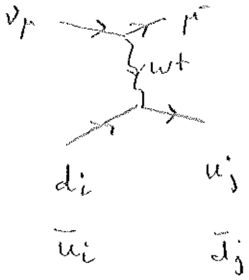
Ex. 1 : Règles de somme

a) $\nu_\mu + p \to \mu^- + X$ @LO , V^{CKM} supposé diagonal



Processus partoniques : $W^+ + d \to u \quad \sim |V_{ud}|^2 \simeq 1$

$\Big[\; W^+ + d \to c \quad \sim |V_{cd}|^2 \simeq 0$

$W^+ + d \to t$ (seulement si $(\hat{p}+q)^2 \geq m_t^2$) $\sim |V_{td}|^2 \simeq 0 \;\Big]$

$W^+ + s \to c$

$\Big[\; W^+ + s \to u \;,\; W^+ + s \to t \;\Big]$

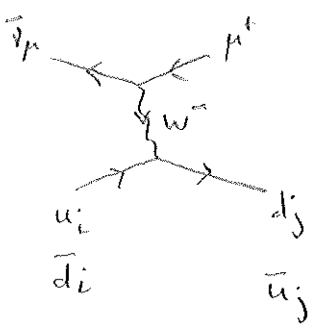
$W^+ + \bar{u} \to \bar{d}$

$\Big[\; W^+ + \bar{u} \to \bar{s} \;,\; W^+ + \bar{u} \to \bar{b} \;\Big]$

$W^+ + \bar{c} \to \bar{s}$

$\Big[\; W^+ + \bar{c} \to \bar{d} \;,\; W^+ + \bar{c} \to \bar{b} \;\Big]$

$\bar{\nu}_\mu + p \to \mu^+ + X$



Processus ($V^{CKM} = \mathbb{1}$) :

$W^- + u \to d \;,\; W^- + c \to s$

$W^- + \bar{d} \to \bar{u} \;,\; W^- + \bar{s} \to \bar{c}$

b)

Isospin : $u^n = d^p \equiv d$, $d^n = u^p \equiv u$

$\bar{u}^n = \bar{d}^p \equiv \bar{d}$, $\bar{d}^n = \bar{u}^p \equiv \bar{u}$

$\underbrace{s^n = s^p}_{\equiv s}$, $\underbrace{\bar{s}^n = \bar{s}^p}_{\equiv \bar{s}}$, $\underbrace{c^n = c^p}_{\equiv c}$, $\bar{c}^n = \bar{c}^p \equiv \bar{c}$

$$\Rightarrow \quad F_2^{en} = x\left[\frac{4}{9}(u^n+\bar{u}^n+c^n+\bar{c}^n) + \frac{1}{9}(d^n+\bar{d}^n+s^n+\bar{s}^n)\right]$$

$$= x\left[\frac{4}{9}(d+\bar{d}+c+\bar{c}) + \frac{1}{9}(u+\bar{u}+s+\bar{s})\right]$$

$$F_2^{\nu n} = 2x\left[u+s+\bar{d}+\bar{c}\right]$$

c)

(i) $$\frac{F_2^{\nu n} - F_2^{\nu p}}{2x} = u+s+\bar{d}+\bar{c} - (d+s+\bar{u}+\bar{c}) = u_V - d_V$$

avec $u_V \equiv u-\bar{u}$, $d_V \equiv d-\bar{d}$

$$\Rightarrow \quad I_A = u_V(n=1) - d_V(n=1) = 2-1 = \underline{\underline{1}}$$

avec $f(n) \equiv \int_0^1 dx\, x^{n-1} f(x)$ la transformée de Mellin

$u_V(1) = \int_0^1 dx\, u_V(x) = 2$, $d_V(1) = \int_0^1 dx\, d_V(x) = 1$

(ii) $$\frac{F_2^{\bar{\nu}p} - F_2^{\nu p}}{2x} = u_V - d_V + c_V - s_V$$, avec $c_V \equiv c-\bar{c}$, $s_V \equiv s-\bar{s}$

$$\Rightarrow \quad I_B = u_V(1) - d_V(1) + c_V(1) - s_V(1)$$

$$= 2 - 1 + 0 - 0 = \underline{\underline{1}}$$

(iii) $$F_3^{\nu p} + F_3^{\bar{\nu}p} = 2(d_V+s_V+u_V+c_V)$$

$$\Rightarrow \quad I_{GLS} = 2\Big(\underbrace{d_V(1)}_{=1} + \underbrace{s_V(1)}_{=0} + \underbrace{u_V(1)}_{=2} + \underbrace{c_V(1)}_{=0}\Big) = \underline{\underline{6}}$$

