Resumé :

$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim (3,2)_{1/3}$ $\quad L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (1,2)_{-1}$ $\quad \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (1,2)_1$ $\quad B_\mu \sim (1,1)_0$

$u_R \sim (3,1)_{-4/3}$ $\quad e_R \sim (1,1)_{-2}$ $\quad W_\mu^i \sim (1,3)_0$

$d_R \sim (3,1)_{-2/3}$ $\quad \nu_R \sim (1,1)_0$ (?) $\quad G_\mu^a \sim (8,1)_0$

$$Q_{e.m.} = T_3 + \frac{Y}{2}$$

$$u_L = P_L u \,,\; u_R = P_R u \,,\; P_L = \frac{1-\gamma_5}{2} \,,\; P_R = \frac{1+\gamma_5}{2} \quad ; \quad u = \begin{pmatrix} u_L \\ u_R \end{pmatrix}$$

Indices :

$Q_{f\,i\,\beta\,\alpha}$

$SU(2)_L$ $\beta = 1,2$

$f = 1,2,3$ génération

$SU(3)_c$ $(i = 1,2,3)$

$\alpha = 1,2,(3,4)$ spineur de Dirac (ou $\alpha = 1,2$: spineur de Weyl)

$u_{L_3} = u_{L_4} = 0 \;;\; u_L = \begin{pmatrix} \chi_\alpha \\ 0 \end{pmatrix} \,,\; \alpha = 1,2$

$u_R = \begin{pmatrix} 0 \\ \bar{\xi}^{\dot\alpha} \end{pmatrix} \,,\; \dot\alpha = 1,2$

$$(u_R)_{f\,i\,\dot\alpha} \,,\; (d_R)_{f\,i\,\dot\alpha}$$

$$L_{f\,\beta\,\alpha} \,,\; (e_R)_{f\,\dot\alpha}$$

$$\mathcal{L} = \mathcal{L}_{Gauge} + \mathcal{L}_{Matière} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

$$\mathcal{L}_{Gauge} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} - \frac{1}{4} W^i_{\mu\nu} W^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$\mathcal{L}_{Mat} = \bar{Q}\, i\not{D}\, Q + \bar{u}_R\, i\not{D}\, u_R + \bar{d}_R\, i\not{D}\, d_R + \bar{L}\, i\not{D}\, L + \bar{e}_R\, i\not{D}\, e_R$$
$$\left(+ \bar{\nu}_R\, i\underbrace{\not{D}}_{=\not\partial}\, \nu_R\right)$$

$$D_\mu Q = \left(\partial_\mu + i g_s \frac{\lambda_a}{2} G_\mu^a + i g \frac{\tau^i}{2} W_\mu^i + i g' \underbrace{\frac{Y}{2}}_{Y/2 = 1/6 \text{ pour } Q} B_\mu\right) Q$$

$$\mathcal{L}_{Higgs} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) \quad \text{avec } V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

$$D_\mu \Phi = \left(\partial_\mu + i g \frac{\tau^i}{2} W_\mu^i + i g' \frac{Y}{2} B_\mu\right) \Phi \,,\quad Y/2 = 1/2 \,,\quad \Phi_\alpha = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$$\mathcal{L}_{Yuk} = -\Gamma_u^{ff'} \bar{Q}^f \tilde{\Phi}\, u_R^{f'} - \Gamma_d^{ff'} \bar{Q}^f \Phi\, d_R^{f'} - \Gamma_\ell^{ff'} \bar{L}^f \Phi\, e_R^{f'} \quad \left(-\Gamma_N^{ff'} \bar{L}^f \tilde{\Phi}\, \nu_R^{f'}\right)$$
$$+ h.c.$$

$$\tilde{\Phi} = \epsilon\, \Phi^* \,,\quad \tilde{\Phi}^{\dot\alpha} = \begin{pmatrix} \phi^0 \\ -\phi^- \end{pmatrix}$$

Mécanisme de Higgs

Lit. : Chaps. 11.1 – 11.4, Maggiore

Brisure spontanée de la symétrie (SSB)

* $SU(2)_L \times U(1)_Y \xrightarrow{SSB} U(1)_{em}$
* $M_W, M_Z \neq 0$

Higgs field $H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix} \sim 2_{SU(2)}$, $h^+(x), h^0(x)$ complexe, scalaire

↑ repr. fond.

