Deep Inelastic Scattering (DIS)

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Lecture 2

CTEQ School on
QCD and Electroweak Phenomenology

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Disclaimer

This lecture has profited a lot from the following resources:

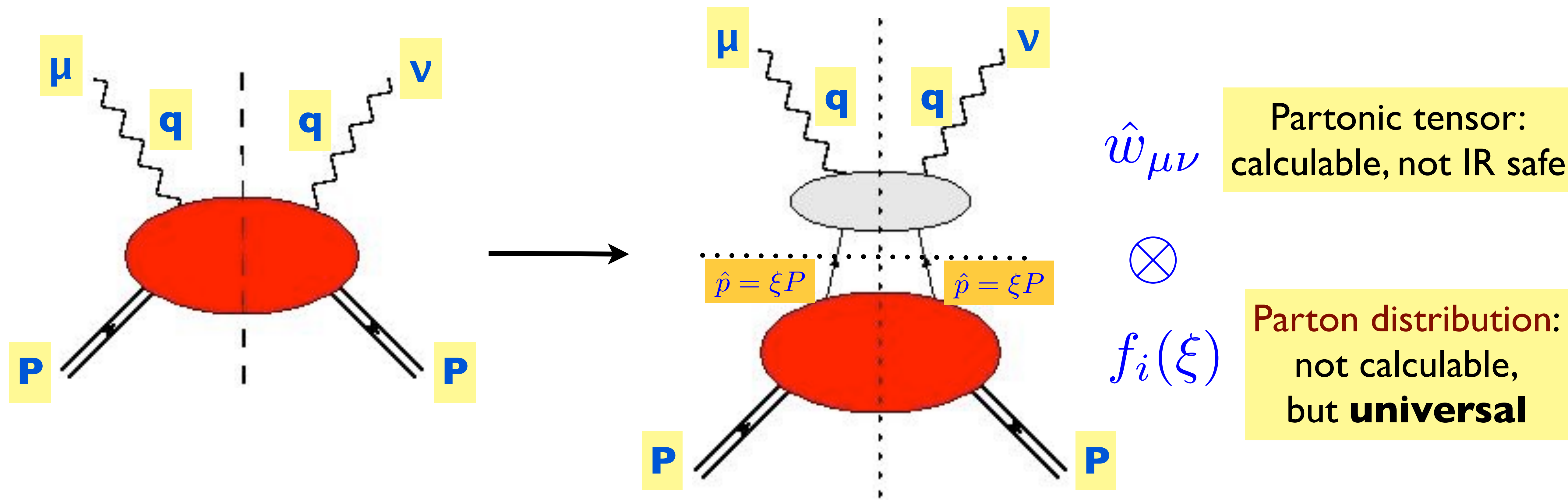
- The text book by **Halzen&Martin**, Quarks and Leptons
- The text book by **Ellis, Stirling & Webber**, QCD and Collider Physics
- The lecture on DIS at the CTEQ school in 2012 by **F. Olness**
- The lecture on DIS given by **F. Gelis** Saclay in 2006

Lecture 11

9. Recap
10. The pQCD formalism
11. NLO corrections to DIS
12. Parton evolution
13. DIS with massive quarks (not covered)
14. Target mass corrections (not covered)
15. Nuclear corrections (not covered)
16. QCD studies with neutrinos (not covered)

IX. Recap

So far: Naive Parton Model



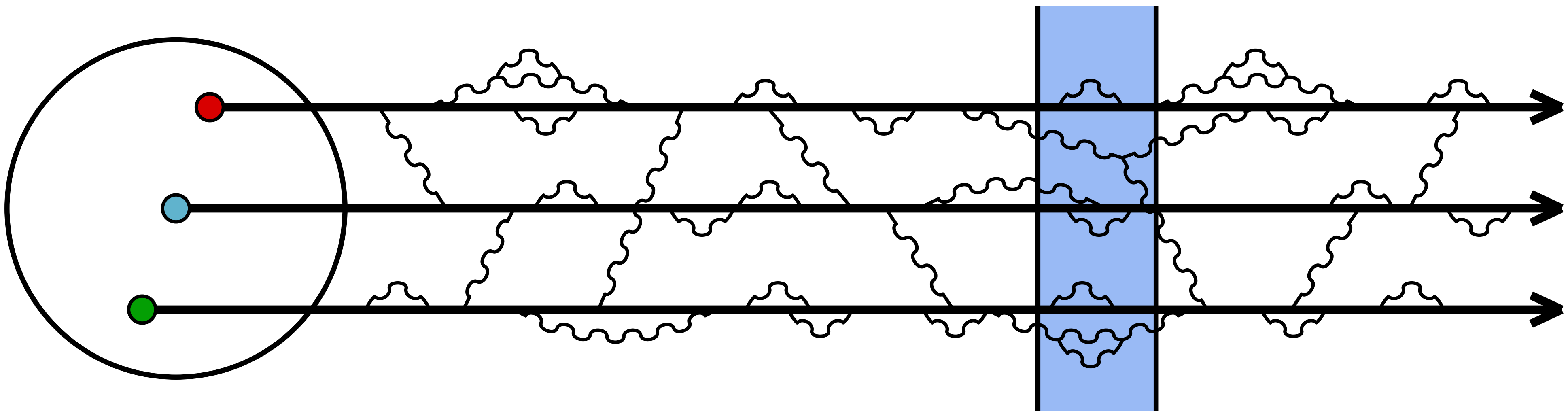
$$W^{\mu\nu}(P,q) = \sum_i \int_x^1 \frac{d\xi}{\xi} f_i(\xi)\ \hat{w}_i^{\mu\nu}(\xi P, q)$$

