RADIOACTIVITY
and
ELEMENTS of NUCLEAR PHYSICS

Exercises

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Make sure to have a pocket computer with you all time!

Series 0

Revision of exponentials and logarithms

Exercise 0.1: Expressions and functions

1) Find which answer is the correct one:

$$\begin{aligned}
\ln(1-x^2) - \ln(1-x) &= \ln(1+x)? \quad \text{or} \quad \ln x(1-x)? \\
\ln(x^n) &= e^{nx}? \quad \text{or} \quad n \ln x? \\
e^{x+1}/e^{1-x} &= e^{x^2-1}? \quad \text{or} \quad e^{2x}?
\end{aligned}$$

2) Give the numerical solutions to:

$$\ln 1 = \ldots?$$

$$\log_{10}(0.1) = \ldots?$$

$$\frac{3 \times 10^{-2} + 5 \times 10^{-3}}{7 \times 10^{-3}} = \ldots?$$

3) Compute the integral of $1/x$ between x_1 and x_2 :

$$\int_{x_1}^{x_2} \frac{dx}{x},$$

and deduce the value of:

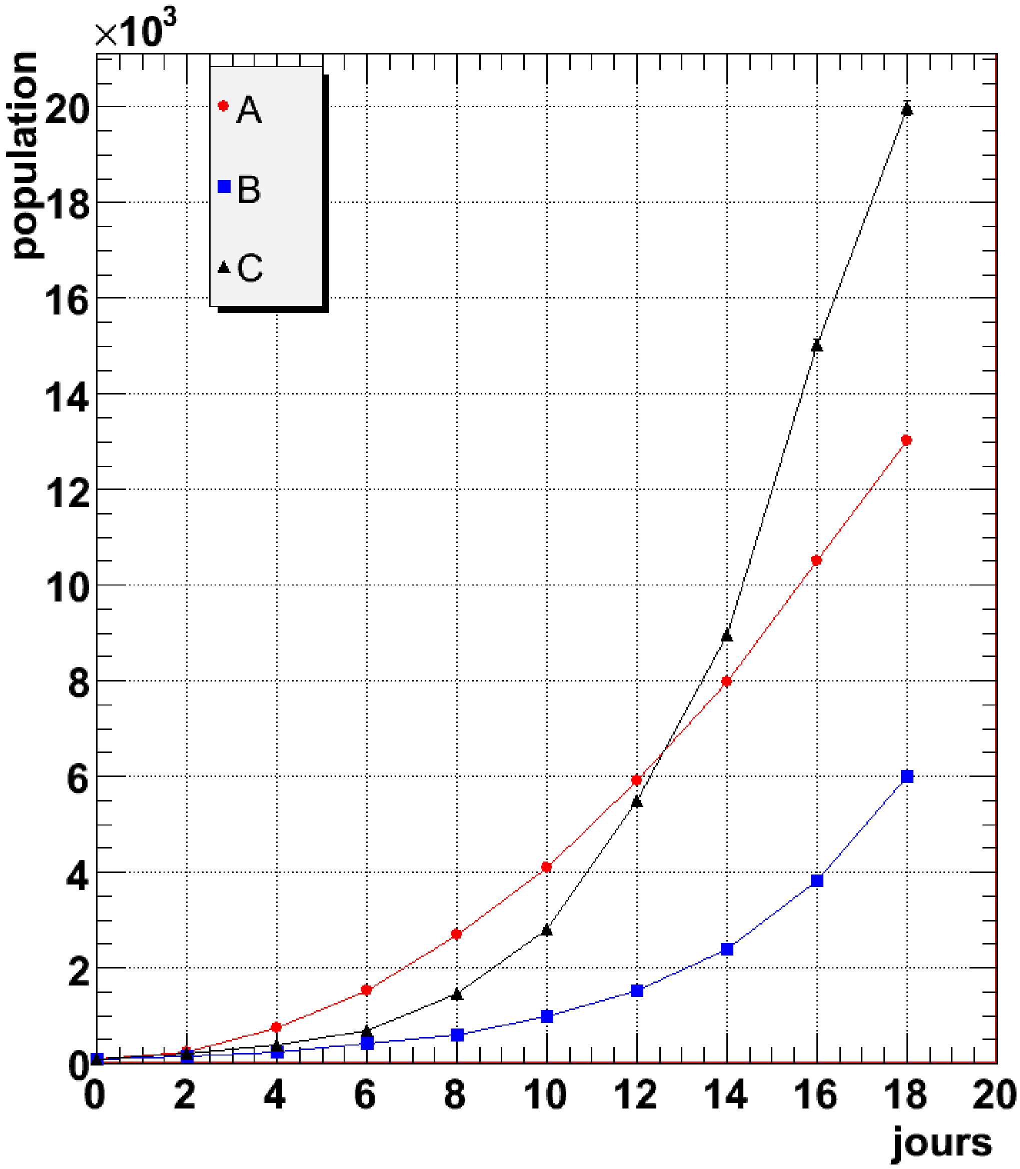
$$\int_1^e \frac{dx}{x}$$

Exercise 0.2: Population growth: illustrating exponential growth in biology

The variations of the populations of 3 microbial cultures A,B and C are being studied in the lab. For this purpose, samples of each of the 3 cultures are taken every 2 days during 18 days and the concentration of microbes in each sample is measured and reported.
The concentration measurements are normalized to the same initial value of 100 microbes per cm^3 in such a was as to make the comparison easier. These normalized values are gathered in the following table:

day nb	0	2	4	6	8	10	12	14	16	18
A	100	247	752	1513	2687	4095	5912	7987	10521	13032
B	100	153	247	406	594	991	1511	2389	3812	5994
C	100	205	402	696	1478	2816	5483	8969	15022	19977

These results have been plotted as a function of time in a linear scale graph:



1. Using these curves, is it possible to characterize and differentiate the evolution laws for the 3 populations?
2. One now wants to represent the variations of the 3 populations in a semi-logarithmic graph (the log scale will represent the populations, use the supplied semi-log paper chart):
