L + \cos\theta\,\widetilde{l}_R\,. \tag{2.71}$$

  The mass eigenstates are on the right-hand, flavor eigenstates on the left-hand side. Usually, the mass eigenstate 1 is chosen to be the lightest. The angle $\theta$ is the mixing angle, different for each flavor.

- Squarks of each flavor, same as sleptons.

- Charged winos and higgsinos → charginos:

$$\chi_i^+ = V_{ij}\Psi_j^+ \ , \quad \chi_i^- = U_{ij}\Psi_j^- \quad i,j = 1,2 \ . \tag{2.72}$$

- Bino, neutral wino and neutral higgsinos → neutralinos:

$$\chi_i^0 = N_{ij}\Psi_j^0 \quad i,j = 1,..,4 \ . \tag{2.73}$$

  Neutralinos are Majorana fermions.

## 2.5 R-Parity

From one point of view, there is a problem within the MSSM. There is a priori no law that forbids lepton number ($L$) and baryon number ($B$) violating terms in the Lagrangian, or more precisely, in the MSSM superpotential[6]. The most general one invariant under transformations of the Lorentz group and the local gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ is the sum of $W_{MSSM}$ given in Eq. (2.70) and the following R-parity violating superpotential [12]

$$W_{\not{R}_p} = \mu_i \widehat{H}_u \widehat{L}_i + \frac{1}{2}\lambda_{ijk}\widehat{L}_i\widehat{L}_j\widehat{E}_k + \lambda'_{ijk}\widehat{L}_i\widehat{Q}_j\widehat{D}_k + \frac{1}{2}\lambda''_{ijk}\widehat{U}_i\widehat{D}_j\widehat{D}_k \ , \tag{2.74}$$

where $i,j,k$ are family indices and the Yukawa couplings $\lambda_{ijk}$ ( $\lambda''_{ijk}$) are antisymmetric in the first (last) two indices because of gauge invariance. However, the operators in Eq. (2.74) induce a too rapid proton decay in contradiction with the experimental observation. Therefore, a discrete symmetry, R-parity [13, 12], is imposed by hand in order to forbid the problematic terms in (2.74). R-parity is defined as

$$R = (-1)^{3B+L+2S} \ , \tag{2.75}$$

where $S$ is the spin of the particle. With this definition one has $R = +1$ for all SM particles and $R = -1$ for their supersymmetric partners. Furthermore, it is a multiplicative quantity allowing supersymmetric particle production only in pairs.

Yet this symmetry is more restrictive than necessary. In order to forbid a too rapid proton decay, it is sufficient to impose either baryon number conservation, allowing the R-parity violating terms $\widehat{L}_i\widehat{L}_j\widehat{E}_k$ and $\widehat{L}_i\widehat{Q}_j\widehat{D}_k$, or lepton number conservation if the proton is lighter than the LSP, allowing the term $\widehat{U}_i\widehat{D}_j\widehat{D}_k$ [13]. In particular, the combination of couplings $\lambda'_{imk}\lambda''_{11k}$ (i = 1, 2, 3; m = 1, 2) would lead to proton decay via tree-level down squark exchange at an unacceptable rate, unless $|\lambda'_{imk}\lambda''_{11k}|$ is smaller than about $10^{-26}$ for a typical squark mass in the 300 $GeV$ range. Further limits are given in tables 2.3 and 2.4. They depend typically on sfermion masses.

[6]Note that in the SM lepton and baryon number violating terms are automatically avoided by demanding local gauge invariance and renormalizability.

| coupling | $\lambda_{ijk}$ | $\lambda'_{1jk}$ | $\lambda'_{2jk}$ | $\lambda'_{3jk}$ |
|---|---|---|---|---|
| limit | 0.07 | 0.28 | 0.56 | 0.52 |

Table 2.3: Limits for R-parity violating Yukawa couplings $(i, j, k = 1, 2, 3)$ using a mSUGRA model [14].

| coupling | $\lambda'_{11k}$ | $\lambda'_{12k}$ | $\lambda'_{2j1}$ | $\lambda'_{21k}$ |
|---|---|---|---|---|
| limit | $0.02\left(\frac{m_{\tilde{d}_{kR}}}{100\ GeV}\right)$ | $0.44\left(\frac{m_{\tilde{d}_{kR}}}{100\ GeV}\right)$ | $0.18\left(\frac{m_{\tilde{d}_{jL}}}{100\ GeV}\right)$ | $0.06\left(\frac{m_{\tilde{d}_{kR}}}{100\ GeV}\right)$ |
| coupling | $\lambda'_{22k}$ | $\lambda'_{31k}$ | $\lambda'_{32k}$ | |
| limit | $0.21\left(\frac{m_{\tilde{d}_{kR}}}{100\ GeV}\right)$ | $0.12\left(\frac{m_{\tilde{d}_{kR}}}{100\ GeV}\right)$ | $0.52\left(\frac{m_{\tilde{d}_{kR}}}{100\ GeV}\right)$ | |

Table 2.4: Limits for R-parity violating Yukawa couplings $(j, k = 1, 2, 3)$ given in [12], p. 156.

One may ask for the origin of R-parity breaking terms in the Lagrangian. There are two possibilities: explicit R-parity breaking terms in the superpotential or R-parity breaking terms generated by supersymmetry breaking. Considering the general superpotential in (2.74) is valid for both cases.

The phenomenology of an R-parity violating supersymmetric theory is very different from the conventional R-parity conserving MSSM and has some interesting features [12]:

- The LSP is no more stable and can decay into SM particles, but if the couplings are weak enough, it remains a good dark matter candidate.
- The $L$-violating couplings generate automatically neutrino masses and mixings and can explain neutrino-flavor transition in matter.
- $B$-violation is one of the three necessary Sakharov conditions [15] for baryogenesis and the $B$-violating couplings can be used to explain the observed baryon asymmetry of the universe within models of baryogenesis. [7]

## 2.6 Lagrangian and Feynman Rules

In this section, we briefly summarize explicit terms in the Lagrangian and Feynman rules that are used in this report.

### Lagrangian

We consider only the R-parity violating term

$$W = \lambda'_{ijk}\widehat{L}_i\widehat{Q}_j\widehat{D}_k \ , \tag{2.76}$$

[7] Though all criteria are already fulfilled in the SM, the generated baryon asymmetry is much too small [16].

which is part of the superpotential in Eq. (2.74). By use of (2.43) the Yukawa-interactions originated by a superpotential is

$$\mathcal{L}_{Yukawa} = -\frac{1}{2}\left[\frac{\partial^2 W}{\partial A^2}\Psi\Psi + \left(\frac{\partial^2 W}{\partial A^2}\right)^\dagger \bar{\Psi}\bar{\Psi}\right] . \tag{2.77}$$

Expressed in 4-component Dirac spinors, this leads to

$$\begin{aligned}\mathcal{L}_{Yuk,LQD} = -\lambda'_{ijk}\big[&\widetilde{\nu}_i \bar{d}_k P_L d_j - \widetilde{e}_{L,i}\bar{d}_k P_L u_j \\ &\widetilde{d}_{L,j}\bar{d}_k P_L \nu_i - \widetilde{u}_{L,j}\bar{d}_k P_L e_i \\ &\widetilde{d}^*_{R,k}\bar{\nu}^c_i P_L d_j - \widetilde{d}^*_{R,k}\bar{e}^c_i P_L u_j\big] + h.c. ,\end{aligned} \tag{2.78}$$

with the projectors $P_L = \frac{1-\gamma_5}{2}$ and $P_R = \frac{1+\gamma_5}{2}$.

### Feynman Rules

The resulting Feynman rules corresponding to the first two terms of this Lagrangian can now be easily derived and are given in Fig. 2.1. The other vertices contained in (2.78)

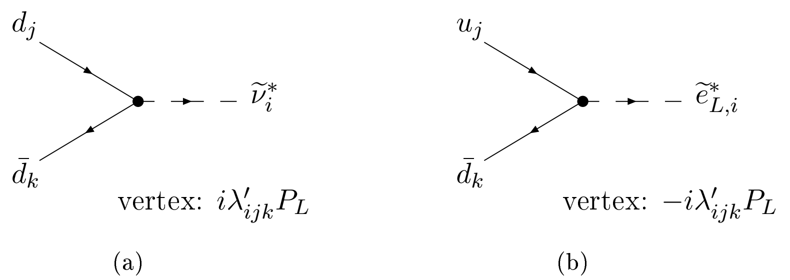



Figure 2.1: Feynman rules for quark-quark-slepton vertices.

are analogous. Another Feynman rule used later for decays is a slepton-neutralino-lepton coupling conserving R-parity. The rule written in Fig. 2.2 is taken from [17].

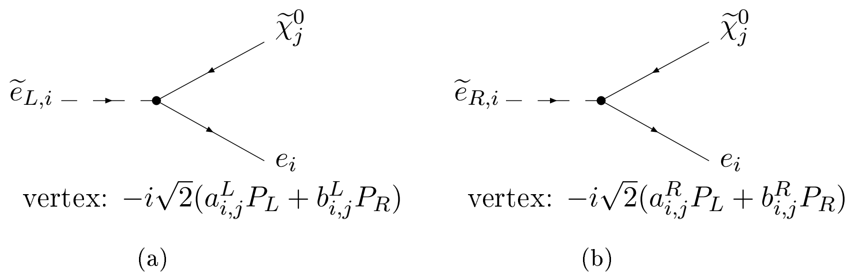



Figure 2.2: Feynman rules for neutralino-electron-selectron vertices.