F_2 and F_L in the Parton Model

- This model provides an explicit realization of **Bjorken scaling**
- The **Callan-Gross relation** $F_L=F_2-F_T=0$ implies $F_2=2x\,F_1=F_T$
  - The observation of this property provides further support of the fact that the **partons are spin-1/2 fermions**
  - If the partons were spin-0 particles, we would have $F_T=0$ and hence $F_2=F_L$
- Caveats and puzzles (to be addressed later):
  - The naive parton model assumes that the partons are free.
    How can this be true in a strongly bound state?
  - One would like to have a field theoretic description (QCD) of what is going on, including the effects of interactions and quantum fluctuations.

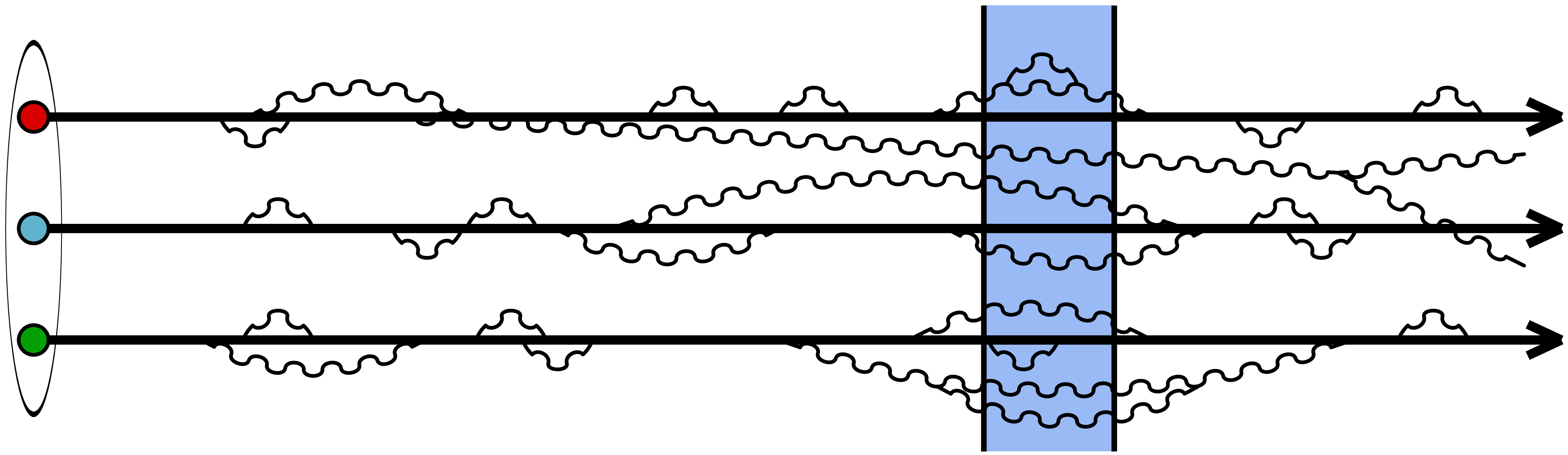
Field theory point of view

Lecture by
F. Gelis 2006



- A **nucleon at rest** is a very complicated object
- Contains **fluctuations at all space-time scales** smaller than its own size
- Only the fluctuations that are longer lived than the external probe participate in the interaction process
- The only role of **short-lived fluctuations** is to renormalize the masses and couplings
- Interactions are very complicated if the constituents of the nucleon have a non-trivial dynamics over time-scales comparable to those of the probe

