QCD and collider physics - Problem sheet

Problem 1 : Color factor

Determine the ratio of the cross sections $\sigma(\gamma + g \to c + \bar{c})/\sigma(\gamma + \gamma \to c + \bar{c})$ at leading order.
a) Draw first the Feynman diagrams indicating all relevant information (couplings, vertices, colors).
b) Discuss the color factor associated to each of these two cross sections.
c) Determine the ratio and estimate its numerical value using $\alpha_s = 0.2$, $\alpha = 1/137$.

Problem 2 : Gluon propagator in an axial gauge

a) Write down the vertex function V_{GG} and the gluon propagator in a covariant gauge, i.e., with a gauge fixing term $\mathcal{L}_{GF} = -\frac{1}{2\xi}(\partial^\mu G^a_\mu)^2$.

b) An axial gauge is defined by $\mathcal{L}_{GF} = -\frac{1}{2\xi}(n^\mu G^a_\mu)^2$ with an arbitrary 4-vector n^μ. Derive the vertex function V_{GG} and the gluon propagator in this gauge.
Hint : The propagator has the general form $i\tilde{D}^{ab}_{\mu\nu} = P_{\mu\nu} = Ag_{\mu\nu} + Bp_\mu p_\nu + Cn_\mu n_\nu + D(p_\mu n_\nu + p_\nu n_\mu)$. Determine the coefficients A, B, C, D so that $P_{\mu\nu}V^{\nu\rho}_{GG} = -\delta^\rho_\mu$.

c) In order to understand the interest of such a gauge consider the special case of a light cone gauge ($\xi = 0, n^2 = 0$). The propagator takes the form

$$i\tilde{D}^{ab}_{\mu\nu}(p) = \delta^{ab}\frac{i}{p^2}\mathrm{d}_{\mu\nu}\,.$$

Show that $d_{\mu\nu} = -g_{\mu\nu} + \frac{n_\mu p_\nu + p_\mu n_\nu}{n\cdot p}$ and that in the limit $p^2 \to 0$

$$n^\mu d_{\mu\nu} = 0\,,\; p^\mu d_{\mu\nu} = 0\,.$$

In this limit, the numerator of the propagator can therefore be decomposed in a sum over two polarisations :

$$d_{\mu\nu} = \sum_{i=1}^{2} \epsilon^{(i)}_\mu(p)\epsilon^{(i)}_\nu(p)$$

with $p^\mu\epsilon^{(i)}_\mu(p) = 0$ and $n^\mu\epsilon^{(i)}_\mu(p) = 0$.
In conclusion, only the two physical polarization states of the gluon propagate.

d) Specify the Faddeev-Popov Lagrangian $\mathcal{L}_{FP}$ in this gauge.