$H^\dagger H$ inv. sous $H \to UH$, U unitaire

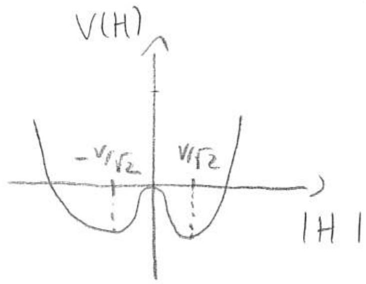
$\Rightarrow$ une fonction $f(H^\dagger H)$ est inv. de jauge

$$\mathcal{L}_{SU(2)-Higgs} = (D_\mu H)^\dagger (D^\mu H) - V(H^\dagger H)$$

$$V(H^\dagger H) = \mu^2 H^\dagger H + \lambda (H^\dagger H)^2 + cte$$

Vacuum = État fond. = Minimum de V

* Si $\mu^2 > 0$ (part. massive) : $H_{min} = 0$ pas de brisure de symétrie
* Si $\mu^2 < 0$
  chapeau Mexicain



$H = |H| e^{i\varphi}$

$V(|H|) = \mu^2 |H|^2 + \lambda |H|^4$

$V'(|H|) \overset{!}{=} 0 \Leftrightarrow 2\mu^2 |H| + 4\lambda |H|^3 = 0$

$\Leftrightarrow |H| = 0 \vee |H|^2 = \frac{-\mu^2}{2\lambda}$

$|H|_{min} = \pm \frac{v}{\sqrt{2}} = \pm \sqrt{\frac{-\mu^2}{2\lambda}}$

$\langle \varphi \rangle_{min}$ arbitrary

$$H_{min} = \pm \frac{v}{\sqrt{2}} = \pm \sqrt{\frac{-\mu^2}{2\lambda}}$$, $H = 0$ n'est pas stable

En choisissant le minimum, par ex., $H_{min} = + \frac{v}{\sqrt{2}}$
donne le vide une direction préférée dans l'espace d'isospin
$\longrightarrow$ SSB

Perturbations autour du minimum

$$H = \begin{pmatrix} \xi_1 + i\,\xi_2 \\ \xi_3 + i\,\xi_4 \end{pmatrix} \overset{\text{sans restr.}}{=} e^{i\,\Theta^k(x)\,\frac{\tau^k}{2}} \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix} \Rightarrow \langle |H| \rangle = \frac{v}{\sqrt{2}}$$

($h(x)$ real scalar field)

$$\xi_1(x),\, \xi_2(x),\, \xi_3(x),\, \xi_4(x) \longrightarrow \underbrace{\Theta^1(x),\, \Theta^2(x),\, \Theta^3(x)}_{\text{Bosons de Goldstone; } m_\Theta = 0\text{, rotation dans la vallée}} \;;\; \underbrace{h(x)}_{\text{Boson de Higgs; fluctuations radiales; } \Rightarrow m_h \neq 0}$$

$\Theta^{1,2,3}(x)$ vont être mangé par W^+, W^-, Z (peuvent être éliminé grace à une transf. de jauge pour une symétrie locale)

* Degre de liberté long. de W^+, W^-, Z
* Rend W^+, W^-, Z massiV

Jauge unitaire $\quad H \longrightarrow e^{-i\,\Theta^k(x)\,\frac{\tau^k}{2}} H = \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix}$

$h(x)$ champs scalaire <u>réel</u>

SSB:

$$H \to \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix} \qquad (\text{jauge unitaire}) \qquad \langle H^0 H^{0+} \rangle = \frac{v^2}{2},$$

$$\langle |H^0| \rangle = \frac{v}{\sqrt{2}} \quad \text{vev}$$

$$\tau^{\pm} := \frac{1}{\sqrt{2}} (\tau^1 \pm i\tau^2), \qquad \tau^+ = \sqrt{2}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \tau^- = \sqrt{2}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