The parameters $a$ and $b$ depend on mixings (see appendix D) and are given by

$$a^L_{i,j} = \frac{gm_{e_i}}{2m_W \cos\beta} N^*_{j3} \; , \qquad b^L_{i,j} = -eN'_{j1} - \frac{g}{c_W}\left(\frac{1}{2} - s^2_W\right) N'_{j2} \; , \tag{2.79}$$

$$b^R_{i,j} = \frac{gm_{e_i}}{2m_W \cos\beta} N_{j3} \; , \qquad a^R_{i,j} = eN'^*_{j1} - \frac{gs^2_W}{c_W} N'^*_{j2} \; , \tag{2.80}$$

where $c_W$ and $s_W$ are cosine and sine of the Weinberg angle $\theta_W$ and $\tan\beta = \frac{\langle H_u \rangle}{\langle H_d \rangle}$ is the ratio of the vacuum expectation value of the two Higgs fields. $N$ is the $4 \times 4$ neutralino mixing matrix of Eq. (2.4) and $N'$ are given by

$$\begin{aligned} N'_{j1} &= c_W \; N_{j1} + s_W \; N_{j2} \; , \qquad & N'_{j3} &= N_{j3} \; , \\ N'_{j2} &= -s_W \; N_{j1} + c_W \; N_{j2} \; , \qquad & N'_{j4} &= N_{j4} \; . \end{aligned} \tag{2.81}$$

Computations of the Feynman rules of the MSSM can be found in [4, 18] and for R-parity violating rules we refer to [12].

# 3 Single Slepton Production

In this chapter we are going to analyze resonant single charged slepton production at polarized and unpolarized hadron colliders [19, 20, 21]:

$$\text{hadron}_1 + \text{hadron}_2 \to \tilde{e}_i \to \text{decay products} .$$

If not otherwise stated, the family index $i$ is suppressed. There are two points to be outlined. First it is an R-parity violating process, wherefore we investigate physics beyond the MSSM. Secondly the produced slepton is in principle a virtual particle, but if the energy of the incoming particles equals the slepton mass, it is produced on-shell, as a quasi real particle, that lives and decays afterwards. Technically it is the narrow width approximation which can be employed when the width of the slepton resonance $\Gamma$ is much smaller than the mass (cf. appendix B).

## 3.1 Analytical Results

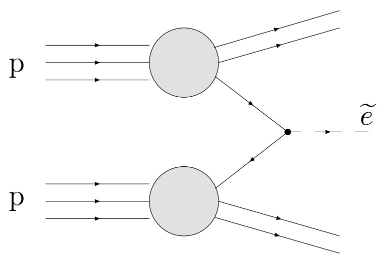



Figure 3.1: Drell-Yan process.

The cross section for resonant single slepton production in hadron–hadron collisions (see Fig. 3.1) can be computed in the QCD-improved parton model according to the following formula:

$$\sigma = \sum_{j,k} \int_0^1 \mathrm{d}x_A \mathrm{d}x_B \; f_j(x_A, Q) \; f_k(x_B, Q) \hat{\sigma}_{jk \to \tilde{e}+X} \; . \tag{3.1}$$

Here $x_A$ and $x_B$ are the fraction of the hadron momenta carried by the corresponding partons, i.e.

$$p_{A,B} = x_{A,B} P_{A,B} \; , \tag{3.2}$$

where $P_{A,B}$ ($p_{A,B}$) denote the hadron (parton) momenta. Furthermore, $f_i(x, Q)$ is the parton distribution function (PDF) of the parton $i$, which depends on the factorization scale $Q$, which is typically chosen to be close to the hard scale of the process. In this case, the hard scale is set by the slepton mass such that $Q^2 \simeq \tilde{m}^2 = \hat{s}$. Finally, $\hat{\sigma}_{jk \to \tilde{e}+X}$ is the cross section for the partonic subprocess $j + k \to \tilde{e} + X$, where $X$ denotes an arbitrary final state and an incoherent sum over all possible subprocesses has to be performed. In leading order (LO) of quantum chromo dynamics (QCD) only the subprocess $u_j + \bar{d}_k \to \tilde{e}^*_{L,i}$ contributes, which will be computed in the next section.

Similarly, the cross section for polarized hadronic collisions reads

$$\Delta\sigma = \sum_{j,k} \int_0^1 \mathrm{d}x_A \mathrm{d}x_B \; \Delta f_j(x_A, Q) \; \Delta f_k(x_B, Q) \Delta\hat{\sigma}_{jk\to\tilde{e}+X} \; , \tag{3.3}$$

where $\Delta f_i(x, Q)$ are polarized PDFs and $\Delta\hat{\sigma}_{jk\to\tilde{e}+X}$ is the polarized cross section for the process $j + k \to \tilde{e} + X$.

Finally, the cross sections in Eqs. (3.1) and (3.3) are needed to compute the double-polarized asymmetry

$$A_{LL} = \frac{\Delta\sigma}{\sigma} \; . \tag{3.4}$$

The polarized and unpolarized hadronic and partonic cross sections are defined as usual. Nevertheless, we list all relevant definitions including all possible asymmetries in App. C starting from experimental event rates/cross sections and derive in addition the relations between hadron level and parton level cross sections and asymmetries.

### 3.1.1 Parton Level Results

The next step is to compute the partonic cross section of the reaction (see Fig. 3.2)

$$u_j + \bar{d}_k \to \tilde{e}^*_{L,i} \; . \tag{3.5}$$

With the Feynman rules of Fig. 2.1 the transition matrix element $\mathcal{M}$ at leading order (LO), its square, the phase space element $\mathrm{d}PS_{2\to1}$, and the flux $F$ are

$$i\mathcal{M} = \bar{v}(p_B, h_B) \left(-i\lambda'_{ijk} P_L\right) u(p_A, h_A) \; , \qquad \mathrm{d}PS_{2\to1} = \frac{2\pi}{\hat{s}}\delta(1-\tau) \; ,$$

$$|\mathcal{M}|^2 = \frac{|\lambda'_{ijk}|^2 \hat{s}}{12}(1-2h_A)(1-2h_B) \; , \qquad F = 2\hat{s} \; ,$$

where $h_{A,B} = \pm\frac{1}{2}$ are the helicities, $\hat{s}$ is the partonic center of mass energy squared, $\tau = \frac{\widetilde{m}^2}{\hat{s}}$ and $\widetilde{m}$ the mass of the slepton. We have used the approximation of massless quarks, so

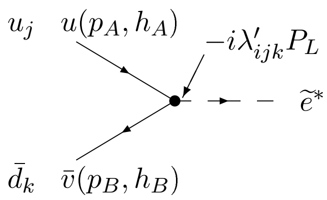



Figure 3.2: Feynman diagram for single slepton production at leading oder of QCD.

that the helicity projection becomes $u(p,\lambda)\bar{u}(p,\lambda) = \frac{1}{2}(1+2\lambda\gamma_5)\not{p}$ and $v(p,\lambda)\bar{v}(p,\lambda) = \frac{1}{2}(1-2\lambda\gamma_5)\not{p}$ [22]. Putting all this together yields the partonic cross section

$$\mathrm{d}\hat{\sigma}(\hat{s},h_A,h_B) = \frac{\pi|\lambda'_{ijk}|^2}{12\hat{s}}\delta(1-\tau)(1-2h_A)(1-2h_B). \tag{3.6}$$

Before turning to the cross section at the hadronic level let us briefly discuss the quite particular structure of the partonic cross section in Eq. (3.6). As can be seen, $\mathrm{d}\hat{\sigma}$ vanishes when $h_A = \frac{1}{2}$ or $h_B = \frac{1}{2}$ such that the only contributing cross section is (cf. Eq. (C.5)–(C.8))

$$\hat{\sigma}(\hat{s},h_A = -\frac{1}{2},h_B = -\frac{1}{2}) =: \hat{\sigma}^{--} \ . \tag{3.7}$$

The explanation for this result is that the produced scalar particle carries no polarization (implying $\hat{\sigma}^{+-} = \hat{\sigma}^{-+} = 0$ for massless quarks) and the vertex is only proportional to $P_L$ ($\hat{\sigma}^{++} = 0$). This implies that the unpolarized partonic cross section $\hat{\sigma}_0$ and the polarized cross section $\Delta\hat{\sigma}$ (cf. appendix C) are given by

$$\hat{\sigma}_0 = \Delta\hat{\sigma} = \frac{1}{4}\hat{\sigma}^{--} = \frac{\pi|\lambda'_{ijk}|^2}{12\hat{s}}\delta(1-\tau) \ . \tag{3.8}$$

### 3.1.2 Hadron Level

Inserting the result in (3.8) into (3.1) and (3.3) we arrive at the following final expressions for the unpolarized respectively polarized hadronic cross sections:

$$\sigma_0 = \sum_{j,k}\int_{\frac{\widetilde{m}^2}{s}}^{1} \mathrm{d}x\ f_j(x)\ f_k\left(\frac{\widetilde{m}^2}{sx}\right)\ \frac{\pi|\lambda'_{ijk}|^2}{12xs}\ , \tag{3.9}$$

$$\Delta\sigma = \sum_{j,k}\int_{\frac{\widetilde{m}^2}{s}}^{1} \mathrm{d}x\ \Delta f_j(x)\ \Delta f_k\left(\frac{\widetilde{m}^2}{sx}\right)\ \frac{\pi|\lambda'_{ijk}|^2}{12xs}\ . \tag{3.10}$$

A nice feature of the cross section is its relation to the parton luminosity function

$$\frac{\mathrm{d}L_{jk}}{\mathrm{d}\hat{s}} = \int \mathrm{d}x_A\mathrm{d}x_B\ f_j(x_A)\ f_k(x_B)\ \delta(x_Ax_Bs-\hat{s})\ , \tag{3.11}$$

where a similar expression holds for the polarized luminosity.

$$\sigma \approx \sum_{jk}\hat{\sigma}_{jk}(x = \frac{\hat{s}}{s})\frac{\mathrm{d}L_{jk}}{\mathrm{d}\hat{s}}\ . \tag{3.12}$$

In our application, it is easy to see that Eq. (3.9) holds exactly if we use $\hat{\sigma}'_{jk}$

$$\sigma_0 = \sum_{jk}\hat{\sigma}'_{jk}\frac{\mathrm{d}L_{jk}}{\mathrm{d}\hat{s}}\Big|_{\hat{s}=\widetilde{m}^2} = \frac{\pi|\lambda'_{ijk}|^2}{12}\frac{\mathrm{d}L_{jk}}{\mathrm{d}\widetilde{m}^2}\ , \tag{3.13}$$

such that the hadronic cross section is nothing other than the partonic luminosity function times a constant. Analogous expressions hold for the polarized case.
Instead of positively charged slepton $\widetilde{e}_L^*$ production we could consider producing a negatively charged slepton $\widetilde{e_L}$. On the other hand, there is no $\widetilde{e}_R$ production possible through the superpotential (2.74). The negative charged slepton production corresponds to the conjugate process $\bar{u}_j + d_k \rightarrow \widetilde{e}_L$. The only difference to the former case is that "left is changed to right," and thus $\hat{\sigma}^{++}$ is the only non-vanishing partonic cross section. An essential difference occurs, however, at the hadronic level because the parton distribution functions change. Yet, according to equation (3.13), these differences are restricted to the parton luminosity function[1].