   a) How many modules must the log scale have?
   b) Define yourself proper labels for the log scale.
   c) Plot the variations of populations A,B and C. Evaluate the precision with which you were able to plot the values from the table and cross out the digits in the table that you could not reasonably represent on the graph.
   d) From the semi-log plots, what can you deduce on the behavior of the 3 populations?
3. Give the evolution laws $N_B(t)$ and $N_C(t)$ of populations B (on the entire measurement period) and C (in the time interval where the plot is rectilinear). One will call $N_0 = 100$ the initial population.
4. Compute the growth coefficients λ_B and λ_C on the evolution laws of populations B and C.
5. Biologically, how would you interpret the evolution of population C in its later part?

One wishes to estimate the uncertainty on the exponential growth coefficient of population B. The uncertainty on each individual measurement $N(t)$ will be taken as $2\sqrt{N(t)}$ (95%(CL)).

6. Plot the corresponding "error bars" around each measurement point on the semi-log graph.
7. Determine the straight lines with maximum and minimum slope that can match all the error bars.
8. From these two extreme cases, compute the mean value of λ and its associated error $\lambda \pm \Delta\lambda$.

Exercise 0.3: Radioactive decay of a plutonium source: a physical example of an exponential law

A probe of a radioactive element with mass m contained in a sealed source will decrease with time according the following exponential law.[1] The mass $m(t)$ will follow an exponential law similar to that describing the number of atom $N(t)$ hence:

$$m(t) = m_0 e^{-\lambda t}$$

where m_0 is the initial mass of the radio-element, λ is the radioactive constant which is related to the decay period T of this nuclear species.