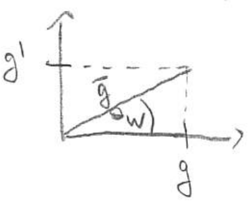
$$W_\mu^{\mp} = \frac{1}{\sqrt{2}} (W_\mu^1 \pm i W_\mu^2)$$

$$\Rightarrow \boxed{\tau^i W^i = \tau^+ W^+ + \tau^- W^- + \tau^3 W^3}$$

$$D_\mu H = \left(\partial_\mu + ig \frac{\tau^i}{2} W_\mu^i + ig' \frac{1}{2} B_\mu\right) \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \partial_\mu h \end{pmatrix} + \frac{ig}{2} \frac{v+h}{\sqrt{2}} \begin{pmatrix} \sqrt{2} W_\mu^+ \\ -W_\mu^3 \end{pmatrix} + ig' \frac{1}{2} \frac{v+h}{\sqrt{2}} \begin{pmatrix} 0 \\ B_\mu \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \partial_\mu h \end{pmatrix} + \frac{i}{2} \frac{v+h}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\, g W_\mu^+ \\ -g W_\mu^3 + g' B_\mu \end{pmatrix}$$



$$\bar{g} := \sqrt{g^2 + g'^2}, \qquad \frac{g}{\bar{g}} = \cos\theta_W, \quad \frac{g'}{\bar{g}} = \sin\theta_W$$

$$\Rightarrow \quad gW^3 - g'B = \bar{g}\,(W^3 \cos\theta_W - B \sin\theta_W) =: \bar{g}\, Z$$

$$(W^3 \sin\theta_W + B\cos\theta_W) =: A$$

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W^3 \\ B \end{pmatrix}$$

Mélange de W^3, B car mêmes nombres quantiques par rapport au générateurs non-brisés ($SU(3)$, $U(1)_{e.m.}$)

$$\Rightarrow \quad D_\mu H = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\partial_\mu h \end{pmatrix} + \frac{i}{2}\frac{v+h}{\sqrt{2}}\begin{pmatrix} \sqrt{2}\, g W_\mu^+ \\ -\bar{g}\, Z_\mu \end{pmatrix}$$

$$(D_\mu H)^\dagger = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\partial_\mu h \end{pmatrix} - \frac{i}{2}\frac{v+h}{\sqrt{2}}\begin{pmatrix} \sqrt{2}\, g W_\mu^- \\ -\bar{g}\, Z_\mu \end{pmatrix}$$

car $h(x)$ réel, $(W_\mu^-)^\dagger = W_\mu^+$

$$\Rightarrow \quad (D_\mu H)(D^\mu H)^\dagger = \frac{1}{2}(\partial_\mu h)(\partial^\mu h) + \frac{1}{4}\frac{(v+h)^2}{2}\left\{ 2g^2 W_\mu^- W^{+\mu} + \bar{g}^2 Z_\mu Z^\mu \right\}$$

$$= \frac{1}{2}(\partial_\mu h)(\partial^\mu h) + \frac{1}{4} g^2 (v+h)^2 W_\mu^- W^{+\mu} + \frac{1}{2}\frac{1}{4}\bar{g}^2 (v+h)^2 Z_\mu Z^\mu$$

$$= \frac{1}{2}(\partial_\mu h)(\partial^\mu h) + m_W^2 W_\mu^- W^{+\mu} + \lambda_{hWW}\, h W_\mu^- W^{+\mu} + \lambda_{hhWW}\, h^2 W_\mu^- W^{+\mu}$$
$$+ \frac{1}{2} m_Z^2 Z_\mu Z^\mu + \lambda_{hZZ}\, h Z_\mu Z^\mu + \lambda_{hhZZ}\, h^2 Z_\mu Z^\mu$$

$$\Rightarrow \quad m_Z^2 = \frac{1}{4}\bar{g}^2 v^2 \quad , \quad m_W^2 = \frac{1}{4} g^2 v^2 \quad , \quad m_A = 0$$