(iv)

$$\frac{F_2^{ep} - F_2^{en}}{x} = \frac{4}{9}\left(u+\bar{u}+c+\bar{c}\right) + \frac{1}{9}\left(d+\bar{d}+s+\bar{s}\right)$$

$$-\frac{4}{9}\left(d+\bar{d}+c+\bar{c}\right) - \frac{1}{9}\left(u+\bar{u}+s+\bar{s}\right)$$

$$= \frac{1}{3}\left(u+\bar{u}-d-\bar{d}\right) \qquad ; \quad u = u_V + \bar{u} \ , \ d = d_V + \bar{d}$$

$$= \frac{1}{3}\left(u_V - d_V\right) + \frac{2}{3}\left(\bar{u}-\bar{d}\right)$$

$$\Rightarrow I_G = \frac{1}{3}\left(\underbrace{u_V(1)}_{2} - \underbrace{d_V(1)}_{1}\right) + \frac{2}{3}\left(\bar{u}-\bar{d}\right)(1)$$

$$= \frac{1}{3} + \frac{2}{3}\int_0^1 dx\, (\bar{u}-\bar{d})(x)$$

$$= \frac{1}{3} \qquad \text{si} \quad \bar{u}(x) = \bar{d}(x) \quad \text{ou} \quad \bar{u}(1) = \bar{d}(1)$$

(symétrie $SU(2)_I$, Isospin fort)

d)

$$I_G^{exp} = 0.235 \pm 0.026 \qquad \Rightarrow \qquad \frac{2}{3}\left(\bar{u}-\bar{d}\right)(1) \simeq -0.1$$

$\Rightarrow$ $\exists$ une asymétrie considérable dans la mer des quarks légers.

e)

$$\frac{\frac{5}{18} F_2^{\nu N} - F_2^{eN}}{x} = \frac{5}{18}\,\frac{1}{2}\left(F_2^{\nu p} + F_2^{\nu n}\right) - \frac{1}{2}\left(F_2^{ep} + F_2^{en}\right)$$

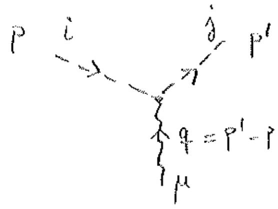
$$= \frac{5}{18}\left(\cancel{d}+s+\cancel{\bar{u}}+\bar{c} + \cancel{u}+s+\cancel{\bar{d}}+\bar{c}\right) - \frac{4}{18}\left(\cancel{u+\bar{u}}+c+\bar{c} + \cancel{d+\bar{d}}+c+\bar{c}\right)$$

$$-\frac{1}{18}\left(\cancel{d+\bar{d}}+s+\bar{s} + \cancel{u+\bar{u}} + s+\bar{s}\right)$$

$$= \frac{5}{9}s + \frac{5}{9}\bar{c} - \frac{1}{9}(s+\bar{s}) - \frac{4}{5}(c+\bar{c})$$

$$= \frac{1}{9}\left(4s - \bar{s} + \bar{c} - 4c\right)$$

Ex. 2 : spin des partons



$$= -i\, e\, e_q\, (p' + p)^\mu\, \delta_{ij}$$

a) Projecteurs:

$$g^{\mu\nu} g^{\perp}_{\mu\nu} = 4 - 1 = 3 \quad , \quad g^{\mu\nu} p^{\perp}_{\mu} p^{\perp}_{\nu} = \left(p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)\left(p^\mu - \frac{p\cdot q}{q^2} q^\mu\right) \quad ; \; p^2 = M_{prot}^2 \to 0$$

$$= -2\frac{p\cdot q}{q^2} p\cdot q + \frac{p\cdot q}{q^2} p\cdot q = -\frac{(p\cdot q)^2}{q^2}$$