### 3.1.3 Slepton Decays

There are the following decay channels for charged sleptons:

$$u_j \bar{d}_k \rightarrow \widetilde{e}_{L,i}^* \rightarrow \begin{cases} u_m \bar{d}_n, & (a) \\ \bar{e}_i \tilde{\chi}_m^0, & (b) \\ \bar{\nu}_i \tilde{\chi}_n^+, & (c) \end{cases} . \tag{3.14}$$

The channel (a) is highly suppressed because it is proportional to $|\lambda'|^2$. In particular the corresponding $2 \rightarrow 2$ process is even proportional to $|\lambda'|^4$. If the lightest neutralino is the LSP, then channel (b) becomes the preferred one, which could be detected by its signal of an outgoing charged lepton and missing energy. Using the corresponding Feynman rule of Fig. 2.1(c) its decay width can be calculated as

$$\begin{aligned} \Gamma_{\widetilde{e}_i^* \rightarrow \bar{e}_i \tilde{\chi}_j^0} = & \frac{g^2}{64\pi \widetilde{m}^3} \sqrt{\widetilde{m}^4 + m_l^4 + m_{\tilde{\chi}^0}^4 - 2\widetilde{m}^2(m_l^2 + m_{\tilde{\chi}^0}^2) - 2m_l^2 m_{\tilde{\chi}^0}^2} \\ & \left[ (\widetilde{m}^2 - m_l^2 - m_{\tilde{\chi}^0}^2)(|a_j|^2 + |b_j|^2) - 2m_l m_{\tilde{\chi}^0}(a_j b_j^* + b_j a_j^*) \right] . \end{aligned} \tag{3.15}$$

This formula is valid if interaction states are equal to mass states.

### 3.1.4 Cross Section $2 \rightarrow 2$

Finally we give the partonic cross section formula for the process $u_j + \bar{d}_k \rightarrow \widetilde{e}_i^* \rightarrow \bar{e}_i + \tilde{\chi}^0$ without selectron mixing:

$$\begin{aligned} \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\Omega_{CMS}} = & \frac{|\lambda'_{ijk}|^2}{1536\pi^2 \hat{s}} \frac{(1-2h_A)(1-2h_B)}{(\hat{s} - \widetilde{m}^2)^2 + (\widetilde{m}\Gamma)^2} \\ & \left[ \frac{1}{2}(\hat{s} - m_l^2 - m_{\tilde{\chi}^0}^2)(|a|^2 + |b|^2) - 2m_l m_{\tilde{\chi}^0}(ab^* + ba^*) \right] . \end{aligned} \tag{3.16}$$

It is interesting to mention that $\hat{\sigma}_{CMS,tot} = 4\pi \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\Omega_{CMS}}$ and thus there is no direction dependence in the center of mass frame.

[1] For $p\bar{p}$–colliders (Tevatron) the parton luminosity functions are identical in both cases. However, for $pp$-colliders (LHC) they are different.

## 3.2 Numerical Results

In this section we present numerical results for polarized and unpolarized single slepton production discussed in the previous section. Following Refs. [23, 24], we consider three (polarized) collider options:

**RHIC:** At present, the Relativistic Heavy Ion Collider (RHIC) is the only polarized hadron collider. It has a center of mass energy range of $\sqrt{s} = 200 \ldots 500\ GeV$ and a polarization of $P = 70\%$ for both beams. In our numerical studies, we take the upper limit $500\,GeV$.

**Tevatron:** The Fermilab Tevatron is an unpolarized proton–antiproton collider operating at $\sqrt{s} = 1.96\ TeV$. We assume a hypothetical polarization of $P = 70\%$ for both beams.

**LHC:** The Large Hadron Collider (LHC) will enter service this year (2008) providing unpolarized proton–proton collisions at $\sqrt{s} = 14\ TeV$. Again, we assume a polarization option of $P = 70\%$ for both beams.

### 3.2.1 Unpolarized Production

We begin to calculate the unpolarized cross section $\sigma_0$ of equation (3.9) also needed to build later the asymmetry $A_{LL}$ as in (3.4). For the following numerical results the coupling strength was always assumed to be $\lambda'_{i12} = \lambda'_{i21} = 0.01$ neglecting all other $\lambda'_{ijk}$ couplings. We adopted the CTEQ6L1 [25] PDFs at LO evaluated at the scale factor $Q = \widetilde{m}$ and the Vegas algorithm of the CUBA library [26] for numerical integration. A possible squark mixing would only appear as a global factor that was set to $\cos^2\theta = 1$. The result, seen in Fig. 3.3 is in complete agreement with the LO curves in Fig. 2 of Ref. [19].

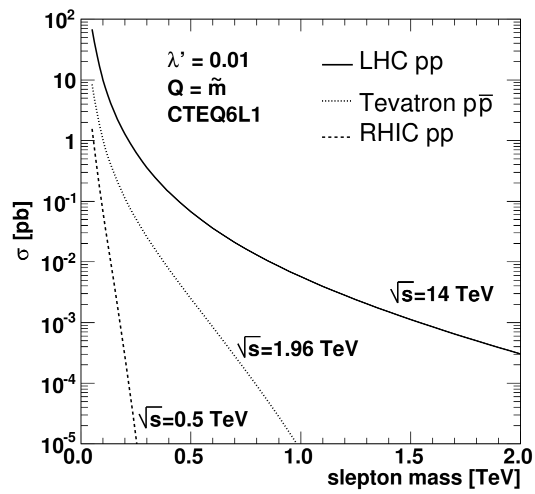


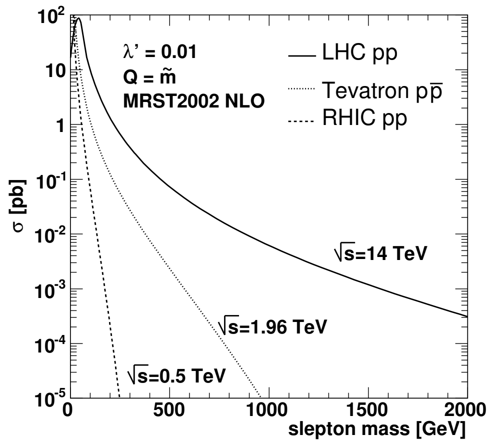

Figure 3.3: Hadronic cross sections for single slepton production via $u+\bar{d} \to \widetilde{e}^*$. Shown are results for LHC, Tevatron, and RHIC energies utilizing a scale factor $Q = \widetilde{m}$, Yukawa couplings $\lambda'_{i12} = \lambda'_{i21} = 0.01$, and CTEQ6L1 [25] LO PDFs.

### 3.2.2 Polarized Production

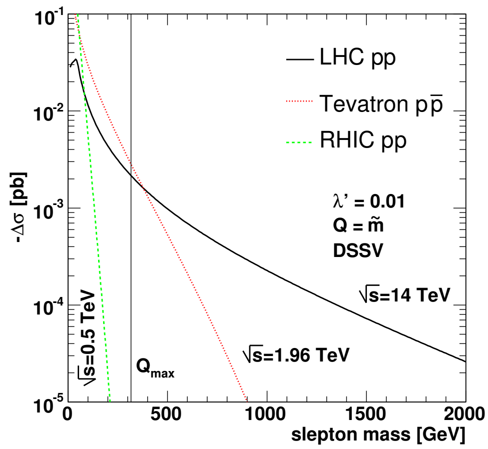
The next step is to calculate $\Delta\sigma$ according to (3.10). We use the very recent polarized DSSV next-to-leading order (NLO) PDFs [27] resulting in Fig. 3.4 together with its corresponding unpolarized MRST2002 NLO PDFs [28][2]. Strictly speaking, it would be necessary to combine our LO partonic cross sections with LO PDFs, whereas the use of scheme dependent NLO PDFs together with LO partonic cross sections produces a scheme dependent and in principle unphysical result. Unfortunately, all recent polarized PDFs are available in NLO only and we have to chose between consistent but oldish LO PDFs and recent NLO PDFs comprising all the relevant experimental information. Since we plan to include the NLO hard scattering cross section in the future, we prefer to use here the PDFs at NLO in the modified minimal subtraction scheme ($\overline{\mathrm{MS}}$), i.e., we assume for the time being (schematically)

$$PDF_{NLO}^{\overline{\mathrm{MS}}} \otimes \hat{\sigma}_{NLO}^{\overline{\mathrm{MS}}} = PDF_{NLO}^{\overline{\mathrm{MS}}} \otimes \left(\hat{\sigma}_{LO}^{(0)} + \frac{\alpha_S}{2\pi}\hat{\sigma}^{(1),\overline{\mathrm{MS}}}\right) \overset{!}{\simeq} PDF_{NLO}^{\overline{\mathrm{MS}}} \otimes \hat{\sigma}_{LO}^{(0)} \,.$$

However, for comparison, we compute the asymmetry also with the LO GRV98/GRSV2000 PDFs.




(a)



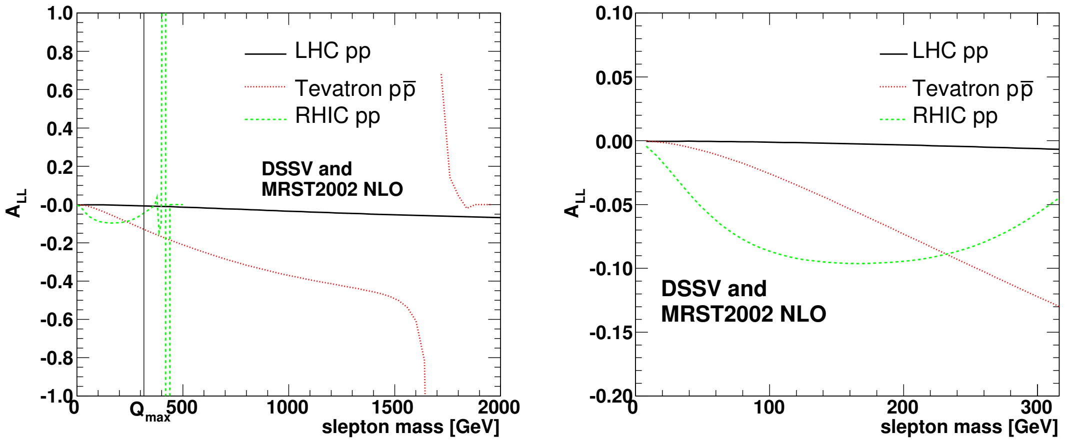

(b)

Figure 3.4: (a) Same as in Fig. 3.3 using MRST2002 NLO PDFs [28]. (b) Corresponding polarized cross sections obtained with the DSSV PDFs [27]. The vertical line indicates the maximal value of $Q$ given by the DSSV PDFs. For $\tilde{m} > Q_{max}$ the scale $Q$ has been frozen at $Q_{max}$.