$$\boxed{\frac{m_W}{m_Z} = \frac{g}{\bar{g}} = \cos\theta_W}$$

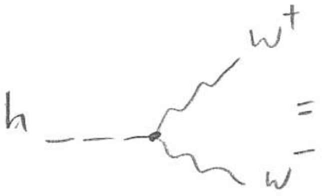
prédiction du MS

$$\rho = \frac{m_W}{m_Z \cos\theta_W}$$

$\rho^{SM} = 1$ à LO

$$\boxed{\begin{aligned} \lambda_{hWW} &= \frac{1}{2} g^2 v = 2\,\frac{m_W^2}{v} \\ \lambda_{hhWW} &= \frac{1}{4} g^2 = \frac{m_W^2}{v^2} \\ \lambda_{hZZ} &= \frac{1}{4}\bar{g}^2 v = \frac{m_Z^2}{v} \\ \lambda_{hhZZ} &= \frac{1}{8}\bar{g}^2 = \frac{m_Z^2}{2v^2} \end{aligned}}$$

Interactions Higgs avec boson W, Z

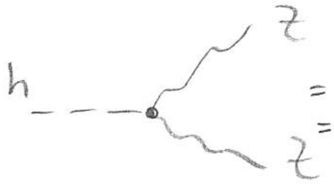


$= i\lambda_{hWW} = \frac{2i m_W^2}{v}$

2 h identiques

$= 2i\,\lambda_{hhWW}$

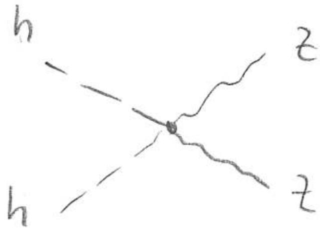
$= 2i\,\frac{m_W^2}{v^2}$



Facteur de symétrie: part. identiques

$= i2\lambda_{hZZ}$

$= 2i\,\frac{m_Z^2}{v}$



Fact. de symétrie 2×2

$= 4i\,\lambda_{hhZZ}$

$= 2i\,\frac{m_Z^2}{v^2}$

Ex. 1 Dérivez les règles de Feynman (facile) en termes de m_Z, m_W, v

$$V(H) = \mu^2 H^\dagger H + \lambda (H^\dagger H)^2$$

$$V(H) \xrightarrow{H = \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}} \frac{1}{2}\mu^2 (v+h)^2 + \frac{\lambda}{4}(v+h)^4$$

$$= \frac{1}{2}\mu^2 \left[v^2 + 2vh + h^2 \right] + \frac{\lambda}{4}\left[v^4 + 4v^3 h + 6v^2h^2 + 4vh^3 + h^4 \right]$$

$$= \frac{1}{2}\mu^2 v^2 + \frac{\lambda}{4} v^4$$

$$+ h\left[\mu^2 v + \lambda v^3 \right]$$

$$+ h^2 \left[\frac{1}{2}\mu^2 + \frac{3}{2}\lambda v^2 \right] + h^3 \lambda v + h^4 \frac{\lambda}{4}$$

avec $\frac{v^2}{2} = -\frac{\mu^2}{2\lambda} = \langle |H| \rangle^2$

$$\Rightarrow \boxed{\mu^2 = -v^2 \lambda}$$

$$\Rightarrow V(H) \to V(h) = \overbrace{\frac{1}{2}\mu^2 v^2 + \frac{\lambda}{4} v^4}^{V(H = H_{min})} + h\left[0 \right] +$$

$$h^2 \left[\underbrace{-\frac{1}{2} v^2 \lambda + \frac{3}{2} v^2 \lambda}_{\lambda v^2} \right] + h^3 \lambda v + h^4 \frac{\lambda}{4}$$

$$= \frac{1}{2} m_h^2 h^2 + \lambda_{hhh} h^3 + \lambda_{4h} h^4 + \text{const.}$$

$$\Rightarrow \boxed{m_h^2 = 2\lambda v^2 \overset{v^2 = -\frac{\mu^2}{\lambda}}{=} -2\mu^2}$$

$$\boxed{\begin{aligned} \lambda_{3h} &= \lambda v = \frac{1}{2v} m_h^2 = \sqrt{\frac{\lambda}{2}}\, m_h \\ \lambda_{4h} &= \frac{\lambda}{4} = \frac{m_h^2}{8v^2} \end{aligned}}$$