$$\underset{\substack{\uparrow \\ x = \frac{Q^2}{2p\cdot q}}}{=} -\left(\frac{Q^2}{2x}\right)^2 \frac{1}{q^2} = \frac{Q^2}{4x^2}$$

$$\Rightarrow g^{\mu\nu} W_{\mu\nu} = -3F_1 + \frac{Q^2}{4x^2}\frac{2x}{Q^2} F_2 = -3F_1 + \frac{1}{2x} F_2 = \frac{1}{2x}\left(F_2 - 6x F_1\right)$$

$$p^\mu p^\nu g^{\perp}_{\mu\nu} = -\frac{(p\cdot q)^2}{q^2} = \frac{Q^2}{4x^2}$$

$$p^\mu p^\nu p^{\perp}_\mu p^{\perp}_\nu = \left(-\frac{(p\cdot q)^2}{q^2}\right)^2 = \left(\frac{Q^2}{4x^2}\right)^2 \qquad ; \quad \frac{1}{p\cdot q} = \frac{2x}{Q^2}$$

$$\Rightarrow p^\mu p^\nu W_{\mu\nu} = -\frac{Q^2}{4x^2} F_1 + \left(\frac{Q^2}{4x^2}\right)^2 \frac{2x}{Q^2} F_2$$

$$= \frac{Q^2}{4x^2}\left(-F_1 + \frac{1}{2x} F_2\right) = \frac{Q^2}{4x^2}\frac{1}{2x}\left(F_2 - 2x F_1\right)$$

$$\Rightarrow \left(\frac{Q^2}{4x^2} g^{\mu\nu} - p^\mu p^\nu\right) W_{\mu\nu} = \frac{Q^2}{4x^2}\frac{1}{2x}\Big(\underbrace{-6x F_1 + 2x F_1}_{-4x F_1}\Big) = -\frac{Q^2}{2x^2} F_1$$

$$\Rightarrow \boxed{P_1^{\mu\nu} = -\frac{1}{2} g^{\mu\nu} + \frac{2x^2}{Q^2} p^\mu p^\nu}$$

$$\left(\frac{Q^2}{4x^2} g^{\mu\nu} - 3p^\mu p^\nu\right) W_{\mu\nu} = \frac{Q^2}{4x^2}\frac{1}{2x}\Big(\underbrace{F_2 - 3F_2}_{= -2F_2}\Big) = -\frac{Q^2}{4x^2}\frac{1}{x} F_2$$

$$\Rightarrow \boxed{P_2^{\mu\nu} = -x\, g^{\mu\nu} + \frac{12\, x^3}{Q^2} p^\mu p^\nu}$$

b)

$$\frac{d^3p'}{2E'} = \int d^4p' \, \delta(p'^2 - m^2)\, \Theta(E') \quad ; \quad \delta(p'^2-m^2)\Theta(E') \equiv \delta_+(p'^2-m^2)$$

$$dPS = (2\pi)^4 \delta^{(4)}(\hat{p}+q-p') \frac{d^3p'}{(2\pi)^3 2E'} = 2\pi \, \underline{d^4p'} \, \delta_+(p'^2-m^2)\, \delta^{(4)}(\hat{p}+q-p')$$

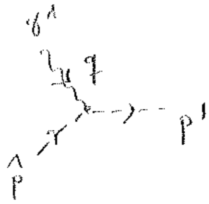
$$= 2\pi \, \delta_+((\hat{p}+q)^2 - m^2) \quad , \; m=0 \quad , \; \hat{p} = \xi p$$

$$= 2\pi \, \delta_+(q^2 + \xi \underbrace{2p\cdot q}_{=\frac{Q^2}{x}})$$

$$= 2\pi \, \delta_+\left(Q^2 \frac{\xi}{x} - Q^2\right) = \frac{2\pi}{Q^2} \delta_+\left(\frac{\xi}{x} - 1\right) = \frac{2\pi}{Q^2} \times \delta(\xi - x)$$

c)

$$M_\mu = -i e \, e_q (p' + \hat{p})_\mu \quad , \; q = p' - \hat{p}$$



$$\Rightarrow \quad M_\mu M_\nu^* = e^2 e_q^2 (p'+\hat{p})_\mu (p'+\hat{p})_\nu$$