[2]Note that global fits of polarized PDFs require unpolarized PDFs in order to compute the experimental asymmetries. The DSSV PDFs have been determined using the unpolarized MRST2002 PDFs, such that these two sets go together for consistency.

### 3.2.3 Asymmetry

Building the asymmetry $A_{LL}$ of Eq. (3.4) results in Fig. 3.5. For comparison, we also show the results of the same calculation using the LO PDFs GRV98lo [29] and GRSV2000 [30]. The maximal value of the factorization scale is here $Q_{max} = 1000\ GeV$. As one

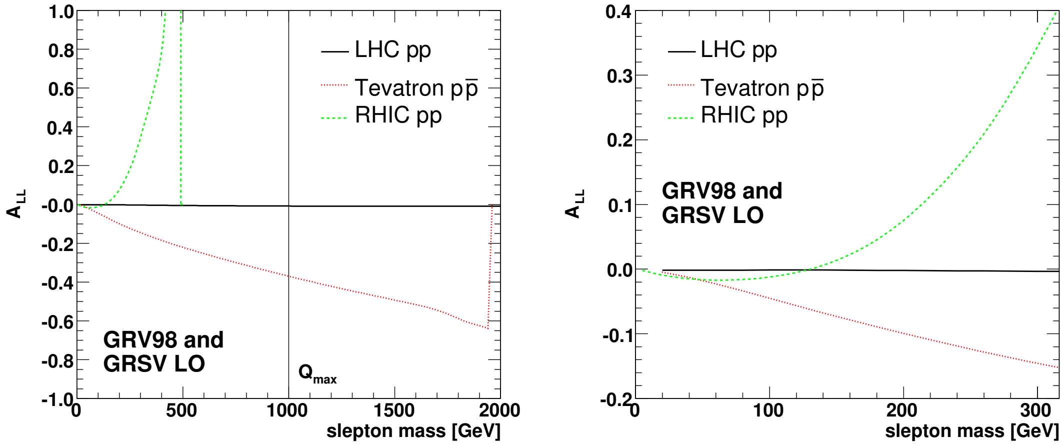



(a) (b)

Figure 3.5: (a) Double spin asymmetry $A_{LL}$ for resonant single slepton production at LHC, Tevatron and RHIC energies. The factorization scale $Q$ has been identified with the slepton mass $\widetilde{m}$ and has been frozen for $\widetilde{m} > Q_{max}$ $Q = Q_{max}$ at $Q_{max} = 316\ GeV$. We applied the NLO PDFs DSSV [27] and MRST2002 [28]. (b) Zoom into the region of $\widetilde{m} = Q < Q_{max}$.

can see, for LHC the asymmetry is negligible for the slepton mass range shown in Fig. 3.5. For Tevatron energies, $A_{LL}$ is negative ranging between 0 and $\sim -50$ % at $\widetilde{m} \sim 1500\ GeV$. We also note that in this mass range from 0 to 1500 $GeV$ the Tevatron results obtained with DSSV/MRST and with GRSV/GRV are very similar. On the other hand, for even larger slepton masses, reaching the kinematical limit, the numerical results for the asymmetries are unstable (see also discussion of Fig. 3.7). Finally, the asymmetry at RHIC is also negative reaching up to $-10$ %. Fig. 3.6(b) zooms into this region 0 to 316 $GeV$ relevant for RHIC. Different from the Tevatron case, the RHIC asymmetry obtained by GRSV/GRV PDFs, which is slightly negative until 120 $GeV$ and then increases up to $+40$ % at $\widetilde{m} \sim 316\ GeV$, differs strongly from the results obtained with DSSV/MRST. The RHIC results start to be unstable for $\widetilde{m} \gtrsim 350\ GeV$ approaching kinematical limit $\hat{s} = 500\ GeV$. In order to explain this behavior a little better, we plot in Fig. 3.7 the unpolarized and polarized PDFs versus the scale $Q$ at $x = \frac{Q}{\sqrt{s}}$. This way, we sample the typical PDF values entering the cross sections/parton luminosity functions[3]. The curves are shown for RHIC energies and are similar for the Tevatron case. As expected, for

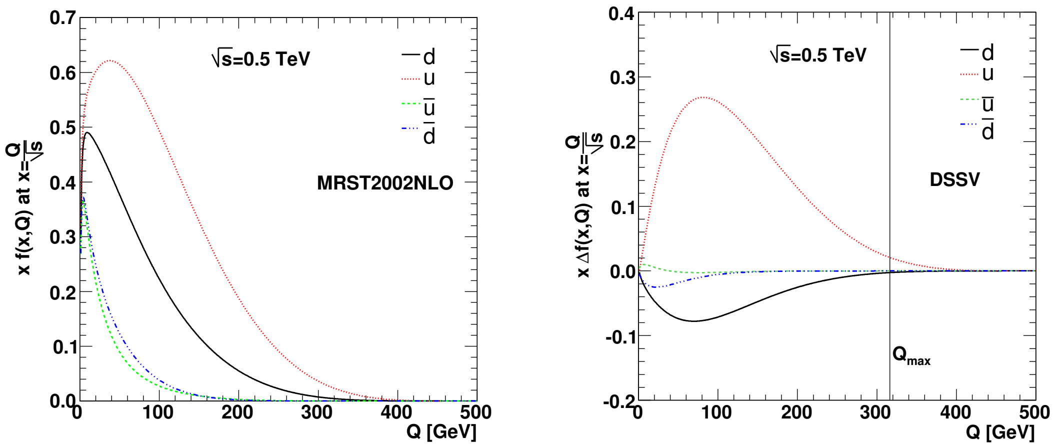
[3] In a Drell-Yan process, one has the relation $x_{1,2} = \frac{Q}{\sqrt{s}} e^{\pm y}$ with pseudo-rapidity $y$. Setting $y = 0$, one obtains typical $x$-values, at which the PDFs are probed.




(a) (b)

Figure 3.6: Same as in Fig. 3.5 employing the GRV98lo [29] and GRSV2000 [30] LO PDFs.

$Q > 350\ GeV$ the PDFs are tiny explaining possible numerical instabilities.




(a) (b)

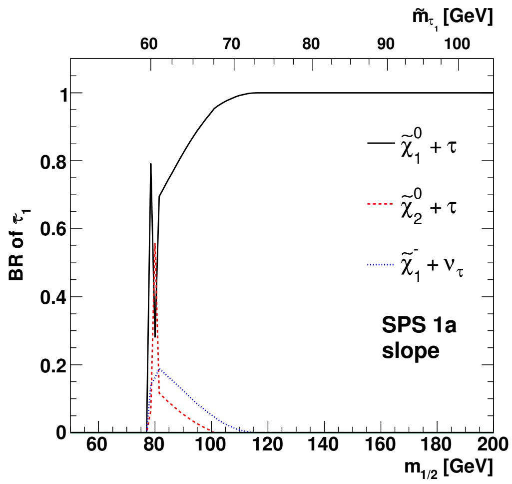
Figure 3.7: PDFs evaluated at $x = \frac{Q}{\sqrt{s}}$ vs. $Q$ at CMS energy of $s = 500\ GeV$. (a) Unpolarized and (b) polarized case.

### 3.2.4 Decay

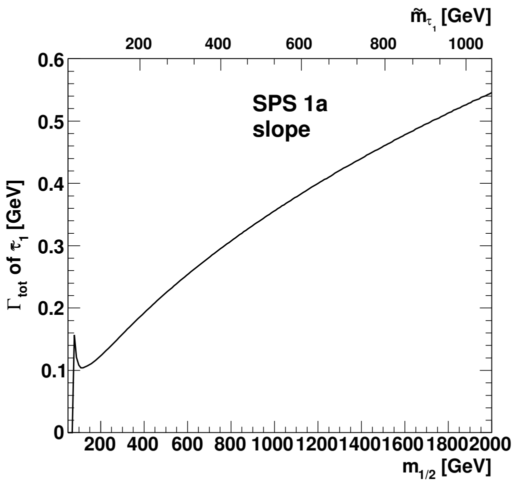
Now we are going to examine the decay channels of charged sleptons discussed in section 3.1.3. In order to evaluate the total decay width and branching rations $BR = \frac{\Gamma_{ij}}{\Gamma_{\text{tot}}}$ of the different channels of Eq. (3.14), masses, mixing parameters, and coupling constants are needed, see Eq. (3.15). For this purpose, we apply the SUSY spectrum generator SUSPECT2 [31] using a minimal supergravity (mSUGRA) SUSY breaking scenario with 5 parameters: $m_0$, $m_{1/2}$, $A_0$, $\tan\beta$ and $\mu$. In order to obtain results depending on the slepton mass as before, we use the benchmark points SPS 1a and SPS3 [32] and their slopes:

- SPS 1a: $\tan\beta = 10$, $\mu = +1$, $m_0 = -A_0 = 0.4\, m_{1/2}$ and $m_{1/2}$ varies.
- SPS 3: $\tan\beta = 10$, $\mu = +1$, $A_0 = 0$, $m_0 = 0.25\, m_{1/2} - 10\, GeV$ and $m_{1/2}$ varies.

