III. QCD perturbative (pQCD)

1. Le modèle des partons

* Liberté asymptotique $\Rightarrow$ α_s petit pour $Q^2 \gtrsim 2\ GeV^2$

$\Rightarrow$ théorie de perturbations possible pour l'interaction forte * *

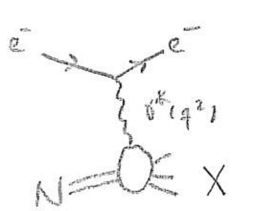
* Problème : souvent $\exists$ hadrons à l'état initial | final $\Rightarrow$ structure des hadrons : physique non-pert.

* Solution : Factorisation : (Phys. à longue distance) x (Phys. à courte distance)

universelle, non-pert. — dépend du processus, calculable

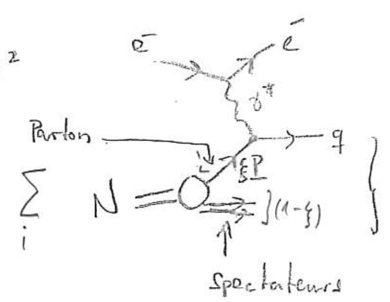
Historiquement : Modèle " naïf " des partons (1965) Feynman, Bjorken, Paschos

Diffusion profondément inélastique $eN \to eX$:



$Q^2 \geq 2\ GeV^2$

$\simeq$



Interaction ponctuelle du γ^* avec un parton libre

$$f_i^N(\xi) = [\, q^N(\xi),\ \bar{q}^N(\xi),\ G(\xi)\,]$$

Partons — Quarks — Anti-quarks — Gluons dans le nucléon

$f_i^N(\xi)\,d\xi$: nombre de partons i avec une fraction ξ de l'impulsion du nucléon

$\hat{p} = \xi P$

$q = u, d, s, \ldots$

$$\boxed{d\sigma = \sum_i \int d\xi\, f_i(\xi)\, d\hat{\sigma}}$$

$$\boxed{\sum_i \int d\xi\, f_i(\xi)\,\xi = 1}$$

Cinématique :

$\hat{P} = \xi P$

$\hat{E} = \xi E$

$\hat{P}_L = \xi P_L$

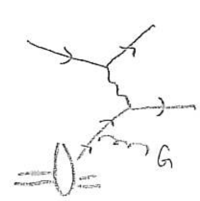
$\hat{P}_T = 0 = P_T$

$\hat{m} = \xi M$?

IMF : $|\vec{p}| \gg M, m$

time dilatation ; partons don't interact with each other during interaction with γ^*

+



Corrections QCD :

$f_i^N(\xi) \to f_i^N(\xi, Q^2)$

Violation d'échelle (scaling violations)

2. DIS dans le modèle des partons

(§ 2.2) :

$$\frac{d^2\sigma}{dx\,dy} = \frac{4\pi\alpha^2 s}{Q^4}\left[(1-y)\,F_2 + y^2 x F_1\right]$$

Bjorken scaling

$F_i(x) \xrightarrow{\text{corr. QCD}} F_i(x, Q^2)$ ← violation de Bjorken-scaling

pour ν, Q^2 large, x = fixe

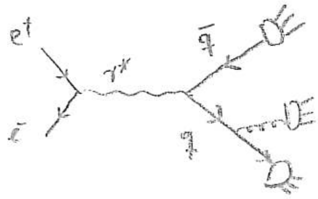
$$F_2(x, Q^2) = x \sum_q e_q^2 \left[q(x,Q^2) + \bar{q}(x,Q^2) \right] = x\left[\frac{4}{9}(u+\bar{u}) + \frac{1}{9}(d+\bar{d}) + \frac{1}{9}(s+\bar{s}) + \ldots \right]$$

(→ Exercice)

* *

Prédiction :

$e^+ e^- \to q \bar{q} g$ "Jet de gluon" (exp. PETRA)



$\exists$ événements avec 3 jets $\sim 10\%$ ($\alpha_s \sim 0.1$) de tous les événements