$m_h = \sqrt{2\lambda}\, v$

$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2} = \frac{1}{2v^2} \Rightarrow v^2 = \frac{1}{\sqrt{2} G_F}$

SM: $v = 246$ GeV

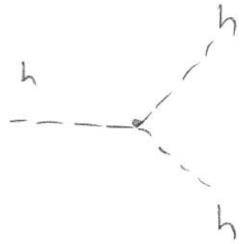
$m_h = 125$ GeV

connu!

$$\Rightarrow \boxed{\begin{aligned} \lambda^{SM} &= \frac{m_h^2}{2v^2} \\ \lambda_{3h}^{SM} &= \frac{m_h^2}{2v} \end{aligned}}$$

prediction to test!

Règles de Feynman



$$= 3!\, i\, \lambda_{hhh}$$

$$= 4!\, i\, \lambda_{4h}$$

$$\lambda = -\frac{\mu^2}{v^2} = \frac{1}{2}\frac{m_h^2}{v^2} = \frac{1}{2}\frac{(125)^2}{(246)^2} = 0.13$$

$$\frac{\lambda_{3h}}{m_h} = \sqrt{\frac{\lambda}{2}} = 0.25$$

$$\lambda_{4h} = \frac{\lambda}{4} = 0.0325$$

Masses pour les fermions

$$\mathcal{L}_{Yuk} = -\Gamma_u \, \bar{Q} \, \tilde{H} \, u_R - \Gamma_d \, \bar{Q} \, H \, d_R - \Gamma_e \, \bar{L} \, H e_R + \text{H.c.}$$

$$H = \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix} \, , \quad \tilde{H} = i\sigma_2 H = \begin{pmatrix} \frac{v+h(x)}{\sqrt{2}} \\ -0 \end{pmatrix}$$

$$L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} , \; \bar{L} = \begin{pmatrix} \bar{\nu}_{eL} \\ \bar{e}_L \end{pmatrix} \quad Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix} , \; \bar{Q} = \begin{pmatrix} \bar{u}_L \\ \bar{d}_L \end{pmatrix}$$

$$\mathcal{L}_{Yuk} = -\Gamma_u \, \overline{u_L} \left(\frac{v+h}{\sqrt{2}} \right) u_R - \Gamma_d \, \overline{d_L} \left(\frac{v+h}{\sqrt{2}} \right) d_R - \Gamma_e \, \overline{e_L} \, \frac{v+h}{\sqrt{2}} \, e_R + \text{H.c.}$$

$$= -\frac{\Gamma_u v}{\sqrt{2}} \, \overline{u_L} u_R - \frac{\Gamma_u}{\sqrt{2}} \overline{u_L} h u_R + \ldots$$

$$= -M_u^{f'f} \, \overline{u_L}^{f'} u_R^f - M_d^{f'f} \, \overline{d_L}^{f'} d_R^f - M_e^{f'f} \, \overline{e_L}^{f'} e_R^f + \text{H.c.}$$

$$- \frac{M_u}{v} \, \bar{u}_L h u_R \quad \ldots \quad (\text{Interactions})$$

avec

$$M_u = \frac{\Gamma_u v}{\sqrt{2}} \, , \quad M_d = \frac{\Gamma_d v}{\sqrt{2}} \, , \quad M_e = \frac{\Gamma_e v}{\sqrt{2}}$$

$$\frac{\Gamma}{\sqrt{2}} = \frac{\tilde{M}}{v}$$

(matrices complexes 3×3)

$\Rightarrow$ Masses des fermions = effet dynamique à cause du couplage des fermions avec le champ de Higgs

* Il faut diagonaliser les matrices de masse !