1. The decay period or half lifetime T is defined as the interval of time after which half of the nuclei will have decayed statistically:
$$N(t+T) = \frac{N(t)}{2}.$$
Show that the radioactive constant λ can be related to the period of the radioactive element by:
$$\lambda = \frac{\ln 2}{T}.$$

[1] All mass m of a given element (radioactive of not) is related to the number of atoms N of this element which is obtained from $N = N_{AV}(m/M_A)$, where N_{AV} is Avogadro's number M_A is the molar mass of the element.
When the considered element is radioactive, the mass m will hence vary with time like the number of remaining (not yet decayed) nuclei of this element.

2. Show that

$$N_0 e^{-\lambda t} = N_0 \left(\frac{1}{2}\right)^{t/T} .$$

A radioactive source of plutonium of initial mass $m_0 = 50$ mg is enclosed inside a sealed container. The radioactive decay period of plutonium is $T = 24000$ years.

3. Give a numerical expression for the mass variation of radioactive plutonium inside the container. One will use time units of thousand years.
4. Plot on a semi-log graph the variation $m(t)$ over 100 000 years:
   (a) first by computing the value of m for $t = 100\ 000$ years.
   (b) this time using directly the value of T.
5. How long do we have to wait until the mass of the remaining plutonium is less than 1% of the initial mass?

Series 1

Radioactive nuclei, nuclear reactions, activity, ^{14}C dating

Exercise 1.1:

Indicate the number of protons and of neutrons in each of the following nuclei:

$$^{40}_{20}\mathrm{Ca} \qquad ^{52}_{24}\mathrm{Cr} \qquad ^{132}_{54}\mathrm{Xe}$$

Exercise 1.2: Understanding conservation laws

The radioactive isotope $^{218}_{84}$Po can decay via α emission, the residual nucleus being Pb. Write down the corresponding decay reaction.

Among the following nuclear reactions, which ones are possible, which ones are not? Assuming that the error hides in the residual atom, establish the correct equation (modify A or/and Z of the residual nuclei in the right hand term]:

a) $^{18}_{8}\mathrm{O}(\mathrm{p},\alpha)^{15}_{7}\mathrm{N}$ that is $^{18}_{8}\mathrm{O} + \mathrm{p} \longrightarrow \alpha +^{15}_{7}\mathrm{N}$

b) $^{9}_{4}\mathrm{Be}(\alpha,^{3}_{1}\mathrm{H})^{10}_{4}\mathrm{Be}$

c) $^{6}_{3}\mathrm{Li}(\mathrm{p},\mathrm{d})\alpha$

d) $^{27}_{13}\mathrm{Al}(\mathrm{p},\gamma)^{28}_{14}\mathrm{Si}$

p, proton or hydrogen nucleus; d, deuton or deuterium nucleus; α, helium 4 nucleus; γ, gamma-ray (chargeless and massless) emitted during the de-excitation of an excited nucleus.

Exercise 1.3: Activity

The carbon isotope $^{11}_{6}$C has a period T equal to 20.4 minutes.

1. How does one define the radioactive period ?

2. Establish the relation between the period and the radioactive constant λ.

3. Compute λ and give its units.

We want to find out what is the activity of a sample of this isotope.

1. Give the definition of the activity and how it is related to the number of atoms at a given time.

2. How many nuclei (atoms) are there in a 6.2μg sample of this isotope?

3. Deduce what is its activity. Use an approximate value for the mass of an atom of this isotope.

4. How many remaining nuclei are there one hour later? Find first the order of magnitude and then the exact value.

5. What is the remaining activity of the sample at this moment?

Exercise 1.4: Activity

The activity of a sample of the $^{131}_{53}\mathrm{I}$ isotope has decreased by a factor 16 in 32 days.

1. On a graph with two linear scales, plot the qualitative behavior of the decrease of activity as a function of time: take T as the time unit; mark $a(t=0)=a_0$ as well as the values of $a(t=nT)$ for $n=1,\ 2,\ 3$ and 4 as a function of a_0, n and powers of 2.