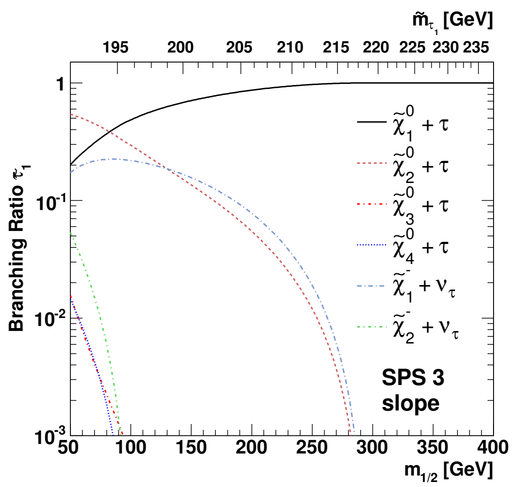
SUPSECT2 assumes no mixing between selectrons and smuons, whereas staus can mix and $\widetilde{\tau}_1$ denotes the lighter mass eigenstate (cf. Eq. (2.71)). Furthermore, we used SDECAY [33] to calculate branching ratios and total decay widths resulting in Fig. 3.8 for $\widetilde{\tau}_1$–decays and in Fig. 3.9 for $\widetilde{e}_L$–decays. First, we note that in every case, the decay into the lightest neutralino becomes the only one for high slepton masses. In Fig. 3.8(a), showing the SPS1 a slope, $\widetilde{\tau}_1$ decays only in the first two neutralinos and the lightest chargino, where the decay in the lightest neutralino is very dominant. The anomaly at $m_{1/2} \approx 80\, GeV$ are thought to be unphysical and caused by numerical uncertainties. The total decay width in Fig. 3.8(b) increases approximately linearly between 0.1 $GeV$ at $\tilde{m}_{\tau_1} \approx 70\, GeV$ and 0.5 $GeV$ at $\tilde{m}_{\tau_1} \approx 1000\, GeV$. The SPS3 slope of $\widetilde{\tau}_1$ gives quite a different picture, where decays in all the gauginos are possible, see Fig. 3.8(c). Particularly $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^-$ give non-negligible contributions. Contrary to the SPS1 a slope, the decay width in 3.8(d) decreases very fast from a value of $\sim 0.8\, GeV$ down to $\sim 0.01\, GeV$ and then continues decreasing slowlier until it becomes zero at $\tilde{m}_{\tau_1} \approx 500\, GeV$. The possible decay channels for $\widetilde{e}_L$, shown in Fig. 3.9(a) and Fig. 3.9(c), are exactly the same as for $\widetilde{\tau}_1$ with the difference that the decay into the lightest neutralino becomes dominant not sooner than for $\tilde{m}_{e_L} \gtrsim 180\, GeV$ in the case of SPS1 a, and for $\tilde{m}_{e_L} \gtrsim 300\, GeV$ in the case of SPS3. Similar to the case of $\widetilde{\tau}_1$, the total decay width of $\widetilde{e}_L$ for SPS1 a increases, seen in Fig. 3.9(b), from $\Gamma_{tot} \sim 0.2\, GeV$ up to $\sim 0.9\, GeV$, whereas the decay width for SPS3 decreases from $\sim 6\, GeV$ down to $\sim 1\, GeV$ and continues almost constantly with a value of $\sim 0.6\, GeV$.



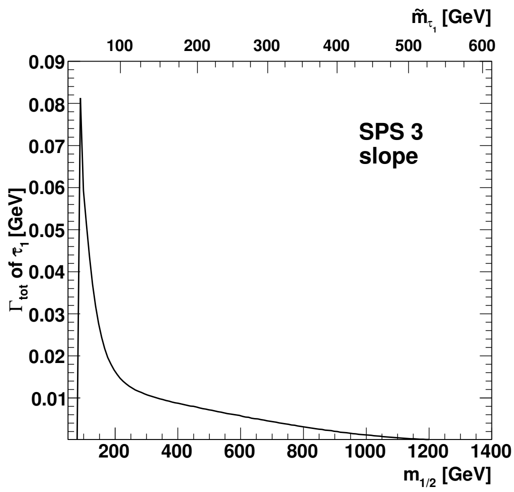

(a) $\tilde{\tau}_1$ branching ratios, SPS 1a.




(b) $\tilde{\tau}_1$ total decay with, SPS 1a.

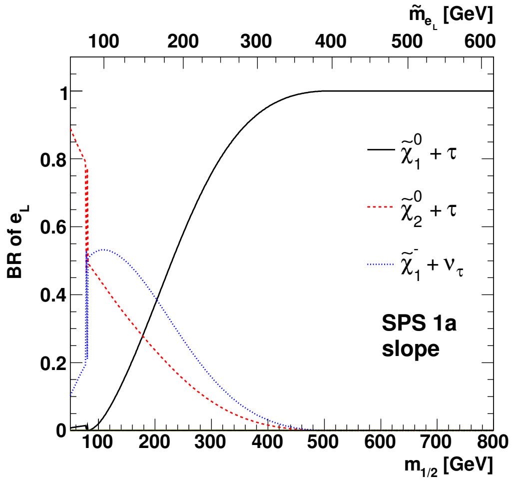



(c) $\tilde{\tau}_1$ branching ratios, SPS 3.

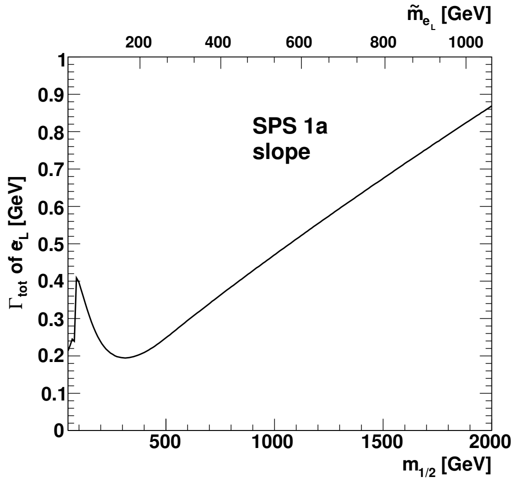



(d) $\tilde{\tau}_1$ total decay with, SPS 3.

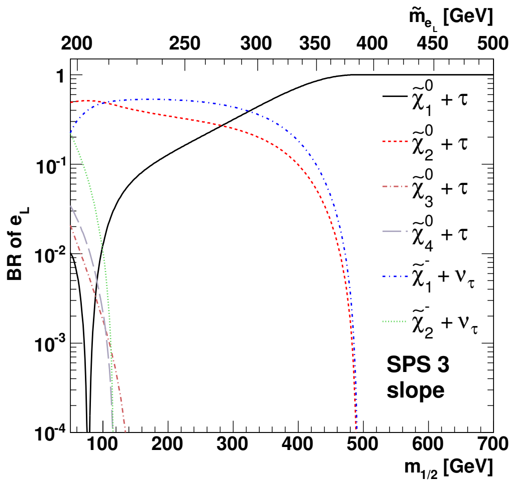
Figure 3.8: Branching ratios and total decay width for $\tilde{\tau}_1$ in dependence of $m_{1/2}$ on the lower x-axis and its mass $m_{\tilde{\tau}_1}$ on the upper x-axis. The SUSY spectrum is generated by SUSPECT2 [31], decays are evaluated by SDECAY [33]. (a) $BR$ and (b) total decay width for $\tilde{\tau}_1$ a with SPS1 a slope, (c) and (d) same for SPS3 slope.



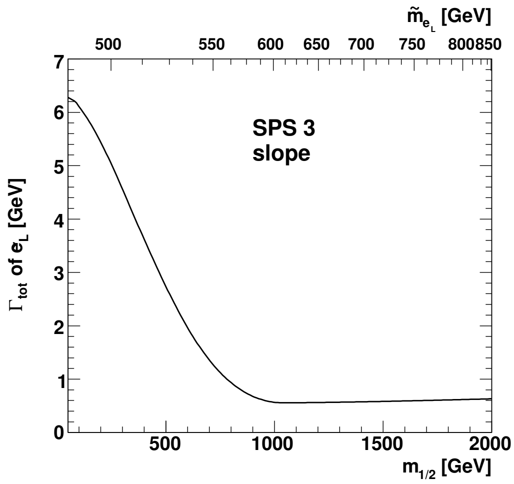

(a) $\tilde{e}_L$ branching ratios, SPS 1a.




(b) $\tilde{e}_L$ total decay with, SPS 1a.




(c) $\tilde{e}_L$ branching ratios, SPS 3.




(d) $\tilde{e}_L$ total decay with, SPS 3.

Figure 3.9: Branching ratios and total decay width for $\tilde{e}_L$ in dependence of $m_{1/2}$ on the lower x-axis and its mass $m_{\tilde{e}_L}$ on the upper x-axis. The SUSY spectrum is generated by SUSPECT2 [31], decays are evaluated by SDECAY [33]. (a) $BR$ and (b) total decay width for $\widetilde{e}_L$ a with SPS1 a slope, (c) and (d) same for SPS3 slope.

# 4 Conclusions

At the end we want to summarize results pointing out some characteristics and future prospects. Initially, we started with the introduction of supersymmetry and arrived at the MSSM. Subsequently, we discussed R-parity violation and derived the corresponding Feynman rules. Then we studied single slepton production at polarized and unpolarized hadron colliders (LHC, Tevatron and RHIC). We worked out the analytical results at LO and used them for our numerical computations, where we confirmed results in the literature for the unpolarized cross sections. We went on to polarized cross sections and double spin asymmetries, where we discussed in some details the use of chosen PDFs and the breakdown of the results in the build asymmetries. In order to complete our discussion, the decay modes of the produced charged slepton was analyzed, wherefore it was necessary to chose a SUSY breaking scenario (mSUGRA) and benchmark points as well.

In particular, the additional information in polarized collisions might be interesting. The combination of a propagating scalar particle together with a vertex only containing $P_L$, but not $P_R$ (reverse for the conjugate process) results in the fact that $\Delta\hat{\sigma} = \hat{\sigma}_0 = \frac{1}{4}\hat{\sigma}^{--}$ and thus the asymmetry $A_{LL}$ becomes just the ratio between the polarized and the unpolarized parton luminosity, which is in principle well known ($\lambda'$ cancels). Therefore $A_{LL}$ could be useful to discriminate models of new physics with the same signature (charged lepton plus missing energy). For futher studies both, the SM backgrounds as well as more precision, reached by a NLO calculation and resummation, are needed.