- $u_L^f, u_R^f \to u_L'^f, u_R'^f$; mass eigenstates (propagating particles)
- $\bar{u}_L \not{Z} u_L \to \bar{u}_L' \not{Z} u_L'$ etc. : NC pas changé
- $\bar{d}_L \not{W} u_L \to \bar{d}_L' V_{CKM} \not{W} u_L'$

$\mathcal{L}_{masse} = -\bar{u}_L \underset{\sim}{M_u} u_R - \bar{d}_L \underset{\sim}{M_d} d_R - \bar{e}_L \underset{\sim}{M_e} e_R \; (-\bar{\nu}_L \underset{\sim}{M_N} \nu_R) + h.c.$

état d'int. $\qquad$ état de masse

$$u_L^f = \underset{\sim}{V_{u_L}}^{ff'} u_L'^{f'} \quad , \; f,f' = 1,2,3 \quad \text{génération}$$

(redéfinition unitaire des champs)

$$u_R = V_{u_R} u_R' \quad , \quad d_L = V_{d_L} d_L' \quad , \quad d_R = V_{d_R} d_R' \quad , \ldots$$

tel que :

$$\underset{\sim}{M_u^{Diag}} = V_{u_L}^{+} \underset{\sim}{M_u} V_{u_R} \quad , \quad \underset{\sim}{M_d^{Diag}} = V_{d_L}^{+} \underset{\sim}{M_d} V_{d_R}$$

etc.

$$\Rightarrow \mathcal{L}_{masse} = -\bar{u}_L' V_{u_L}^{+} M_u V_{u_R} u_R' - \bar{d}_L' V_{d_L}^{+} M_d V_{d_R} d_R' \ldots$$

$$= -\bar{u}_L' M_u^{Diag} u_R' - \bar{d}_L' M_d^{Diag} d_R' \ldots + h.c.$$

$$= -m_u \bar{u}_L u_R - m_c \bar{c}_L c_R - m_t \bar{t}_L t_R$$

$$- m_d \bar{d}_L d_R - m_s \bar{s}_L s_R - m_b \bar{b}_L b_R$$

$$- \ldots + h.c.$$

Effet de la redéf. sur $\mathcal{L}_{matière}$!

$$\mathcal{L}_{mat} = \bar{Q}\, i \not{D}\, Q + \bar{u}_R\, i\not{D}\, u_R + \bar{d}_R\, i\not{D}\, d_R + \dots$$

$$= \begin{pmatrix}\bar{u}_L\\ \bar{d}_L\end{pmatrix}^f i\left[\not{\partial} + i g_s \underset{\sim}{\not{G}} + i\frac{g}{2}\underset{\sim}{\not{W}}^i \tau^i + i g' \frac{1}{6}\not{B}\right]\begin{pmatrix}u_L\\ d_L\end{pmatrix}^f$$

$$+ \bar{u}_R^f\, i\left[\not{\partial} + i g_s \underset{\sim}{\not{G}} \qquad\qquad + i g' \tfrac{2}{3}\not{B}\right] u_R^f$$

$$+ \bar{d}_R^f\, i\left[\not{\partial} + i g_s \underset{\sim}{\not{G}} \qquad\qquad + i g' (-\tfrac{1}{3})\not{B}\right] d_R^f$$

$$+ \dots$$

$$\underset{SU(2)_L \times U(1)_Y}{\supset} \quad i\begin{pmatrix}\bar{u}_L\\ \bar{d}_L\end{pmatrix}^f \left[\not{\partial} + \frac{i}{2} g \left(\overset{\sqrt{2}\begin{pmatrix}0&1\\0&0\end{pmatrix}}{\overset{\shortparallel}{\tau^+}} \not{W}^+ + \overset{\sqrt{2}\begin{pmatrix}0&0\\1&0\end{pmatrix}}{\overset{\shortparallel}{\tau^-}} \not{W}^- + \overset{\begin{pmatrix}1&0\\0&-1\end{pmatrix}}{\overset{\shortparallel}{\tau^3}} \not{W}^3\right) + i\frac{g'}{6}\not{B}\right]\begin{pmatrix}u_L\\ d_L\end{pmatrix}^f$$

$$+ i\, \bar{u}_R^f \left[\not{\partial} \quad + i g' \frac{2}{3}\not{B}\right] u_R^f$$

$$+ i\, \bar{d}_R^f \left[\not{\partial} \quad + i g' (-\tfrac{1}{3})\not{B}\right] d_R^f$$