Field theory point of view



- **Dilation** of all internal time-scales for a **high-energy nucleon**

- Interactions among constituents now take place over time-scales that are longer than the characteristic time-scale of the probe
  ➢ **the constituents behave as if they were free**, the probability of having interactions between the constituents **during the time-scale of the probe** is suppressed

- Many fluctuations live long enough to be seen by the probe. The nucleon appears to be denser at higher energy (it containts more gluons)

- Proofs from first principles of **QCD** show that the parts involving long and short time-scales can be **separated into independent factors** (Factorization theorems)

X. The pQCD formalism

Quantum Chromodynamics (QCD)

QCD: A QFT for the strong interactions

- Statement: Hadronic matter is made of spin-1/2 quarks [$\leftrightarrow$ $SU(3)_{\rm fl}$]
- Baryons like $\Delta^{++} = |u^\uparrow u^\uparrow u^\uparrow\rangle$ forbidden by Pauli exclusion/Fermi-Dirac stat.
  Need additional colour degree of freedom!
- Local SU(3)-color gauge symmetry:

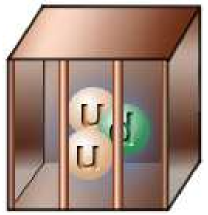
$$\mathcal{L}_{\rm QCD} = \sum_{q=u,d,s,c,b,t} \bar{q}(i\not{\partial} - m_q)q - g\bar{q}\not{G}q - \frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu} + \mathcal{L}_{gf} + \mathcal{L}_{ghost}$$

- Fundamental d.o.f.: quark and gluon fields
- Free parameters:
  - gauge coupling: g
  - quark masses: $m_u, m_d, m_s, m_c, m_b, m_t$

Quantum Chromodynamics (QCD)

Properties:

- **Confinement and Hadronization:**
  - Free quarks and gluons have not been observed:

    A) They are confined in color-neutral hadrons of size $\sim$ 1 fm.

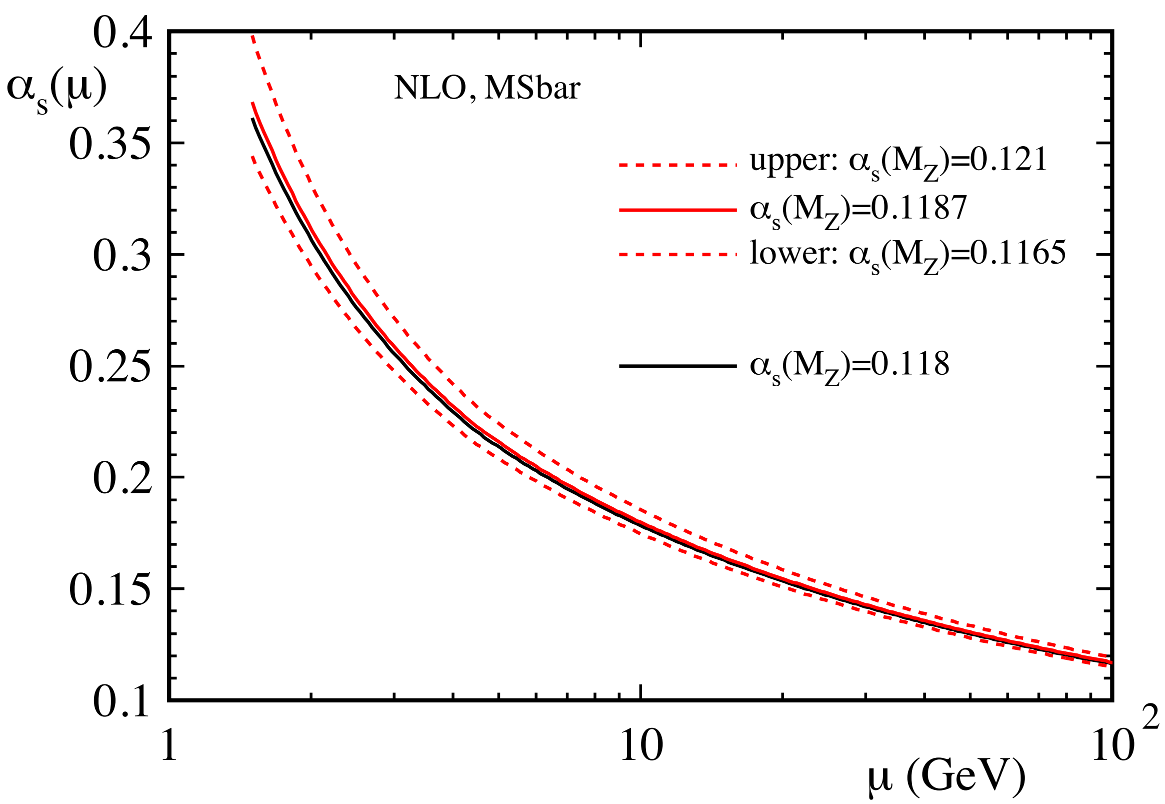
    B) They hadronize into the observed hadrons.
  - Hadronic energy scale: a few hundred MeV [1 fm $\leftrightarrow$ 200 MeV]
  - Strong coupling large at long distances ($\gtrsim$ 1 fm): 'IR-slavery'
  - Hadrons and hadron masses enter the game