At the very end, we can say that the proceeding discussion brings some different topics together: R-parity violation as physics beyond the MSSM, QCD, polarization in hadron collisions and programming—all together making it a rich subject to be continued.

# A Conventions, Definitions and Notation

## A.1 Metric and Matrices

The omnipresent metric throughout this report is

$$g = \text{diag}[1, -1, -1, -1] \; . \tag{A.1}$$

We also use the antisymmetric tensors

$$\varepsilon^{\alpha\beta} = \varepsilon^{\dot{\alpha}\dot{\beta}} = i\sigma^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \text{and} \quad \varepsilon_{0123} = +1 \; , \tag{A.2}$$

which fulfil the useful identity

$$\varepsilon_{\alpha\beta}\varepsilon^{\beta\gamma} = -\delta_\alpha^\gamma \; . \tag{A.3}$$

The Pauli matrices are

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \; , \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \; , \quad \sigma^2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \; , \tag{A.4}$$

$$\sigma^\mu := (1, \vec{\sigma}) \; , \qquad \bar{\sigma} := (1, -\vec{\sigma}) \; , \tag{A.5}$$

$$\sigma^{\mu\nu} := \frac{1}{4}\left(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu\right) \; , \qquad \bar{\sigma}^{\mu\nu} := \frac{1}{4}\left(\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu\right) \; . \tag{A.6}$$

## A.2 Weyl Spinors and Grassmann variables

In quantum field theory spinors, as well as Grassmann variables, fulfil anti-commutator relations. In the superfield formulation they belong together so that we use the same notation for both of them.

Weyl spinors under a Lorentz transformation $M \in SO(3,1)$:

$$\begin{aligned} &\xi_\alpha \to \xi'_\alpha = M_\alpha^{\;\beta}\,\xi_\beta \; , && \xi^\alpha = \varepsilon^{\alpha\beta}\,\xi_\beta \; , && \xi_\alpha = \bar{\xi}_{\dot{\alpha}}^{\;*} \; , \\ &\bar{\eta}^{\dot{\alpha}} \to \bar{\eta}^{\dot{\alpha}\,\prime} = \left(M^{-1}\right)^{\dagger\alpha}{}_{\beta}\,\xi^\beta \; , && \bar{\eta}^{\dot{\alpha}} = \varepsilon^{\dot{\alpha}\dot{\beta}}\,\bar{\eta}_{\dot{\beta}} \; , && \bar{\eta}^{\dot{\alpha}} = \eta^{\alpha *} \; . \end{aligned} \tag{A.7}$$

Products of anticommuting Weyl spinors are the same as products of Grassmann variables.

$$\theta\theta := \theta^\alpha\theta_\alpha = \varepsilon_{\alpha\beta}\theta^\alpha\theta^\beta = -2\theta^1\theta^2 \; , \qquad \theta^\alpha\theta^\beta = -\frac{1}{2}\varepsilon_{\alpha\beta}\theta\theta \; , \tag{A.8}$$

$$\bar{\theta}\bar{\theta} := \bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}}\bar{\theta}^{\dot{\beta}}\theta^{\dot{\alpha}} = +2\bar{\theta}^{\dot{1}}\bar{\theta}^{\dot{2}} \; , \qquad \bar{\theta}^{\dot{\alpha}}\bar{\theta}^{\dot{\beta}} = \frac{1}{2}\varepsilon_{\dot{\alpha}\dot{\beta}}\bar{\theta}\bar{\theta} \; , \tag{A.9}$$

$$\partial_\alpha := \frac{\partial}{\partial\theta^\alpha} \,, \qquad \partial^\alpha := \frac{\partial}{\partial\theta_\alpha} \,, \qquad \partial_\alpha\theta^\beta = \delta_\alpha{}^\beta \tag{A.10}$$

$$\bar{\partial}^{\dot{\alpha}} := \frac{\partial}{\partial\bar{\theta}_{\dot{\alpha}}} \,, \qquad \bar{\partial}_{\dot{\alpha}} := \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} \,, \qquad \bar{\partial}^{\dot{\alpha}}\bar{\theta}_{\dot{\beta}} = \delta^{\dot{\alpha}}{}_{\dot{\beta}} \,. \tag{A.11}$$

Remark that

$$\partial_\alpha\theta_\beta = \varepsilon_{\beta\alpha} = -\varepsilon_{\alpha\beta} \quad \Rightarrow \quad \partial^\alpha = -\varepsilon^{\alpha\beta}\partial_\beta \,. \tag{A.12}$$

Integration of Grassmann variables works as follows.

$$\int \mathrm{d}\theta^\alpha = \int \mathrm{d}\bar{\theta}^{\dot{\alpha}} = 0 \,, \tag{A.13}$$

$$\int \mathrm{d}\theta^\alpha \; \theta^\alpha = \int \mathrm{d}\bar{\theta}^{\dot{\alpha}} \; \bar{\theta}^{\dot{\alpha}} = 0 \quad \text{no summation here.} \tag{A.14}$$

Volume elements in superspce are defined by

$$\mathrm{d}^2\theta := -\frac{1}{4}\mathrm{d}\theta\mathrm{d}\theta = -\frac{1}{4}\mathrm{d}\theta^\alpha\mathrm{d}\theta_\alpha \,, \tag{A.15}$$

$$\mathrm{d}^2\bar{\theta} := -\frac{1}{4}\mathrm{d}\bar{\theta}\mathrm{d}\bar{\theta} = -\frac{1}{4}\mathrm{d}\bar{\theta}^{\dot{\alpha}}\mathrm{d}\bar{\theta}_{\dot{\alpha}} \tag{A.16}$$

$$\mathrm{d}\theta^4 := \mathrm{d}^2\theta\mathrm{d}^2\bar{\theta} \,. \tag{A.17}$$

It follows

$$\int \mathrm{d}^2\theta^2 = \int \mathrm{d}^2\bar{\theta}^2 = \int \mathrm{d}^4\theta^4 = 1 \,. \tag{A.18}$$

# B Narrow Width Approximation

## B.1 Factorization of Phase Space

We consider a process $p_A + p_B \to p_1 + p_2$. The Lorentz-invariant phase space can be written as

$$dPS_{2\to 2} = (2\pi)^4 \delta^4 \left(p_A + p_B - p_1 - p_2\right) \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2} \ . \tag{B.1}$$

This formula can be rewritten in the phase spaces of the processes $p_A + p_B \to q$ and $q \to p_1 + p_2$:

$$dPS_{2\to 1} = (2\pi)^4 \delta^4 \left(p_A + p_B - q\right) \cdot \frac{d^3\vec{q}}{(2\pi)^3 2E_q} \ , \tag{B.2}$$

$$dPS_{1\to 2} = (2\pi)^4 \delta^4 \left(q - p_1 - p_2\right) \cdot \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2} \ . \tag{B.3}$$

Using the three identities

$$1 = \int ds \ \delta(q^2 - s) \ , \tag{B.4}$$

$$1 = \int d^4q \ \delta^4 \left(q - p_A - p_B\right) \ , \tag{B.5}$$

$$\delta^4(q^2 - s) = \frac{1}{2E_q} \cdot \left[\delta(q^0 - E_q) \cdot \theta(E_q) + \delta(q^0 - E_q) \cdot \theta(-E_q)\right]_{E_q = \vec{q}^2 + s} \tag{B.6}$$

yields

$$
\begin{aligned}
dPS_{2\to 2} = & \int d^4q\, \delta^4\left(q - p_A - p_B\right) \int ds\, \delta(s - q^2) \\
& (2\pi)^4\, \delta^4\left(p_A + p_B - p_1 - p_2\right) \cdot \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1} \cdot \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2} \\
= & \int ds \int d^4q\, \delta(s - q^2) \cdot \delta^4\left(q - p_A - p_B\right) \\
& (2\pi)^4\, \delta^4\left(p_A + p_B - p_1 - p_2\right) \cdot \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1} \cdot \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2} \\
= & \int ds \int d^3\vec{q}\, dq^0\, \frac{1}{2E_q} \cdot \left[\delta(q^0 - E_q) \cdot \theta(E_q)\right]_{E_q = \vec{q}^2 + s} \cdot \delta^4\left(q - p_A - p_B\right) \\
& (2\pi)^4\, \delta^4\left(p_A + p_B - p_1 - p_2\right) \cdot \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1} \cdot \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2} \\
= & \int ds \int \frac{d^3\vec{q}}{2E_q}\, \delta^4\left(q - p_A - p_B\right) \\
& (2\pi)^4\, \delta^4\left(q - p_1 - p_2\right) \cdot \frac{d^3 p_1}{(2\pi)^3 2E_1} \cdot \frac{d^3 p_2}{(2\pi)^3 2E_2} \\
= & \int \frac{ds}{2\pi}\, dPS_{2\to 1}(p_A, p_B; s) \cdot dPS_{1\to 2}(s; p_1, p_2)\ . \qquad \text{(B.7)}
\end{aligned}
$$

For a $2 \to 2$ s-channel process with an exchanged virtual scalar particle with propagator

$$
P = \frac{-i}{s - M^2 + iM\Gamma}\ , \qquad \text{(B.8)}
$$

where $s = q^2 = (p_A + p_B)^2$ is the 4-momentum squared, M the mass and $\Gamma$ the decay rate of the exchanged particle. The cross section can be written as

$$
\begin{aligned}
d\sigma = & \frac{1}{F}\, dPS_{2\to 2}\, |M|^2_{2\to 2} \\
= & \int \frac{dq^2}{2\pi}\, dPS_{2\to 1}(p_A, p_B; q) \cdot dPS_{1\to 2}(q; p_1, p_2) \cdot \frac{1}{F} \cdot |M|^2_{2\to 1} \cdot |P|^2 \cdot |M|^2_{1\to 2}\ , \qquad \text{(B.9)}
\end{aligned}
$$

where $F = 2s$ is the flux of the incoming particles, $|M|^2_{2\to 1}$ is the squared matrix element of the production of a virtual particle and $|M|^2_{1\to 2}$ is the matrix element of its decay. Together with the definition of the decay rate

$$
d\Gamma_{1\to 2} = \frac{1}{2M}\, dPS_{1\to 2}\, |M|^2_{1\to 2}\ , \qquad \text{(B.10)}
$$

this equation looks simpler in the following form

$$
d\sigma_{2\to 2} = \int \frac{ds}{2\pi}\, d\sigma_{2\to 1}(p_A, p_B; s) \cdot d\Gamma_{1\to 2}(s; p_1, p_2) \cdot 2M \cdot |P|^2\ . \qquad \text{(B.11)}
$$