$$= \bar{u}_L^f\, i\not{\partial}\, u_L^f + \bar{d}_L^f\, i\not{\partial}\, d_L^f + \bar{u}_R^f\, i\not{\partial}\, u_R^f + \bar{d}_R^f\, i\not{\partial}\, d_R^f$$

$$- \frac{g}{2}\left(\sqrt{2}\,\bar{u}_L^f \not{W}^+ d_L^f + \sqrt{2}\,\bar{d}_L^f \not{W}^- u_L^f + \bar{u}_L^f \not{W}^3 u_L^f - \bar{d}_L^f \not{W}^3 d_L^f\right)$$

$$- \frac{g'}{6}\left(\bar{u}_L^f \not{B}\, u_L^f + \bar{d}_L^f \not{B}\, d_L^f\right)$$

$$- \frac{2}{3} g'\, \bar{u}_R^f \not{B}\, u_R^f$$

$$+ \frac{1}{3} g'\, \bar{d}_R^f \not{B}\, d_R^f$$

Courant chargé :

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\left[\bar{u}_L \gamma^\mu d_L W_\mu^+ + \bar{d}_L \gamma^\mu u_L W_\mu^-\right]$$

$$= -\frac{g}{\sqrt{2}}\left[\bar{u}_L' \gamma^\mu \underbrace{V_{u_L}^+ V_{d_L}}_{= V_{CKM}} d_L' W_\mu^+ + \bar{d}_L' \gamma^\mu \underbrace{V_{d_L}^+ V_{u_L}}_{=: V_{CKM}^+} u_L' W_\mu^-\right]$$

$$= -\frac{g}{\sqrt{2}}\left[\bar{u}_L' \not{W}^+ V_{CKM} d_L' + \bar{d}_L' \not{W}^- V_{CKM}^+ u_L'\right]$$

$$\boxed{V_{CKM} = V_{u_L}^+ V_{d_L}}$$

Courant neutre :

$$\mathcal{L}_{NC} = -\frac{g}{2}\left(\bar{u}_L \not{W}^3 u_L - \bar{d}_L \not{W}^3 d_L\right)$$

$$-\frac{g'}{2}\left(\frac{1}{3}\bar{u}_L \not{B} u_L + \frac{1}{3}\bar{d}_L \not{B} d_L\right)$$

$$-\frac{g'}{2}\left(\frac{4}{3}\bar{u}_R \not{B} u_R - \frac{2}{3}\bar{d}_R \not{B} d_R\right)$$

$g = \bar{g} c_w$, $e = g s_w$

$g' = \bar{g} s_w$

$$= -\frac{\bar{g}}{2}\Big\{ c_w \bar{u}_L (c_w \not{Z} + s_w \not{A}) u_L$$
$$- c_w \bar{d}_L (c_w \not{Z} + s_w \not{A}) d_L$$
$$+ s_w \frac{1}{3} \bar{u}_L (-s_w \not{Z} + c_w \not{A}) u_L$$
$$+ s_w \frac{1}{3} \bar{d}_L (-s_w \not{Z} + c_w \not{A}) d_L$$
$$+ s_w \frac{4}{3} \bar{u}_R (-s_w \not{Z} + c_w \not{A}) u_R - \frac{2}{3} s_w \bar{d}_R (-s_w \not{Z} + c_w \not{A}) d_R \Big\}$$

$$\begin{pmatrix} W^3 \\ B \end{pmatrix} = \begin{pmatrix} c_w & s_w \\ -s_w & c_w \end{pmatrix}\begin{pmatrix} Z \\ A \end{pmatrix}$$

$W^3 = c_w Z + s_w A$

$B = -s_w Z + c_w A$

$$= -\frac{\bar{g}}{2} Z_\mu \Big\{ c_w^2 \bar{u}_L \gamma^\mu u_L - c_w^2 \bar{d}_L \gamma^\mu d_L - \frac{1}{3} s_w^2 \bar{u}_L \gamma^\mu u_L - \frac{1}{3} s_w^2 \bar{d}_L \gamma^\mu d_L$$
$$- \frac{4}{3} s_w^2 \bar{u}_R \gamma^\mu u_R + \frac{2}{3} s_w^2 \bar{d}_R \gamma^\mu d_R \Big\}$$