- **Asymptotic freedom:**
  - Strong couling small at short distances: perturbation theory
  - Quarks and gluons behave as free particles at asymptotically large energies

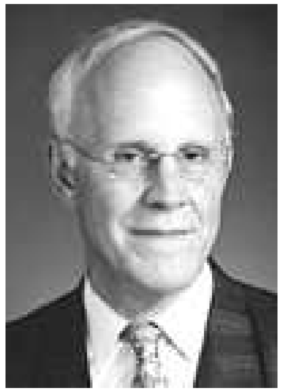
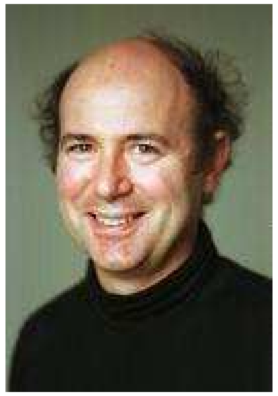
Asymptotic Freedom

Renormalization of UV-divergences:
Running coupling constant $a_s := \alpha_s/(4\pi)$

$$a_s(\mu) = \frac{1}{\beta_0 \ln(\mu^2/\Lambda^2)}$$



- Gross, Wilczek ('73); Politzer ('73)


Non-abelian gauge theories:
negative beta-functions

$$\frac{da_s}{d \ln \mu^2} = -\beta_0 a_s^2 + \ldots$$

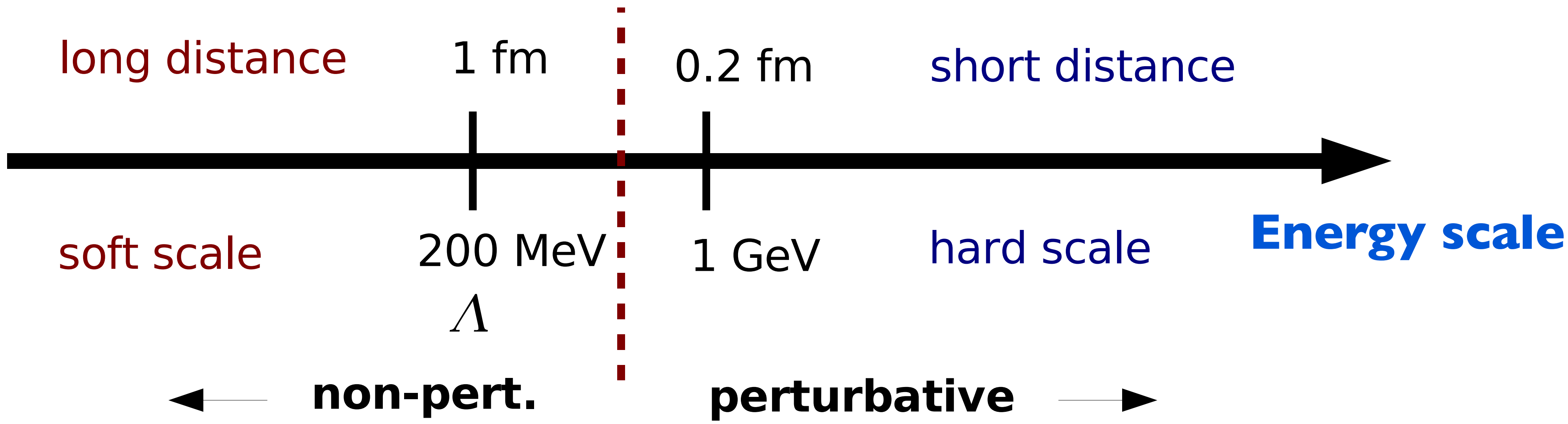
where $\beta_0 = \frac{11}{3} C_A - \frac{2}{3} n_f$