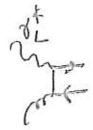
Interaction ponctuelle + Quark-partons avec spin $= \frac{1}{2}$:

$$\boxed{F_L(x,Q^2) = F_2(x,Q^2) - \underbrace{2x\, F_1(x,Q^2)}_{= F_T} = 0}$$

Relation Callan-Gross

Rem. : • Spin 0 $\Rightarrow$ $F_1(x,Q^2) = 0$

• NLO $F_L(x) \sim \alpha_s \neq 0$



← dominant ($\sim 85\%$) : distribution du gluon

• Violation de scaling : $F(x),\ q(x) \rightarrow F(x,Q^2),\ q(x,Q^2)$

$\Rightarrow$ tests de QCD

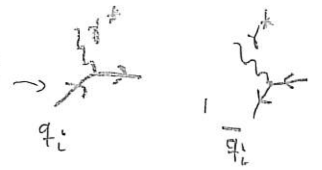
Exercice : Montrez que $F_2(x,Q^2) = x\sum_q e_q^2\,(q+\bar q)(x,Q^2) + O(\alpha_s)$

$$\frac{d^2\sigma}{dx\,dy} = \frac{4\pi\alpha^2 S}{Q^4}\Big[(1-y)F_2 + \frac{y^2}{2}\underbrace{2xF_1}_{\overset{LO}{=}F_2}\Big] = \frac{2\pi\alpha^2 S}{Q^4}\Big[\underbrace{y^2+2(1-y)}_{1+(1-y)^2}\Big]F_2$$

$$d\sigma = \sum_i \int_0^1 d\xi\, f_i(\xi)\; d\hat\sigma(e\,i\to eX)$$

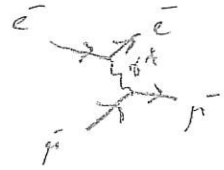
LO: $i = u, d, s, \dots$
$\bar u, \bar d, \bar s, \dots$

$\hat p = \xi P \rightarrow$



q_i , $\bar q_i$

$d\hat\sigma(eq\to eq)$: de $e^-\mu^-\to e^-\mu^-$



$$|M_{e\mu\to e\mu}|^2 = 2e^4 q_\mu^2\,\frac{\hat s^2+\hat u^2}{\hat t^2}\quad ; \quad e^4 = (4\pi\alpha)^2 = 16\pi^2\alpha^2$$

en bas

$$\begin{cases} \hat s := (k+\xi P)^2 = \xi\, 2k\cdot P = \xi\cdot S = \xi\, Q^2 x^{-1} y^{-1} \\ \hat t := (k-k')^2 = q^2 = -Q^2 = t \\ \hat u := (k'-\xi P)^2 = \xi(-2k'\cdot P) = \xi\cdot u \end{cases}$$

$$\Rightarrow\quad \frac{d\hat\sigma_q}{d\hat t} = \frac{1}{16\pi\hat s^2}\,\overline{|M|^2} = \frac{2\pi\alpha^2}{\hat s^2}\,\overline{|M|} = \frac{2\pi\alpha^2}{\hat s^2}\, e_q^2\,\frac{\hat s^2+\hat u^2}{\hat t^2}$$

$$\frac{d\hat\sigma_q}{d\hat t\, d\hat u} = \frac{2\pi\alpha^2 e_q^2}{\underset{=(Q^2)^2}{\hat t^2}}\;\underset{=\frac{s^2+u^2}{s^2}}{\frac{\hat s^2+\hat u^2}{\hat s^2}}\;\delta(\hat s+\hat t+\hat u)$$

$$\frac{d\sigma}{dx\,dy} = \sum_i\int_0^1 d\xi\, f_i(\xi)\,\frac{d\hat\sigma}{dx\,dy} = \sum_i\int_0^1 d\xi\, f_i(\xi)\,\underbrace{\frac{d\hat\sigma}{d\hat t\, d\hat u}\,J}_{\frac{d\hat\sigma}{dx\,dy}}$$

avec le Jacobien $J = \frac{\partial(\hat t,\hat u)}{\partial(x,y)}$

Cinématique : $\hat{s}, \hat{t}, \hat{u} \to s, t, u \to s, x, y$

$$x = \frac{Q^2}{2p\cdot q} \quad , \; y = \frac{p\cdot q}{p\cdot k} \qquad , \; q = k - k'$$