$$-\frac{\bar{g}}{2} A_\mu \{ \dots \}$$

$$\mathcal{L}_{mat} \to \mathcal{L}_{NC} = -\frac{g}{2}\left(\bar{u}_L \not{W}^3 u_L - \bar{d}_L \not{W}^3 d_L\right)$$

$$-\frac{g'}{2}\left(Y_{u_L}\bar{u}_L \not{B} u_L + Y_{d_L}\bar{d}_L \not{B} d_L\right)$$

$$-\frac{g'}{2}\left(Y_{u_R}\bar{u}_R \not{B} u_R + Y_{d_R}\bar{d}_R \not{B} d_R\right)$$

$$W^3 = c_W Z + s_W A$$

$$B = -s_W Z + c_W A$$

$$= -g W^3_\mu \Big(\overbrace{T^3_{u_L}\bar{u}_L\gamma^\mu u_L + T^3_{d_L}\bar{d}_L\gamma^\mu d_L}^{J^\mu_3}\Big)$$

$$-\frac{g'}{2} B_\mu \Big(\underbrace{Y_{u_L}\bar{u}_L\gamma^\mu u_L + Y_{d_L}\bar{d}_L\gamma^\mu d_L + Y_{u_R}\bar{u}_R\gamma^\mu u_R + Y_{d_R}\bar{d}_R\gamma^\mu d_R}_{= J^\mu_Y}\Big)$$

$$= -g W^3_\mu J^\mu_3 - \frac{g'}{2} B_\mu J^\mu_Y$$

$$\frac{g'}{g} = \frac{s_W}{c_W} \qquad \begin{aligned} e &= g s_W \\ &= \bar{g} s_W c_W \\ &= g' c_W \end{aligned}$$

$$= -g c_W Z^\mu J^\mu_3 - \overbrace{g s_W}^{e} A_\mu J^\mu_3 + \frac{g'}{2} s_W Z_\mu J^\mu_Y - \frac{g'}{2} c_W A_\mu J^\mu_Y$$

$$= -g Z_\mu \underbrace{\left(c_W J^\mu_3 - \frac{g'}{2g} s_W J^\mu_Y\right)}_{=: J^\mu_Z} - e A_\mu \underbrace{\left(J^\mu_3 + \frac{1}{2} J^\mu_Y\right)}_{=: J^\mu_{e.m.}}$$

$$\left(Q = T_3 + \frac{Y}{2}\right)$$

$$J^\mu_Z = c_W J^\mu_3 - \frac{s_W^2}{2c_W} J^\mu_Y$$

$$= -g Z_\mu J^\mu_Z - e A_\mu J^\mu_{e.m.}$$

Ne change pas sous $u_L = V_{u_L} u_L'$, $d_L = V_{d_L} d_L'$, $u_R = V_{u_R} u_R'$, $d_R = V_{d_R} d_R'$

À faire :

$\mathcal{L}_{Mat}$ en termes de $W^{\pm}, Z, A$,

$\rightsquigarrow$ Courants neutres

Courants chargés

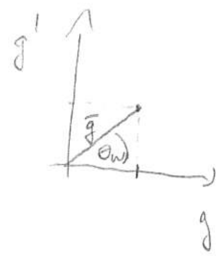
Règles de Feynman

$\mathcal{L}_{Gauge}$ en termes de $W^{\pm}, Z, A$

$\to e = g s_W = g' c_W$

$$\bar{g}^2 = g^2 + g'^2 = \frac{e^2}{s_W^2} + \frac{e^2}{c_W^2} = e^2 \left(\frac{1}{s_W^2} + \frac{1}{c_W^2} \right) = e^2 \frac{1}{s_W^2 c_W^2}$$

$$e = \bar{g} s_W c_W$$



$$\sin\theta_W = \frac{g'}{\bar{g}} = \frac{e}{\bar{g} c_W} = s_W$$

$$c_W = \frac{g}{\bar{g}}$$

$$\tan\theta_W = \frac{g'}{g}$$

$$\bar{g}^2 = g^2 + g'^2$$

$$(g, g') \to (\bar{g}, \theta_W) \to (e, \theta_W)$$