$\Rightarrow$ asympt. freedom: $a_s \searrow$ for $\mu \nearrow$

- Nobel Prize 2004

The perturbative QCD formalism

Two key ingredients:



Asympt. freedom → pQCD possible if **all** scales hard

Factorisation → Possible to **separate** hard and soft scales
soft part : **universal**
hard part : **perturbative**

The perturbative QCD formalism

QCD factorization theorems:

$$\mathrm{d}\sigma = \mathrm{PDF} \otimes \mathrm{d}\hat{\sigma} + \text{remainder}$$

- PDF:
  - Proton composed of partons = quarks, gluons
  - Structure of proton described by parton distribution functions (PDF)
  - Factorization theorems provide field theoretic definition of PDFs
  - PDFs universal $\rightarrow$ **PREDICTIVE POWER**
- Hard part $\mathrm{d}\hat{\sigma}$:
  - depends on the process
  - calculable order by order in perturbation theory
  - Factorization theorems prescribe how to calculate $\mathrm{d}\hat{\sigma}$:
    "$\mathrm{d}\hat{\sigma} =$ partonic cross section - mass factorization"
- Statement about error: remainder suppressed by hard scale

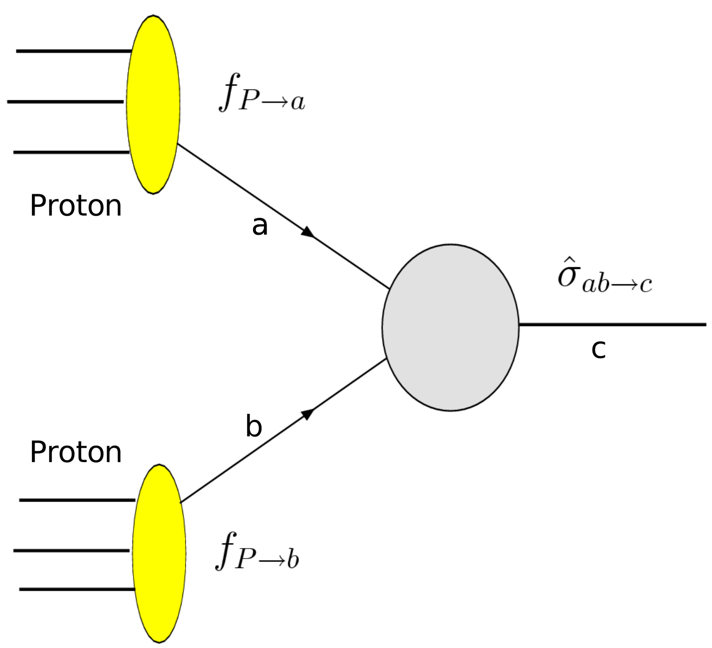
Original factorization proofs considered massless partons

Factorization theorems

Inclusive DIS:

$$W^{\mu\nu} = \sum_i \int_x^1 \frac{d\xi}{\xi} f_i(\xi, \mu_F^2, \mu_R^2) \hat{w}_i^{\mu\nu}(\xi, Q^2, \mu_F^2/Q^2, \mu_R^2/Q^2, \alpha_s(\mu_R^2)) + \mathcal{O}(Q^2/\Lambda^2)$$

In addition to inclusive DIS, there are factorization formulas for other processes



$$\sigma = f_{P\to a} \otimes f_{P\to b} \otimes \hat{\sigma}_{ab\to c}$$

From experiment

Calculable from theoretical model

Parton Distribution Functions (PDFs) $f_{P\to a,b}(x, \mu^2)$

- Universal
- Describe the structure of hadrons
- The key to calculations involving hadrons in the initial state!!!

The hard part $\hat{\sigma}_{ab\to c}(\mu^2)$

- Free of short distance scales
- Calculable in perturbation theory
- Depends on the process

Predictive Power

Universality: same PDFs/FFs enter different processes:

- DIS: $$F_2^A(x,Q^2) = \sum_i \left[f_i^A \otimes C_{2,i}\right](x,Q^2)$$
- DY: $$\sigma_{A+B\to\ell^+ + \ell^- + X} = \sum_{i,j} f_i^A \otimes f_j^B \otimes \hat{\sigma}^{i+j\to\ell^+ + \ell^- + X}$$
- A+B -> H + X: $$\sigma_{A+B\to H+X} = \sum_{i,j,k} f_i^A \otimes f_j^B \otimes \hat{\sigma}^{i+j\to k+X} \otimes D_k^H$$
- Predictions for unexplored kinematic regions and for your favorite new physics process

How to compute the hard cross section?

