2.)

Identité de Gordon :

Avec :

Dirac : $\not{p}\, u(p) = M\, u(p)$

$\bar{u}(p')\, \not{p}' = \bar{u}(p')\, M$

$\gamma^\mu \not{p}' = - \not{p}' \gamma^\mu + 2 p'^\mu$

$\not{p}\, \gamma^\mu = - \gamma^\mu \not{p} + 2 p^\mu$

$$u(p')\, \sigma_{\mu\nu}\, (p \pm p')^\nu\, u(p)$$

$$= \frac{i}{2}\, \bar{u}' \left[\gamma_\mu\, (\not{p} \pm \not{p}') - (\not{p} \pm \not{p}')\, \gamma_\mu \right] u$$

$$= \frac{i}{2}\, \bar{u}' \left[\gamma_\mu\, (M \pm \not{p}') - (\not{p} \pm M)\, \gamma_\mu \right] u$$

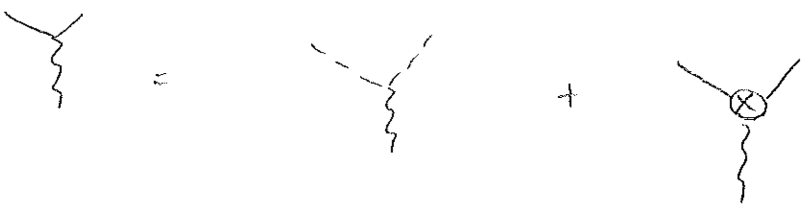
$$= \frac{i}{2}\, \bar{u}' \left[(M \mp M)\, \gamma_\mu \pm (2 p'_\mu - \not{p}' \gamma_\mu) - 2 p_\mu + \gamma_\mu \not{p} \right] u$$

$$= \begin{cases} \frac{i}{2}\, \bar{u}' \left[2 p'_\mu - \cancel{M \gamma_\mu} - 2 p_\mu + \cancel{M \gamma_\mu} \right] u \\ \frac{i}{2}\, \bar{u}' \left[2M \gamma_\mu - 2 p'_\mu + M \gamma_\mu - 2 p_\mu + M \gamma_\mu \right] u \end{cases}$$

$$= \begin{cases} i\, \bar{u}'\, (p' - p)_\mu\, u \\ i\, \bar{u}' \left[2M \gamma_\mu - (p' + p)_\mu \right] u \end{cases} \qquad (*)$$

En particulier :

$$\bar{u}'\, \gamma_\mu\, u = \frac{1}{2M}\, \bar{u}' \left[(p + p')_\mu + i\, \sigma_{\mu\nu}\, q^\nu \right] u$$



$$\left.\begin{aligned} \bar{u}'\, \sigma_{\mu\nu} P^\nu u &= i\, \bar{u}'\, q_\mu u \\ \bar{u}'\, \sigma_{\mu\nu} q^\nu u &= i\, \bar{u}' \left[P_\mu - 2M \gamma_\mu \right] u \end{aligned} \right\} (*)$$

$$\Gamma_\mu = A\, q_\mu + B\, P_\mu + C \gamma_\mu + D\, i \sigma_{\mu\nu} q^\nu + E\, i \sigma_{\mu\nu} P^\nu$$

$$\hat{=} \quad (A - E)\, q_\mu + B\, P_\mu + C \gamma_\mu + (B + D')\, i \sigma_{\mu\nu} q^\nu$$

$$\hat{=} \quad (A - E)\, q_\mu + \underbrace{C \gamma_\mu + B 2M \gamma_\mu}_{(C + 2MB)\gamma_\mu} + D'\, i \sigma_{\mu\nu} q^\nu$$

$$\hat{=} \quad A'\, q_\mu + C' \gamma_\mu + D'\, i \sigma_{\mu\nu} q^\nu$$

Conservation du courant : $\partial_\mu j^\mu(x) = 0$

$$0 = \partial_\mu \langle p' | \hat{j}^\mu(x) | p \rangle = \partial_\mu \langle p' | e^{i\hat{P}x}\, \hat{j}^\mu(0)\, e^{-i\hat{P}x} | p \rangle$$

$$= \partial_\mu e^{i(p'-p)x} \langle p' | \hat{j}^\mu(0) | p \rangle = i q_\mu \langle p' | \hat{j}^\mu(0) | p \rangle e^{iqx}$$

$$\Rightarrow \quad q_\mu \langle p' | \hat{j}^\mu(0) | p \rangle = 0 \quad \Rightarrow \quad q_\mu \bar{u}' \Gamma^\mu u = 0$$

$$\bar{u}' q_\mu \Gamma^\mu u = \bar{u}' \Big(A' q^2 + C' \underbrace{\not{q}}_{\text{Dirac } M - M = 0} + D' \underbrace{i \underbrace{\sigma_{\mu\nu}}_{A} \underbrace{q^\nu q^\mu}_{S}}_{= 0} \Big) u$$

$\Rightarrow \quad A' = 0$ car $q^2 \neq 0$ en général

$$\Rightarrow \quad \underline{\underline{\Gamma_\mu \hat{=} C' \gamma_\mu + D'\, i \sigma_{\mu\nu} q^\nu}}$$