$$s = (k+p)^2 = 2kp$$
$$t = (k-k')^2 = q^2 = -Q^2 = -2k\cdot k'$$

$$u = (k'-p)^2 = -2k'\cdot p = -2(k-q)\cdot p = -2kp + 2q\cdot p = -s - \frac{q^2}{x} = -s - \frac{t}{x}$$

$$\Rightarrow \quad x = -\frac{t}{u+s}$$

$$y = \frac{p\cdot q}{p\cdot k} = \frac{-t\,2}{2xs} = -\frac{t}{xs} = \frac{s+u}{s} = 1 + \frac{u}{s} \qquad \Rightarrow \; \frac{u}{s} = -(1-y)$$

$$\Rightarrow \boxed{\begin{aligned} t &= -Q^2 = -sxy \\ u &= -s(1-y) \end{aligned}}$$

$$s + u = s - s + sy = sy$$

$$\hat{t} = t = -sxy$$

$$\hat{u} = \xi u = -\xi s(1-y)$$

$$\hat{s} = \xi s$$

$$\Rightarrow \quad J = \frac{\partial(\hat{t},\hat{u})}{\partial(x,y)} = \begin{vmatrix} \frac{\partial\hat{t}}{\partial x} & \frac{\partial\hat{t}}{\partial y} \\ \frac{\partial\hat{u}}{\partial x} & \frac{\partial\hat{u}}{\partial y} \end{vmatrix} = \begin{vmatrix} -sy & -sx \\ 0 & \xi s \end{vmatrix} = s^2 y \xi$$

$$\delta(\hat{s}+\hat{t}+\hat{u}) = \delta(\underbrace{t + \xi(s+u)}_{-sxy + \xi sy = sy(\xi - x)}) = \frac{1}{s+u}\,\delta\left(\frac{t}{s+u} + \xi\right) \Rightarrow \xi = -\frac{t}{s+u} \; ; \frac{t(x,y)}{u(x,y)}$$
$$= x$$

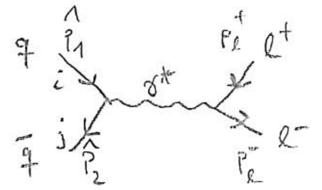
$$\Rightarrow \quad \frac{d\hat{\sigma}_q}{dx\,dy} = s^2 y \xi \, \frac{2\pi\alpha^2 e_q^2}{(sxy)^2} \Bigg[\underbrace{\frac{1\cancel{s^2} + \cancel{s^2}(1-y)^2}{\cancel{s^2}}}_{\frac{s^2+u^2}{s^2}}\Bigg] \frac{1}{sy}\,\delta(\xi - x)$$

$$= s\xi \, \frac{2\pi\alpha^2 e_q^2}{Q^4} \Big[\overbrace{1 + (1-y)^2}^{2(1-y)+y^2}\Big]\, \delta(\xi - x)$$

$$\Rightarrow \quad \frac{d\sigma}{dx\,dy} = \sum_q \int_0^1 d\xi\, (q(x) + \bar{q}(x))\, \frac{d\hat{\sigma}_q}{dx\,dy} = \frac{2\pi\alpha^2 s}{Q^4}\left[1 + (1-y)^2\right] \underbrace{\sum_q x\, e_q^2 (q + \bar{q})(x, Q^2)}_{= F_2(x,Q^2)}$$

Ex. 2 : Drell-Yan

a)



Facteur de couleur : $\dfrac{1}{N_c^2} \displaystyle\sum_{i,j=1}^{N_c} \delta_{i\bar{j}} = \dfrac{1}{N_c}$

b)

TD6.1 : $\overline{|M|^2}_{e^+e^- \to \mu^+\mu^-} = \dfrac{2e^4}{\hat{s}^2} \left[\hat{t}^2 + \hat{u}^2 \right]$ avec $\hat{s} + \hat{t} + \hat{u} = 0$

Modifications :

* quark charge at the vertex $q\bar{q}\gamma^*$: $\to Q_i^2$ (Q_i)
* Colour factor : $\dfrac{1}{N_c}$

$$\Rightarrow \overline{|M|}^2 = \frac{1}{N_c} Q_i^2 \frac{2e^4}{\hat{s}^2} \left[\hat{t}^2 + \hat{u}^2 \right]$$

$$\Rightarrow C = \frac{2e^4}{N_c} = \frac{32\pi^2}{N_c} \alpha^2 \quad ; \; N_c = 3$$

c)