LO: $d\hat{\sigma}^{(0)} = d\sigma^{(0)}$

Remove long distance part
(soft scales)

NLO: $d\hat{\sigma}^{(1)} = d\sigma^{(1)} - collinear\ subtractions$

Mass factorization
Factorization scheme dependent

Renormalized partonic cross section

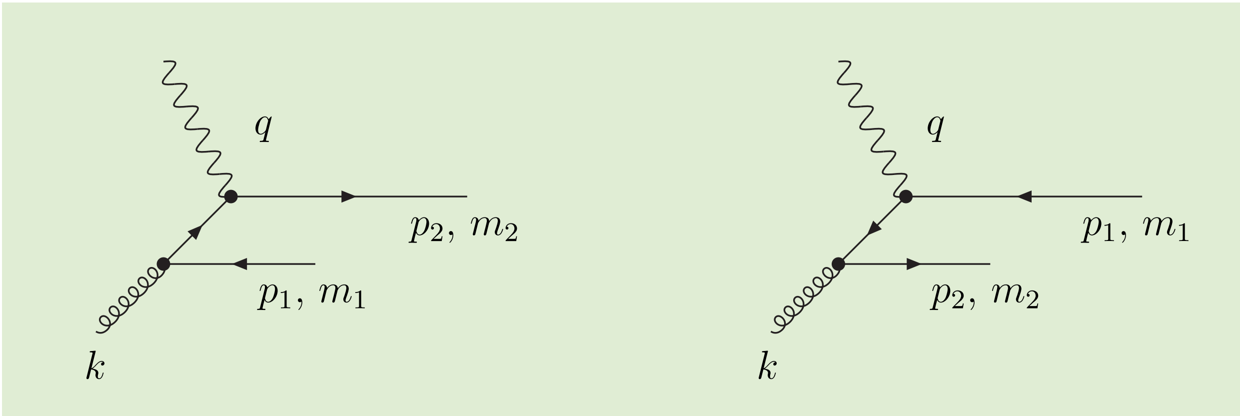
UV div.

(IR div. cancel between real and virtual diagrams)

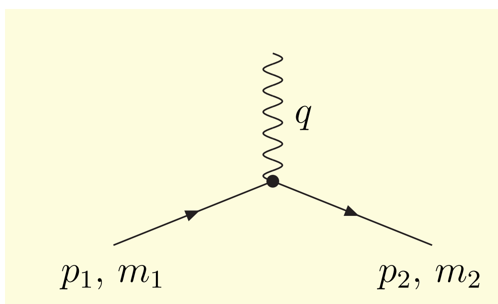
- The good news is at LO the **hard cross section is just the partonic cross section**
- At higher orders, **divergences** appear which have to be **regularized** and then be treated with procedures called **"renormalization"** and **"mass factorization"**