2. Deduce the value of the period T of $^{131}_{53}\mathrm{I}$ in units of days.

3. See if you can find the same value using the exponential decay law $a(t)$.

4. What is the mass of the radio-isotope $^{131}_{53}\mathrm{I}$ corresponding to an activity of 1.85×10^8Bq?

Exercise 1.5: Dilution effects. Blood volume determination

The discovery of artificial radioactivity has allowed to associate to each chemical element a certain number of radio-isotopes that have the very same chemical properties as the stable element. These radio-isotopes are often used in medical applications.

1. One can produce sodium 24 by exposing stable sodium $^{23}_{11}\mathrm{Na}$ to a flux of neutrons. Write down the formation reaction of sodium 24.

2. Sodium 24 is radioactive through the emission of β^- and its period is 15h. Write down the equation for the decay of sodium 24.

3. One injects into the blood of an individual 10 cm^3 of a solution containing initially sodium 24 with a concentration of 10^{-3} mol.l^{-1}. What is the number of moles of sodium 24 that have been injected into the blood? How much of it will remain after 6h?

4. After 6h one takes a sample of 10 cm^3 from the blood of the same individual. One finds then 1.5×10^{-8} mol of sodium 24. Assuming sodium 24 is uniformly spread in the blood and that one can neglect the decrease due to biological elimination, compute the total blood volume of this individual.

Exercise 1.6: Carbone-14 dating

Carbone 14 is a β^- emitter. Its period is 5 570 years.
It is formed in the upper atmosphere in collisions between cosmic rays neutrons with nitrogen atoms ^{14}N in the air. It is assumed that the ratio of the radioactive isotope ^{14}C to that of the stable isotope ^{12}C (the ratio $^{14}C/^{12}C$) has remained constant in the atmosphere for the last 100 000 years.
Plants assimilate atmospheric carbon dioxide containing both isotopes ^{14}C and ^{12}C. During their life, plants (as well as animals feeding on them) maintain the ratio $^{14}C/^{12}C$ constant and identical to that prevailing in the atmosphere.
But when a plant or a living being dies, the exchange process with air stops as well as the carbon assimilation process. The ^{14}C content in the plant will slowly decrease with time because of the radioactive decay. In a dead plant the isotopic distribution between ^{14}C and ^{12}C evolves with time.

1. Write down the reactions:
   (a) of the formation of ^{14}C isotope from ^{14}N,
   (b) of the decay ^{14}C.

One of the methods for determining the age of prehistorical habitats (like the Lascaux or Chauvet caves), consists in measuring the residual radioactivity of wood samples found embedded in the different ground layers. For that purpose, one compares the activity of these samples with that of recent samples of same nature and same mass.

2. Give the expression for the variation of the activity of a wood sample as a function of time (a_0 = activity at the time of the death of the tree).

3. Compute the age of a piece of charcoal found in a prehistorical cave, given that the number of decays measured is 1.6 per minute and that it is 11.5 per minute for a recent sample of charcoal of same mass.

Hypothesis: one consider that the dating obtained from this method are reliable when the remaining activity a_s is still strong enough but differs sufficiently from the activity a_0 of the reference recent sample (itself supposed to be identical to the initial activity of the historical sample). One demands for example that a_s differs by more than 10% from a_0 and that $a_s > 0.1 \times a_0$.

4. Determine the time limits for which the carbon-14 dating is reliable.

Series 2

Preparing a radioactive tracer – nuclear filiations

Exercise 2.1: Filiations of two radio-isotopes, Production and decay of Technetium ^{99m}Tc

The technetium ^{99m}Tc (excited state of ^{99}Tc) is a γ emitter used in nuclear medicine to detect cervical tumors with a γ-camera:

$$^{99m}\text{Tc} \longrightarrow^{99} \text{Tc} + \gamma \tag{2}$$

The decay lifetime (period) T_2 of ^{99m}Tc is of 6 hours.
The ^{99m}Tc is itself a byproduct from the β^- decay of molybdenum ^{99}Mo, whose lifetime T_1 is 66.5 hours:

$$^{99}_{42}\text{Mo} \longrightarrow^{99m} \text{Tc} + e^- + \bar{\nu}_e \tag{1}$$

Indicate with an index 1 the radioactive parameters (period, radioactive constant, activity) relative to the ^{99}Mo element, producing the ^{99m}Tc via its decay, and those relative to technetium ^{99m}Tc with the index 2.