## B.2 Narrow Width Approximation

The factored form of the cross section in Eq. B.9 is useful in the so called narrow width approximation as we will see. Using the well-known result (see [34], Eq. (14.45))

$$\frac{1}{\pi}\frac{\varepsilon}{x^2+\varepsilon^2} \xrightarrow[\varepsilon\to 0]{} \delta(x) \ . \tag{B.12}$$

One can see that the square of the exchange particle propagator can be written as

$$|P|^2 = \frac{1}{(s-M^2)^2+(M\Gamma)^2} \approx \frac{\pi}{M\Gamma}\,\delta(s-M^2) \ , \tag{B.13}$$

in the physical limit $\Gamma \ll M$, which corresponds to a resonance ($s = M$) with a narrow width. With the help of Eq. B.13 we obtain a simplified, factored cross section in the narrow width approximation:

$$\mathrm{d}\sigma_{2\to 2}(s = M^2) = \mathrm{d}\sigma_{2\to 1}(p_A, p_B; M^2) \cdot \frac{\mathrm{d}\Gamma_{1\to 2}(M^2; p_1, p_2)}{\Gamma} \ . \tag{B.14}$$

Thus, this process is completely divided into the production of the virtual on-shell particle and its decay. $\frac{\Gamma_{1\to 2}}{\Gamma}$ is called *branching ratio*.

# C Polarized Hadronic Cross Section

## C.1 Definitions

In polarized scattering experiments in which the initial particles are longitudinally polarized, one measures the polarized cross sections

$$\sigma(\lambda_1, \lambda_2)\ , \quad \sigma(1-\lambda_1, 1-\lambda_2)\ , \quad \sigma(\lambda_1, 1-\lambda_2)\ , \quad \sigma(1-\lambda_1, \lambda_2)\ . \tag{C.1}$$

Here the values $0 \leq \lambda \leq 1$ are the probabilities for a particle to have a right-handed helicity, thus $1-\lambda$ is the probability of left-handedness. In experiments the given quantity is the beam polarization P for each beam, which is connected to the helicity probabilities by

$$\lambda = \frac{1+P}{2}\ . \tag{C.2}$$

In experiments, the outgoing particles are also polarized, but, in general, detectors are not able to measure it. Calculations, however, are done with helicities $h = \pm$ of the initial particles and the cross section $\sigma^{h_1,h_2}$. So there are four quantities

$$\sigma^{++},\ \sigma^{--},\ \sigma^{+-},\ \sigma^{-+} \tag{C.3}$$

The relation between polarization and helicity becomes

$$\sigma(\lambda_1, \lambda_2) = \lambda_1\lambda_2\sigma^{++} + \lambda_1(1-\lambda_2)\sigma^{+-} + (1-\lambda_1)\lambda_2\sigma^{-+} + (1-\lambda_1)(1-\lambda_2)\sigma^{--}\ . \tag{C.4}$$

Now it is useful to change the basis, introducing

$$\sigma_0 := \frac{1}{4}\left(\sigma^{++} + \sigma^{--} + \sigma^{+-} + \sigma^{-+}\right) \tag{C.5}$$

$$\Delta\sigma := \frac{1}{4}\left(\sigma^{++} + \sigma^{--} - \sigma^{+-} - \sigma^{-+}\right) \tag{C.6}$$

$$\sigma^{PV} := \frac{1}{4}\left[\left(\sigma^{++} - \sigma^{--}\right) + \left(\sigma^{+-} - \sigma^{-+}\right)\right] \tag{C.7}$$

$$\Delta\sigma^{PV} := \frac{1}{4}\left[\left(\sigma^{++} - \sigma^{--}\right) - \left(\sigma^{+-} - \sigma^{-+}\right)\right] \tag{C.8}$$

The advantage of this basis will become obvious when dealing with PDFs and the partonic cross section. PV stands for parity violating, since, if parity is not violated, $\sigma^{++} = \sigma^{--}$ and $\sigma^{+-} = \sigma^{-+}$, and so each term cancels and they become zero. The formula (C.4) expressed in this basis becomes

$$\sigma(\lambda_1, \lambda_2) = \sigma_0 + (2\lambda_1 - 1)(2\lambda_2 - 1)\,\Delta\sigma + (2\lambda_1 - 1)\,\sigma^{PV} + (2\lambda_2 - 1)\,\Delta\sigma^{PV}. \tag{C.9}$$

In order to switch between experimental (C.1) and theoretical (C.5)(-C.8) quantities, we write their relations

$$\sigma_0 = \frac{1}{4}\left[\sigma(\lambda_1,\lambda_2) + \sigma(1-\lambda_1, 1-\lambda_2) + \sigma(\lambda_1, 1-\lambda_2) + \sigma(1-\lambda_1,\lambda_2)\right] \quad (C.10)$$

$$\Delta\sigma = \frac{1}{4P_1P_2}\left[\sigma(\lambda_1,\lambda_2) + \sigma(1-\lambda_1, 1-\lambda_2) - \sigma(\lambda_1, 1-\lambda_2) - \sigma(1-\lambda_1,\lambda_2)\right] \quad (C.11)$$

$$\sigma^{PV} = \frac{1}{4P_1P_2}\left[\sigma(\lambda_1,\lambda_2) - \sigma(1-\lambda_1, 1-\lambda_2) + \sigma(\lambda_1, 1-\lambda_2) - \sigma(1-\lambda_1,\lambda_2)\right] \quad (C.12)$$

$$\Delta\sigma = \frac{1}{4P_1P_2}\left[\sigma(\lambda_1,\lambda_2) - \sigma(1-\lambda_1, 1-\lambda_2) - \sigma(\lambda_1, 1-\lambda_2) + \sigma(1-\lambda_1,\lambda_2)\right] \quad (C.13)$$

Furthermore, we build so called asymmetries:

$$A_{LL} = \frac{\sigma^{++} + \sigma^{--} - \sigma^{+-} - \sigma^{-+}}{\sigma^{++} + \sigma^{--} + \sigma^{+-} + \sigma^{-+}} = \frac{\Delta\sigma}{\sigma_0} \quad (C.14)$$

$$A_{L0}^{PV} = \frac{\sigma^{+0} - \sigma^{-0}}{\sigma^{+0} + \sigma^{-0}} = \frac{\sigma^{PV}}{\sigma_0} \quad (C.15)$$

$$A_{0L}^{PV} = \frac{\sigma^{0+} - \sigma^{0-}}{\sigma^{0+} + \sigma^{0-}} = \frac{\Delta\sigma^{PV}}{\sigma_0} \quad (C.16)$$

$$A_{LL,even}^{PV} = \frac{\sigma^{+-} - \sigma^{-+}}{\sigma^{+-} + \sigma^{-+}} = \frac{\sigma^{PV} - \Delta\sigma^{PV}}{\sigma_0 - \Delta\sigma} \quad (C.17)$$

$$A_{LL,odd}^{PV} = \frac{\sigma^{++} - \sigma^{--}}{\sigma^{++} + \sigma^{--}} = \frac{\sigma^{PV} + \Delta\sigma^{PV}}{\sigma_0 + \Delta\sigma} \quad (C.18)$$

Here a zero in the upper index means no polarization, that is $\sigma^{+0} = \frac{1}{2}(\sigma^{++} + \sigma^{+-})$ and so on. As in the unpolarized case, the probability of finding a parton $i$ carrying the momentum fraction $x_i$ and with helicity $h$ in a hadron with helicity $H$ is given by the polarized parton density function (PDF)

$$f_{i,H}^{h}(x_i, Q^2), \quad (C.19)$$

where Q is the scale factor. Since the PDF includes only information about QCD processes, parity is conserved and thus we conclude

$$f_{i,+}^{+} = f_{i,-}^{-} \qquad f_{i,+}^{-} = f_{i,-}^{+} \quad (C.20)$$

Therefore we can always express the PDF with a positive hadron helicity and this is done by convention whenever the lower index is dropped.

## C.2 Relations between hadronic and partonic polarized cross sections

The relation between hadronic and partonic polarized cross section $\sigma^{h_1,h_2}$ and $\widehat{\sigma}^{h_1',h_2'}$ is a straightforward generalization from the unpolarized case (3.1):

$$\sigma_{hadr_1+hadr_2\to k+X}^{h_1,h_2} = \sum_{i,j}\sum_{h_1',h_2'}\int_0^1\int_0^1 \mathrm{d}x_1\mathrm{d}x_2\; f_{i,h_1}^{h_1'}(x_1,Q^2) f_{j,h_2}^{h_2'}(x_2,Q^2)\;\widehat{\sigma}_{i+j\to k+X'}^{h_1',h_2'} \quad (C.21)$$

In order to calculate $\sigma$ and $\Delta\sigma$, it useful to define

$$\begin{aligned} f_i &:= f_i^+ + f_i^- \\ \Delta f_i &:= f_i^+ - f_i^- \end{aligned} \tag{C.22}$$

This yields

$$\begin{aligned} \Delta\sigma &= \frac{1}{4}\left(\sigma^{++} + \sigma^{--} - \sigma^{+-} - \sigma^{-+}\right) \\ &= \frac{1}{4}\sum_{i,j}\sum_{h_1',h_2'}\int_0^1\int_0^1 \mathrm{d}x_1\mathrm{d}x_2\,\left[f_{i,+}^{h_1'}f_{j,+}^{h_2'} + f_{i,-}^{h_1'}f_{j,-}^{h_2'} - f_{i,+}^{h_1'}f_{j,-}^{h_2'} - f_{i,-}^{h_1'}f_{j,+}^{h_2'}\right]\,\widehat{\sigma}_{i,j}^{h_1',h_2'} \end{aligned} \tag{C.23}$$