$$\overline{M_\mu M_\nu^*} = \underbrace{\frac{1}{3} N_c}_{=1} M_\mu M_\nu^* = e^2 e_q^2 \underbrace{(p'+\hat{p})_\mu}_{q+2\hat{p}} \underbrace{(p'+\hat{p})_\nu}_{q+2\hat{p}} = 4 e^2 e_q^2 \left(\hat{p} + \frac{q}{2}\right)_\mu \left(\hat{p}+\frac{q}{2}\right)_\nu$$

scalaire

$$\underset{\hat{p}=\xi p}{=} 4 e^2 e_q^2 \xi^2 \left(p + \frac{q}{2\xi}\right)_\mu \left(p + \frac{q}{2\xi}\right)_\nu$$

$$\Rightarrow \quad \hat{W}_{\mu\nu} = \frac{1}{\pi} e_q^2 \xi^2 \left(p + \frac{q}{2\xi}\right)_\mu \left(p + \frac{q}{2\xi}\right)_\nu \frac{2\pi}{Q^2} \times \delta(\xi - x)$$

$$= \frac{2}{Q^2} e_q^2 x^3 \left(p + \frac{q}{2x}\right)_\mu \left(p + \frac{q}{2x}\right)_\nu \delta(\xi - x) \quad ; \quad x = \frac{-q^2}{2pq} \Rightarrow \frac{1}{2x} = -\frac{pq}{q^2}$$

$$= \frac{2}{Q^2} e_q^2 x^3 \left(p - \frac{p\cdot q}{q^2} q\right)_\mu \left(p - \frac{p\cdot q}{q^2} q\right)_\nu \delta(\xi - x)$$

$$= \underline{\underline{\frac{2}{Q^2} e_q^2 x^3 \, p^\perp_\mu \, p^\perp_\nu \, \delta(\xi - x)}}$$

d)

$$W_{\mu\nu} = \int \frac{d\xi}{\xi} \, (q+\bar{q})(\xi) \, \hat{\omega}_{\mu\nu} \big|_{\hat{p}=\xi p}$$

$$= \underline{\frac{2}{Q^2} \, e_q^2 \, x^2 \, (q+\bar{q})(x) \, p_\mu^\perp p_\nu^\perp}$$

e)

$$\Rightarrow \boxed{F_1(x, Q^2) = 0}$$

$$\frac{1}{p\cdot q} F_2 = \frac{2}{Q^2} e_q^2 x^2 (q+\bar{q})(x) \qquad \frac{1}{p\cdot q} = \frac{2x}{Q^2}$$

$$\Rightarrow \boxed{F_2 = x \, e_q^2 \, (q+\bar{q})(x)}$$ (somme sur q sous-entendu)

Cas du spin $\frac{1}{2}$:

Même F_2 : $F_2 = \sum_q x \, e_q^2 (q+\bar{q})(x)$

$F_2 = F_T + F_L$, $F_T = 2x F_1$

- spin-0 : $F_T = 0$, $F_2 = F_L$
- spin-$1/2$: $F_L = 0$, $F_2 = F_T$ (Callan-Gross)

f)

$$dPS = 2\pi \, \delta_+\left(q^2 + \xi \frac{Q^2}{x} - m^2\right) = \frac{2\pi}{Q^2} \delta_+\left(\frac{\xi}{x} - 1 - \frac{m^2}{Q^2}\right)$$

$$= \underline{\frac{2\pi}{Q^2} \, x \, \delta_+\left(\xi - x\left(1 + \frac{m^2}{Q^2}\right)\right)} \qquad ; \ \chi := x\left(1 + \frac{m^2}{Q^2}\right)$$

$$\Rightarrow \boxed{\frac{F_2(x, Q^2)}{x} = e_q^2 \, (q+\bar{q})(\chi, Q^2)}$$ "slow-rescaling"

(aussi : $F_1 \neq 0$ (spin-0) ; $F_L \neq 0$ (spin-$1/2$))