The nuclear cascade can be sketched in the following way::

$$\begin{array}{ccccc} ^{99}_{42}\text{Mo} & & ^{99m}\text{Tc} & & ^{99}\text{Tc} \\ (1) & \longrightarrow & (2) & \longrightarrow & (3) \\ & \lambda_1 & & \lambda_2 & \end{array}$$

1. Time variation of the number of nuclei $N_1(t)$ of ^{99}Mo and activity $a_1(t)$:
   - (a) Express the variation dN_1 of the number N_1 of nuclei (1) during a time interval dt as a function of N_1 and λ_1.
   - (b) Use this result to derive an expression for the variation da_1 of the activity a_1 (as a function of a_1 and λ_1).
   - (c) Find the variation of a_1 as a function of time (use $a_{1,0}$ as initial activity of the radio element in (1)).

2. Time variation of the number of nuclei $N_2(t)$ of ^{99m}Tc and activity $a_2(t)$:

(a) Express the variation dN_2 of the number N_2 of nuclei (2) during the time interval dt as a function of N_1, λ_1, N_2 and λ_2.

(b) Find the expression for da_2 as a function of a_2, a_1, λ_2 and rearrange it in the form of a differential equation:

$$K_1 \frac{da_2}{dt} + K_2 a_2 = f(t)$$

where K_1 and K_2 are two constants. (*This equation, called "equation 2b" in the following, is the differential equation that governs the evolution with time of the activity* a_2.)

(c) Verify[1] that the solution of the differential equation 2b (which is a first order differential equation with a non zero right hand term) is:

$$a_2(t) = a_{1,0} \frac{\lambda_2}{\lambda_2 - \lambda_1} \left(e^{-\lambda_1 t} - e^{-\lambda_2 t} \right) + a_{2,0} e^{-\lambda_2 t}$$

3. Plotting the activities $a_1(t)$ and $a_2(t)$
Lets assume that the initial activity of a ^{99}Mo source is $a_{1,0} = 8,5$ Ci. This source does not contain any technetium initially: $a_2(t = 0) = 0 = a_{2,0}$.

(a) Compute the activity a_1 of ^{99}Mo and the activity a_2 of ^{99m}Tc at the time $t = 10$h.

The activity a_2 of ^{99m}Tc reaches a maximum. Determine:

(b) the instant t_{max} for which this maximum is reached,

(c) the values $a_2(t_{max})$ and $a_1(t_{max})$.

Let us remind that a function reaches an extremum if its derivative equals to zero (knowing that the activity is zero at the initial time, it will first grow to reach a maximum and then decrease).

(d) Complete the following table:

t [h]	0	10	$t_{max} = \dots$	50	100
$a_1(t)$ [Ci]	8,5	...	...	5,1	3,0
$a_2(t)$ [Ci]	0	...	...	5,5	3,3

and plot on the same graph the variations of a_1 and a_2 with time.

4. For this source to be useful for nuclear medical applications, the activity a_2 must be larger than 5 Ci at the time of the medical exam. Estimate with the help of the curve in which time interval the source can be used.

[1] Verifying that $a_2(t)$ is a solution of equation 2b consist in showing that if one replaces a_2 et da_2/dt by their expressions in the left hand part of the equation, one finds back the right hand term $f(t)$.
Remark: the derivative of $g(t) = e^{\alpha t}$ with respect to t is $g'(t) = \alpha e^{\alpha t}$ if α is a constant.

Series 3

Interaction of radiation and particles with matter

Exercise 3.1: Photoelectric effect

1. Describe in a few lines the 3 main interaction processes of photons with matter and draw a schematic representation for each one.
2. What is the range in air of an α particle of 5.3 MeV and of a β particle of 2 MeV?; explain qualitatively this difference.