- $\dfrac{d\hat{\sigma}}{d\hat{t}} = \dfrac{1}{16\pi \hat{s}^2} \overline{|M|}^2 = \dfrac{2\pi}{3} \alpha^2 \dfrac{1}{\hat{s}^4} Q_i^2 (\hat{t}^2 + (\hat{s}+\hat{t})^2)$

- CMS:

$$\left.\begin{aligned} \hat{p}_1 &= \frac{\sqrt{\hat{s}}}{2}(1,0,0,1) \\ \hat{p}_2 &= \frac{\sqrt{\hat{s}}}{2}(1,0,0,-1) \\ p_{\ell^+} &= \frac{\sqrt{\hat{s}}}{2}(1, \sin\theta, 0, \cos\theta) \end{aligned}\right\} \Rightarrow \begin{aligned} \hat{t} &= (\hat{p}_1 - p_{\ell^+})^2 = -2\,\hat{p}_1 p_{\ell^+} \\ &= -\frac{\hat{s}}{2}(1-\cos\theta) \\ \hat{s}+\hat{t} &= \frac{\hat{s}}{2}(1+\cos\theta) \end{aligned}$$

$$\Rightarrow d\hat{t} = \frac{\hat{s}}{2} d\cos\theta \quad \Rightarrow \quad \frac{d\hat{\sigma}}{d\cos\theta} = \frac{\hat{s}}{2}\frac{d\hat{\sigma}}{d\hat{t}} = \frac{\pi}{3}\alpha^2 Q_i^2 \frac{1}{\hat{s}^3} \frac{\hat{s}^2}{4} \underbrace{\left[(1-\cos\theta)^2 + (1+\cos\theta)^2 \right]}_{2(1+\cos^2\theta)}$$

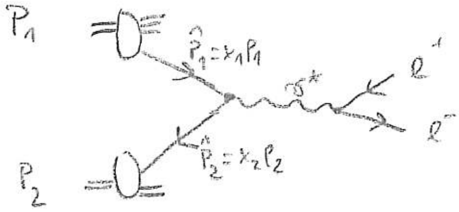
$$= \frac{\pi}{6} \alpha^2 Q_i^2 \frac{1}{\hat{s}} (1 + \cos^2\theta)$$

$$\hat{\sigma}_0 = \int_{-1}^{1} d\cos\theta \; \frac{d\hat{\sigma}}{d\cos\theta} = \frac{\pi}{6} \alpha^2 Q_i^2 \frac{1}{\hat{s}} \underbrace{\int_{-1}^{1} d\cos\theta \; (1+\cos^2\theta)}_{x\big|_{-1}^{1} + \frac{x^3}{3}\big|_{-1}^{1} = 2 + \frac{2}{3} = \frac{8}{3}}$$

$$= \frac{4}{9} \pi \frac{\alpha^2}{\hat{s}} Q_i^2$$

d)

$$d\sigma = \sum_q \int_0^1 dx_1 \, dx_2 \; f_q(x_1) \, f_{\bar{q}}(x_2) \; d\hat{\sigma}_{q\bar{q} \to \ell^+\ell^-} \quad (+ \; q \leftrightarrow \bar{q})$$



e)

$$\frac{d\sigma}{dM^2} = \sum_{i = u,d,s,c,b} \int_0^1 dx_1 \int_0^1 dx_2 \left[q_i(x_1)\bar{q}_i(x_2) + \bar{q}_i(x_1) q_i(x_2) \right] \hat{\sigma}_0 \; \underbrace{\delta(\hat{s} - M^2)}_{\delta(x_1 x_2 s - M^2)}$$

$$= \frac{1}{x_1 s} \delta\left(x_2 - \frac{\tau}{x_1}\right)$$

$$= \sum_i \int_0^1 dx_1 \frac{1}{x_1 s} \left[q_i(x_1) \, \bar{q}_i(\tau/x_1) + \bar{q}_i(x_1) \, q_i(\tau/x_1) \right] \hat{\sigma}_0 \quad \Theta(0 \le \tau/x_1 \le 1)$$