Using (C.20) and (C.22), the expression in brackets simplifies to

$$\begin{aligned} &\frac{1}{4}\sum_{h_1',h_2'}\left[f_i^{h_1'}f_j^{h_2'} + f_i^{-h_1'}f_j^{-h_2'} - f_i^{h_1'}f_j^{-h_2'} - f_i^{-h_1'}f_j^{h_2'}\right]\,\widehat{\sigma}_{i,j}^{h_1',h_2'} \\ =&\frac{1}{4}\sum_{h_1',h_2'}\left[f_i^{h_1'}\left(f_j^{h_2'} - f_j^{-h_2'}\right) - f_i^{-h_1'}\left(f_j^{h_2'} - f_j^{-h_2'}\right)\right]\,\widehat{\sigma}_{i,j}^{h_1',h_2'} \\ =&\frac{1}{4}\sum_{h_1'}\left\{\left[f_i^{h_1'}\Delta f_j - f_i^{-h_1'}\Delta f_j\right]\,\widehat{\sigma}_{i,j}^{h_1',+} - \left[f_i^{h_1'}\Delta f_j - f_i^{-h_1'}\Delta f_j\right]\,\widehat{\sigma}_{i,j}^{h_1',-}\right\} \\ =&\frac{1}{4}\sum_{h_1'}\left\{\left[\left(f_i^{h_1'} - f_i^{-h_1'}\right)\Delta f_j\right]\,\widehat{\sigma}_{i,j}^{h_1',+} - \left[\left(f_i^{h_1'} - f_i^{-h_1'}\right)\Delta f_j\right]\,\widehat{\sigma}_{i,j}^{h_1',-}\right\} \\ =&\frac{1}{4}\Delta f_i\Delta f_j\,\left(\widehat{\sigma}_{ij}^{++} + \widehat{\sigma}_{ij}^{--} - \widehat{\sigma}_{ij}^{+-} - \widehat{\sigma}_{ij}^{-+}\right) \end{aligned}$$

So, we also define for the partonic cross section

$$\Delta\widehat{\sigma}_{ij} := \frac{1}{4}\left(\widehat{\sigma}_{ij}^{++} + \widehat{\sigma}_{ij}^{--} - \widehat{\sigma}_{ij}^{+-} - \widehat{\sigma}_{ij}^{-+}\right)\ . \tag{C.24}$$

The same calculation is valid for $\sigma_0$ by replacing all 'real' minus signs by positive signs. This also justifies our definitions in (C.22). In fact, we define the partonic cross sections in the same way as the hadronic one in Eq. (C.5)-(C.8). We list the resulting relations using a convolution notation $\otimes$ for the integrals and the sum convention:

$$\sigma_0 = f_i \otimes f_j \otimes \widehat{\sigma}_{0,ij}\ , \tag{C.25}$$

$$\Delta\sigma = \Delta f_i \otimes \Delta f_j \otimes \Delta\widehat{\sigma}_{ij}\ , \tag{C.26}$$

$$\sigma^{PV} = \Delta f_i \otimes f_j \otimes \widehat{\sigma}_{ij}^{PV}\ , \tag{C.27}$$

$$\Delta\sigma^{PV} = f_i \otimes \Delta f_j \otimes \Delta\widehat{\sigma}_{ij}^{PV}\ . \tag{C.28}$$

The first line is just a reproduction of the unpolarized hadronic cross section formula.

# D Gaugino and higgsino mixing

This appendix is taken from [24] The soft SUSY-breaking terms in the minimally supersymmetric Lagrangian include a term [4]

$$\mathcal{L} \supset -\frac{1}{2}(\psi^0)^T Y \psi^0 + \text{h.c.}, \tag{D.1}$$

which is bilinear in the (2-component) fermionic partners

$$\psi_j^0 = (-i\tilde{B}, -i\tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0)^T \quad \text{with} \quad j = 1, \ldots, 4 \tag{D.2}$$

of the neutral electroweak gauge and Higgs bosons and proportional to the, generally complex and necessarily symmetric, neutralino mass matrix

$$Y = \begin{pmatrix} M_1 & 0 & -m_Z s_W c_\beta & m_Z s_W s_\beta \\ 0 & M_2 & m_Z c_W c_\beta & -m_Z c_W s_\beta \\ -m_Z s_W c_\beta & m_Z c_W c_\beta & 0 & -\mu \\ m_Z s_W s_\beta & -m_Z c_W s_\beta & -\mu & 0 \end{pmatrix}. \tag{D.3}$$

Here, $M_1$, $M_2$, and $\mu$ are the SUSY-breaking bino, wino, and off-diagonal higgsino mass parameters with $\tan\beta = s_\beta/c_\beta = v_u/v_d$ being the ratio of the vacuum expectation values $v_{u,d}$ of the two Higgs doublets, while $m_Z$ is the SM $Z$-boson mass and $s_W$ ($c_W$) is the sine (cosine) of the electroweak mixing angle $\theta_W$. After electroweak gauge-symmetry breaking and diagonalization of the mass matrix $Y$, one obtains the neutralino mass eigenstates

$$\chi_i^0 = N_{ij}\,\psi_j^0, \quad i = 1, \ldots, 4, \tag{D.4}$$

where $N$ is a unitary matrix satisfying the relation

$$N^* Y N^{-1} = \text{diag}\,(m_{\tilde{\chi}_1^0}, m_{\tilde{\chi}_2^0}, m_{\tilde{\chi}_3^0}, m_{\tilde{\chi}_4^0}). \tag{D.5}$$

In 4-component notation, the Majorana-fermionic neutralino mass eigenstates can be written as

$$\tilde{\chi}_i^0 = \begin{pmatrix} \chi_i^0 \\ \bar{\chi}_i^0 \end{pmatrix}. \tag{D.6}$$

The application of projection operators leads to relatively compact analytic expressions for the mass eigenvalues $m_{\tilde{\chi}_1^0} < m_{\tilde{\chi}_2^0} < m_{\tilde{\chi}_3^0} < m_{\tilde{\chi}_4^0}$ [35]. As we choose them to be real and non-negative, our unitary matrix $N$ is generally complex [36].

The chargino mass term in the SUSY Lagrangian [4]

$$\mathcal{L} \supset -\frac{1}{2}(\psi^+ \psi^-)\begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix}\begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + \text{h.c.} \tag{D.7}$$

is bilinear in the (2-component) fermionic partners

$$\psi_j^{\pm} \;=\; (-i\tilde{W}^{\pm}, \tilde{H}^{\pm}_{2,1})^T \quad \text{with} \quad j = 1, \ldots, 2 \tag{D.8}$$

of the charged electroweak gauge and Higgs bosons and proportional to the, generally complex, chargino mass matrix

$$X \;=\; \begin{pmatrix} M_2 & m_W\,\sqrt{2}\,s_\beta \\ m_W\,\sqrt{2}\,c_\beta & \mu \end{pmatrix}, \tag{D.9}$$

where $m_W$ is the mass of the SM $W$-boson. Since $X$ is not symmetric, it must be diagonalized by two unitary matrices $U$ and $V$, which satisfy the relation

$$U^*\,X\,V^{-1} \;=\; \mathrm{diag}\,(m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}) \tag{D.10}$$

and define the chargino mass eigenstates

$$\begin{aligned} \chi_i^+ &= V_{ij}\,\psi_j^+ \\ \chi_j^- &= U_{ij}\,\psi_j^- \end{aligned}\;, \quad i,j = 1,2. \tag{D.11}$$

In 4-component notation, the Dirac-fermionic chargino mass eigenstates can be written as

$$\tilde{\chi}_i^\pm \;=\; \begin{pmatrix} \chi_i^\pm \\ \bar{\chi}_i^\mp \end{pmatrix}. \tag{D.12}$$

As Eq. (D.10) implies

$$V X^\dagger X V^{-1} \;=\; \mathrm{diag}\,(m^2_{\tilde{\chi}_1^\pm}, m^2_{\tilde{\chi}_2^\pm}), \tag{D.13}$$

the hermitian matrix $X^\dagger X$ can be diagonalized using only $V$, and its eigenvalues

$$\begin{aligned} m^2_{\tilde{\chi}^\pm_{1,2}} = \frac{1}{2}\Big\{ &|M_2|^2 + |\mu|^2 + 2m_W^2 \\ &\mp \sqrt{(|M_2|^2 + |\mu|^2 + 2m_W^2)^2 - 4|\mu M_2 - m_W^2 s_{2\beta}|^2}\Big\} \end{aligned} \tag{D.14}$$

are always real. If we take also the mass eigenvalues $m_{\tilde{\chi}_1^\pm} \le m_{\tilde{\chi}_2^\pm}$ to be real and non-negative, the rotation matrix

$$V \;=\; \begin{pmatrix} \cos\theta_+ & \sin\theta_+\,e^{-i\phi_+} \\ -\sin\theta_+\,e^{i\phi_+} & \cos\theta_+ \end{pmatrix} \tag{D.15}$$

can still be chosen to have real diagonal elements, but the off-diagonal phase $e^{\mp i\phi_+}$ is needed to rotate away the imaginary part of the off-diagonal matrix element in $X^\dagger X$,

$$\Im\Big[(M_2^*\,s_\beta + \mu\,c_\beta)\,e^{i\phi_+}\Big] \;=\; 0. \tag{D.16}$$

The rotation angle $\theta_+ \in [0;\pi]$ is uniquely fixed by the two conditions

$$\tan 2\theta_+ = \frac{2\sqrt{2}m_W\,(M_2^*\,s_\beta + \mu\,c_\beta)\;e^{i\phi_+}}{|M_2|^2 - |\mu|^2 + 2m_W^2 c_{2\beta}} \quad \text{and} \tag{D.17}$$

$$\sin 2\theta_+ = \frac{-2\sqrt{2}m_W\,(M_2^*\,s_\beta + \mu\,c_\beta)\;e^{i\phi_+}}{\sqrt{(|M_2|^2 - |\mu|^2 + 2m_W^2 c_{2\beta})^2 + 8m_W^2\left[(M_2^*\,s_\beta + \mu\,c_\beta)\,e^{i\phi_+}\right]^2}}. \tag{D.18}$$

Once $V$ is known, the unitary matrix $U$ can be obtained from

$$U = \mathrm{diag}\,(m^{-1}_{\tilde\chi^\pm_1}, m^{-1}_{\tilde\chi^\pm_2})\;V^* X^T. \tag{D.19}$$

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