A photon has a wavelength $\lambda = 10^{-1}$nm.

3. Compute its energy in joules knowing that the Plank constant h= $6,62 \times 10^{-34}$ J.s

The photon ejects an atomic electron with a binding energy $We = 100$ eV via the photoelectric effect.

4. What is the kinetic energy (usually noted T by nuclear physicists) of the ejected electron?

Exercise 3.2: Parallel beam of photon attenuation by matter

Consider an X-ray beam with a flux of 10^5photons/second where each of these photons carries a kinetic energy of 100 keV.

1. What would be the flux after the beam had traversed a screen of lead of 1 mm thickness? The linear attenuation coefficient of lead μ is 50 cm^{-1} for X-ray radiations of 100 keV.
2. What is the energy of the emerging photons after having passed through the screen?
3. What is the definition of the half attenuation thickness $x_{\frac{1}{2}}$?
4. What is the relation between $x_{\frac{1}{2}}$ and the linear attenuation coefficient μ.
5. Given that for lead $x_{\frac{1}{2}} = 3$ mm for photons of 300 keV, what is necessary thickness of a screen of lead in order to absorb 95% of the photons of such a beam?
6. Why has the value of $x_{\frac{1}{2}}$ for lead changed between question 1 and question 5?

Consider a screen made of an alloy of copper and aluminum (70% in volume of Cu et 30% in volume of Al) which is inserted on the trajectory of a monochromatic beam of photons of energy $E = 100$ keV. For photons of such energy, the linear attenuation coefficient for copper is $\mu_{\mathrm{Cu}} = 4,5$ cm^{-1} and for Aluminum $\mu_{\mathrm{Al}} = 0,5$ cm^{-1}.

7. In which proportion does the photon flux diminish after having passed through a screen thickness of 1,4 cm ?

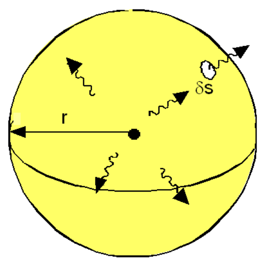
Exercise 3.3:

During the transition from an excited state to the ground state, the $^{60}_{27}$Co nucleus emits a photon γ of energy equal to 1,33 MeV.

1. If one assigns a zero energy to the ground state, what is the energy of the excited state?
2. What is the atomic mass of $^{60}_{27}$Co in this excited state?

Data: Atomic mass of $^{60}_{27}$Co in its ground state: 59.933820 a.m.u.

Exercise 3.4: Photons spatial dispersion



Consider a point like source emitting isotropically N particles per second. Lets assume that this source is placed at the center of a sphere of radius r.

1. How many particles per second exit through the total surface of the sphere?
2. How many particles exit through a little surface of area δS which is on the sphere?
3. Assuming δS represents the entrance surface of a counting detector, deduce the theoretical law describing the counting rate of that detector as a function of the distance r (distance source-detector).

Exercise 3.5: Attenuation et dispersion : Internal point like radioactive source

A point like radioactive source is placed at 2 cm depth inside a muscular tissue. It emits photons isotropically.

1. How many photons per second would exit from a sphere of muscles of 2 cm radius centered on the source if $x_{\frac{1}{2}} = 4$ cm and if the activity of the source corresponds to the emission of $N_0 = 37\,000$ photons s^{-1};
2. A detector with an entrance surface of 5 cm^2 is placed at 20 cm distance from the source. Compute the number of photons that reach the detector entrance each second. (Neglect the attenuation of photons in air).

Series 4

Effective period, dating techniques

Exercise 4.1: Physical period, biological period

Iodine $^{131}_{53}\mathrm{I}$ has a radioactive decay period (or physical period) $T_p = 8$ days. The initial activity of a radioactive source $^{131}_{53}\mathrm{I}$ is $a_0 = 400\mu\mathrm{Ci}$.