XI. NLO corrections to DIS

NLO corrections to DIS

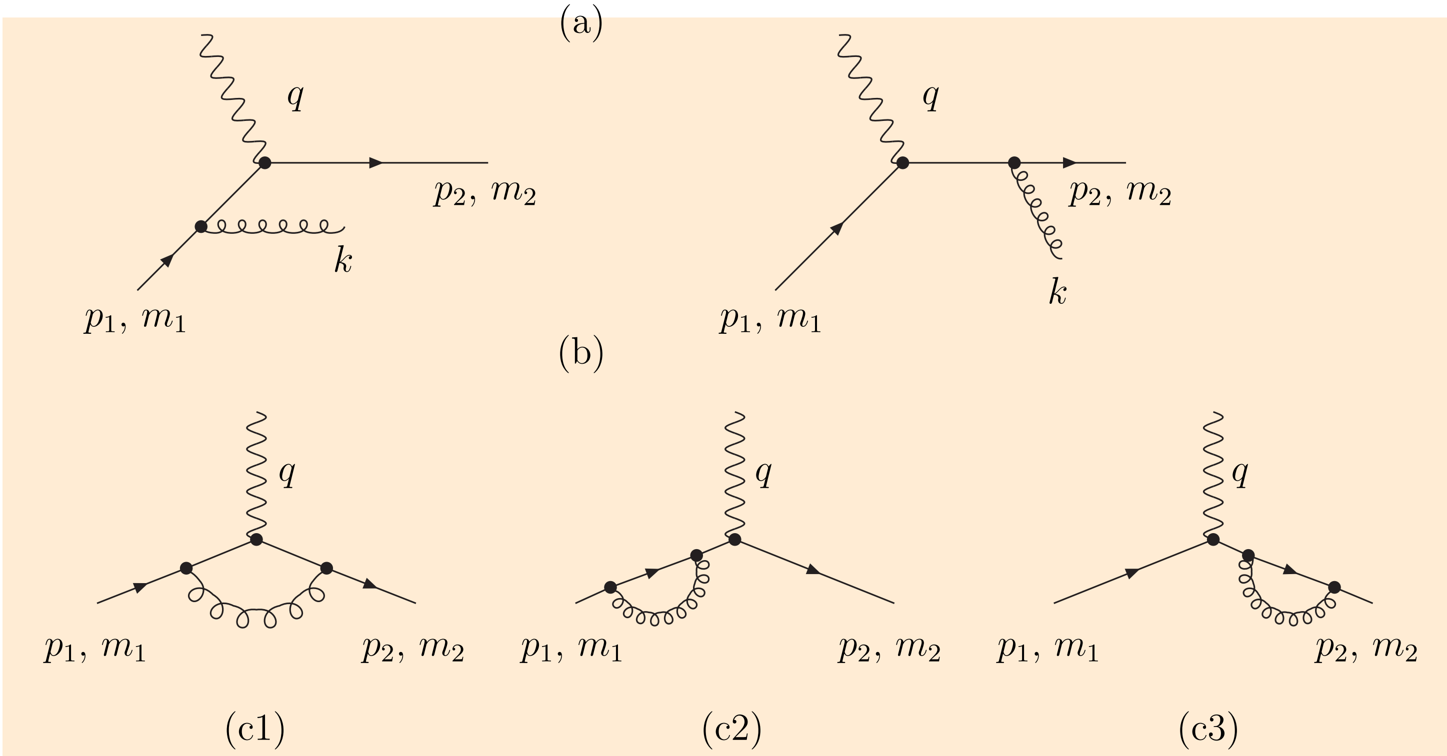


Boson gluon fusion



(a)

LO: quark initiated



(b)

(c1) (c2) (c3)

NLO: quark intiated

(b) NLO real (R)
(c) NLO virtual (V)

The anatomy of divergences

- There are **3 types of divergences** which appear (quite generally at higher orders) and which **need to be regularized** in order to have mathematically well defined objects

  - **UV** divergences (due to high energy modes in loop diagrams)

  - **Soft** divergences (due to the emission of soft/low energy gluons)

  - **Collinear** divergences (due to collinear parton branchings)

- Soft (S) and Collinear (C) divergences involve propagators which are close to their mass shell/which propagate long distances. They are therefore both **IR** divergences.

- The **Collinear** divergences are also called **Mass singularities** because they appear for massless particles. For massive particles the "singularities" are regularized/finite and give just very large contributions to the scattering amplitude for scales much larger than the mass

The anatomy of divergences

- There are **3 types of divergences** which appear (quite generally at higher orders) and which **need to be regularized** in order to have mathematically well defined objects

  - **UV** divergences (due to high energy modes in loop diagrams)

    - Treated by the **renormalization procedure**

    - All parameters of the Lagrangian are redefined in a renormalization scheme, acquire a renormalization scale dependence (running couplings) and need to be 'measured' at one scale

    - We can obtain the runnning parameter at another scale with differential equations called **renormalization group equations** (RGEs)

  - **Soft** divergences (due to the emission of soft/low energy gluons)

  - **Collinear** divergences (due to collinear parton branchings)

The anatomy of divergences

- There are **3 types of divergences** which appear (quite generally at higher orders) and which **need to be regularized** in order to have mathematically well defined objects
  - **UV** divergences? **Renormalization!**
  - **Soft** divergences (due to the emission of soft/low energy gluons)
    - For 'reasonable observables' (IR safe), the soft divergences cancel in the sum of real and virtual contributions: **Real^S + $\mathrm{Virtual}^S$ = finite** (KLN theorem)
  - **Collinear** divergences (due to collinear parton branchings)

The anatomy of divergences

- There are **3 types of divergences** which appear (quite generally at higher orders) and which **need to be regularized** in order to have mathematically well defined objects
  - **UV** divergences? **Renormalization!**
  - **Soft** divergences? **Cancel!**
  - **Collinear** divergences (due to collinear parton branchings)
    - Treated by the **mass factorization procedure** (multiplicative mass factorization is very similar to multiplicative renormalization)
    - PDFs and Fragmentation functions are redefined in a factorization scheme, acquire a factorization scale dependence and we need to 'measure' them at one scale
    - We can obtain the evolution from one scale to another with renormalization group equations known as **DGLAP** equations