Ex. 3 : α_s à NLO

a)

$$\frac{da_s}{d\ln Q^2} = -\beta_0 a_s^2 - \beta_1 a_s^3 = \beta(a_s)$$

b)

$$\frac{da_s}{\beta(a_s)} = \frac{da_s}{\beta_0 a_s^2 (1+\frac{\beta_1}{\beta_0} a_s)} = d\ln Q^2 \quad ; \quad \frac{1}{x^2(1+cx)} = \frac{Ax+B}{x^2} + \frac{C}{1+cx} = \frac{B + x(A+Bc) + x^2(Ac+C)}{x^2(1+cx)}$$

$$\Rightarrow B = 1 \,;\, A = -c \,;\, C = c^2$$

$$= -\frac{1}{\beta_0} da_s \left(\frac{1}{a_s^2} \left(-\frac{\beta_1}{\beta_0} a_s + 1 \right) + \frac{1}{1+\frac{\beta_1}{\beta_0} a_s} \left(\frac{\beta_1}{\beta_0} \right)^2 \right)$$

$$= -\frac{1}{\beta_0} da_s \left(\frac{1}{a_s^2} - \frac{\beta_1}{\beta_0 a_s} + \left(\frac{\beta_1}{\beta_0} \right)^2 \frac{1}{1+\frac{\beta_1}{\beta_0} a_s} \right)$$

$$= \frac{1}{\beta_0} \left(\frac{1}{a_s} \Big|_{a_s(Q_0^2)}^{a_s(Q^2)} + \frac{\beta_1}{\beta_0} \ln a_s \Big| - \underbrace{\frac{\beta_1}{\beta_0} \ln (1 + \frac{\beta_1}{\beta_0} a_s) \Big|}_{\simeq \left(\frac{\beta_1}{\beta_0}\right)^2 a_s \;\; ; \;\; \ln(1+\epsilon) \simeq \epsilon} \right) = \ln \frac{Q^2}{Q_0^2}$$

(négligeable à NLO)

$$\Rightarrow \boxed{\frac{1}{\beta_0} \left(\frac{1}{a_s(Q^2)} - \frac{1}{a_s(Q_0^2)} \right) + \frac{\beta_1}{\beta_0^2} \left(\ln a_s(Q^2) - \ln a_s(Q_0^2) \right) = \ln \frac{Q^2}{Q_0^2}}$$

c)

$$\ln \frac{Q^2}{Q_0^2} = \ln \frac{Q^2}{\Lambda^2} + \ln \frac{\Lambda^2}{Q_0^2} \quad ; \quad \Lambda \text{ tel que : } \frac{1}{\beta_0} \frac{1}{a_s(Q_0^2)} + \frac{\beta_1}{\beta_0^2} \ln a_s(Q_0^2) = \ln \frac{Q_0^2}{\Lambda^2}$$

$$\Rightarrow \boxed{\frac{1}{\beta_0} \frac{1}{a_s(Q^2)} + \frac{\beta_1}{\beta_0^2} \ln a_s(Q^2) = \ln \frac{Q^2}{\Lambda^2}}$$

d)

$$\frac{1}{\beta_0}\frac{1}{a_s(Q^2)} + \frac{\beta_1}{\beta_0^2}\ln a_s(Q^2) = \ln\frac{Q^2}{\Lambda^2} \qquad ; \qquad L_\Lambda := \ln\frac{Q^2}{\Lambda^2}$$

$$\ln a_s(Q^2) \simeq -\ln(\beta_0 L_\Lambda)$$
$$\simeq -\ln L_\Lambda$$

$$\Rightarrow \quad \frac{1}{\beta_0}\frac{1}{a_s(Q^2)} \simeq L_\Lambda + \frac{\beta_1}{\beta_0^2}\ln L_\Lambda$$

$$\Rightarrow \quad a_s(Q^2) \simeq \left[\beta_0 L_\Lambda + \frac{\beta_1}{\beta_0}\ln L_\Lambda\right]^{-1}$$
$$= \left[\beta_0 L_\Lambda\left(1 + \frac{\beta_1}{\beta_0^2}\frac{\ln L_\Lambda}{L_\Lambda}\right)\right]^{-1}$$
$$\simeq \frac{1}{\beta_0 L_\Lambda}\left(1 - \frac{\beta_1}{\beta_0^2}\frac{\ln L_\Lambda}{L_\Lambda}\right)$$