1. How long do you have to wait in order that the activity decreases to $100\mu\mathrm{Ci}$?
2. Give the expression for the time evolution of the activity of the source (as a function of T_p and a_0).

All the iodine present in this source is in fact injected into the thyroid of an individual. The decrease of the activity of the iodine contained inside the thyroid is the results of the radioactive (physical) decay but also of the biological elimination process which satisfies as well an exponential law characterized by a decay period T_b.
The effective decay period of iodine in the thyroid is then T.

1. Can you guess whether the effective T is larger or smaller than the physical period T_p ? Justify your answer.
2. Give the variation dN of the number of $^{131}_{53}\mathrm{I}$ nuclei during a time interval dt.
3. Give the expression (as a function of t, T_b, T_p and a_0), of the evolution of the activity of the iodine contained in the thyroid.
4. The iodine activity in the thyroid as measured 6 days after the injection is $200\mu\mathrm{Ci}$. From this result, deduce what is the value of the biological period T_b.

Exercise 4.2: Geological dating by the Potassium-Argon method

Volcanic rocks contain potassium among which one isotope, the $^{40}_{19}\mathrm{K}$, is radioactive. 10.72% of the $^{40}_{19}\mathrm{K}$ decays into $^{40}_{18}\mathrm{Ar}$ via electron capture. The rest of the potassium $^{40}_{19}\mathrm{K}$ undergoes a β^- decay into $^{40}_{20}\mathrm{Ca}$. The period of the $^{40}_{19}\mathrm{K}$ resulting from these two decay processes is $T = 1.3 \cdot 10^9$ years. Assume that the nuclear masses of $^{40}_{19}\mathrm{K}$ and $^{40}_{18}\mathrm{Ar}$ are equal.

1. Explain the mechanism of electron capture.

2. Write down the capture reaction undergone by the ${}^{40}_{19}\mathrm{K}$ and producing ${}^{40}_{18}\mathrm{Ar}$.

When a volcanic eruption occurs the lava decompression releases most of the gases it contains including the ${}^{40}_{18}\mathrm{Ar}$. At the time of the eruption the lava is free from argon, which slowly reappears with time due to the capture mechanism described above.
The analysis of a sample of basalt of 1 kg mass shows that it contains 1.4900 mg ${}^{40}_{19}\mathrm{K}$ and 0.0218 mg of ${}^{40}_{18}\mathrm{Ar}$.

3. Write down the evolution law for the mass of the ${}^{40}_{19}\mathrm{K}$ as a function of time, and deduce from it the evolution law governing the argon mass. We will call $m_K(0)$ the initial mass of potassium.

4. What was the total mass of ${}^{40}_{19}\mathrm{K}$ per kg of basalt at the time of the volcanic eruption?

5. Can you find the approximate date of the eruption?

Series 5

Radioprotection

Exercise 5.1: Dosimetry

A biological tissue is irradiated using a beam of photons with energy 500 keV (gamma-rays). The quality factor for this type of radiation is equal to 1. The irradiation dose received by the tissue is equal to 10 S.I. (international system of units).

1. What is the S.I. unit for doses?
2. What other (older) unit is still frequently in use and what is its relation to the S.I. one?
3. What is the energy absorbed by 1 g of this tissue?

Exercise 5.2:

During a radiotherapy session to treat a prostate cancer a patient receives locally a dose of 2 Gray which is deposited by photons.

1. Assuming that the prostate can be identified with a sphere of 2.5 cm radius with a density equal to that of liquid water, compute the temperature elevation of the prostate (using the thermal capacity of liquid water which is 4180 $J.kg^{-1}$).
2. What is the physical meaning of what is called the equivalent dose?
3. Compute the equivalent dose received by the tissue.

Exercise 5.3: Dosimetry

An organ of 1.2 kg is being irradiated during 10 minutes. Each second, it receives 10^3 photons of energy $E = 1$ MeV. Only half of these photons are absorbed and contribute to the energy deposition. Compute the dose D absorbed by the irradiated organ.