$$= \frac{4\pi\alpha^2}{9} \sum_i Q_i^2 \int_\tau^1 \frac{dx_1}{x_1} \frac{1}{M^2 s} \left[\dots \right]$$

$$\Rightarrow M^4 \frac{d\sigma}{dM^2} = \frac{4\pi\alpha^2}{9} \sum_i Q_i^2 \int_\tau^1 \frac{dx_1}{x_1} \, \tau \left[q_i(x_1) \bar{q}_i(\tau/x_1) + \bar{q}_i(x_1) q_i(\tau/x_1) \right]$$

$$= \frac{4\pi\alpha^2}{9} \tau f(\tau) \quad \text{with } f(\tau) = \sum_i Q_i^2 \int_\tau^1 \frac{dx_1}{x_1} \left[q_i(x_1) \bar{q}_i(\tau/x_1) + q \leftrightarrow \bar{q} \right]$$

$$\tau = \frac{M^2}{s}$$

f)

$$y = \frac{1}{2} \ln \frac{x_1}{x_2} \quad \Rightarrow \quad e^{y} = \sqrt{\frac{x_1}{x_2}} \quad , \quad e^{-y} = \sqrt{\frac{x_2}{x_1}} \quad , \quad \tau = x_1 x_2$$

$$\Rightarrow \quad e^{y} = \sqrt{\frac{x_1^2}{x_1 x_2}} \overset{x_1>0}{\Rightarrow} x_1 = \sqrt{\tau}\, e^{y}$$

$$e^{-y} = \sqrt{\frac{x_2^2}{x_1 x_2}} \quad \Rightarrow \quad x_2 = \sqrt{\tau}\, e^{-y}$$

$$dx_1\, dx_2 = \left| \frac{\partial(x_1, x_2)}{\partial(\tau, y)} \right| d\tau\, dy$$

$$\left| \frac{\partial(x_1, x_2)}{\partial(\tau, y)} \right| = \left| \begin{vmatrix} \frac{1}{2\tau} x_1 & x_1 \\ \frac{1}{2\tau} x_2 & -x_2 \end{vmatrix} \right| = \left| -\frac{1}{\tau} x_1 x_2 \right| = 1$$

g)

$$\frac{d\sigma}{dx_1 dx_2} = \sum_i \left[q_i(x_1) \bar{q}_i(x_2) + q \leftrightarrow \bar{q} \right] \hat{\sigma}_0 = \frac{d\sigma}{d\tau\, dy}$$

$$\frac{d\sigma}{dM^2 dx_F} = \frac{1}{s\sqrt{x_F^2 + 4\tau}} \frac{d\sigma}{d\tau\, dy} \qquad \hat{s} = M^2 = \tau s$$

$$= \frac{4\pi}{9} \frac{\alpha^2}{M^2} \frac{1}{s\sqrt{x_F^2 + 4\tau}} \sum_i Q_i^2 \left[q_i(x_1) \bar{q}_i(x_2) + q \leftrightarrow \bar{q} \right]$$

On a : $x_1 \cdot x_2 = \frac{M^2}{s}$; $x_{1,2} = \sqrt{\tau}\, e^{\pm y}$ $\Rightarrow$ $\left.\begin{array}{l} x_1 + x_2 = 2\sqrt{\tau} \cosh y \\ x_1 - x_2 = 2\sqrt{\tau} \sinh y \end{array}\right\} \Rightarrow (x_1+x_2)^2 - (x_1-x_2)^2 = 4\tau$

$x_1 - x_2 = x_F$

$$\Rightarrow \quad x_1 + x_2 = \sqrt{x_F^2 + 4\tau}$$

$$\Rightarrow \quad x_{1,2} = \frac{1}{2} \left(\sqrt{x_F^2 + 4\tau} \pm x_F \right)$$

Aussi : $x_F = 2\sqrt{\tau} \sinh y \Rightarrow dx_F = 2\sqrt{\tau} \cosh y\, dy$

$$= (x_1 + x_2)\, dy = \sqrt{x_F^2 + 4\tau}\, dy$$

$$\Rightarrow \quad dM^2 dx_F = d\tau\, dy\, s\sqrt{x_F^2 + 4\tau}$$