The anatomy of divergences

- There are **3 types of divergences** which appear (quite generally at higher orders) and which **need to be regularized** in order to have mathematically well defined objects

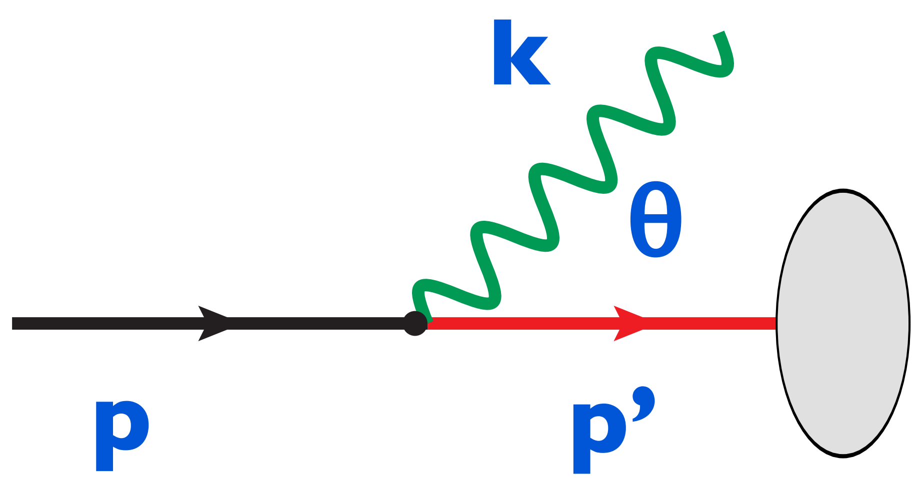
  - **UV** divergences? **Renormalization!**

  - **Soft** divergences? **Cancel!**

  - **Collinear** divergences? **Mass factorization!**

    The collinear divergences will absorbed into a redefinition of the PDFs (or FFs). The redefinition depends on the factorization scheme.

Where do IR divergences come from?



Neglecting the electron mass:

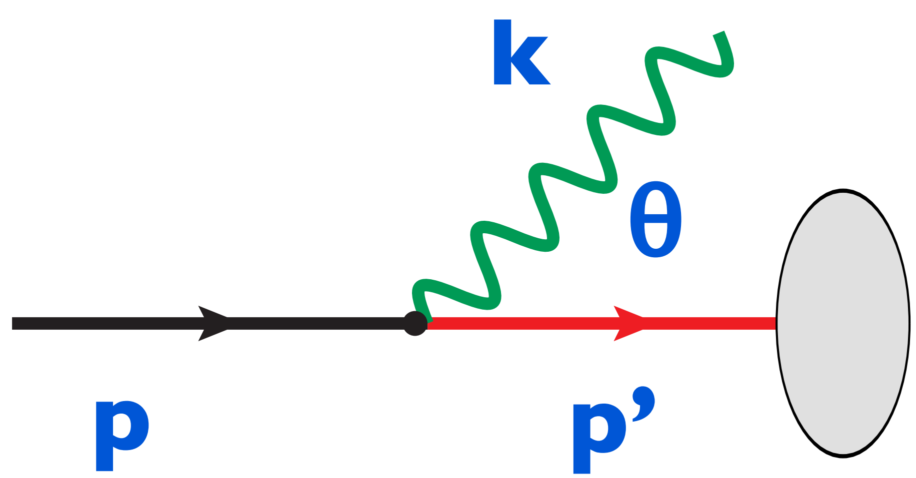
$$p = (E, 0, 0, E)$$
$$k = (\omega, \omega \sin\theta, 0, \omega\cos\theta)$$
$$p' = p - k$$

The red propagator:

$$\frac{i}{\not{p}'} = \frac{i\not{p}'}{p'^2} \sim \frac{1}{p'^2} = \frac{1}{p^2 - 2p\cdot k + k^2} = \frac{-1}{2p\cdot k} = \frac{-1}{E\omega(1-\cos\theta)}$$

We see that the propagator is **divergent** for
a) **ω→0** (soft photon emission)
b) **θ→0** (collinear emission of photon)

Where do IR divergences come from?



Including the electron mass:

$$p = (E, 0, 0, |\vec{p}|) = (E, 0, 0, E\beta)$$

$$\text{with} \quad \beta = \sqrt{1 - m^2/E^2}$$

$$k = (\omega, \omega \sin\theta, 0, \omega\cos\theta)$$

$$p' = p - k$$

The red propagator:

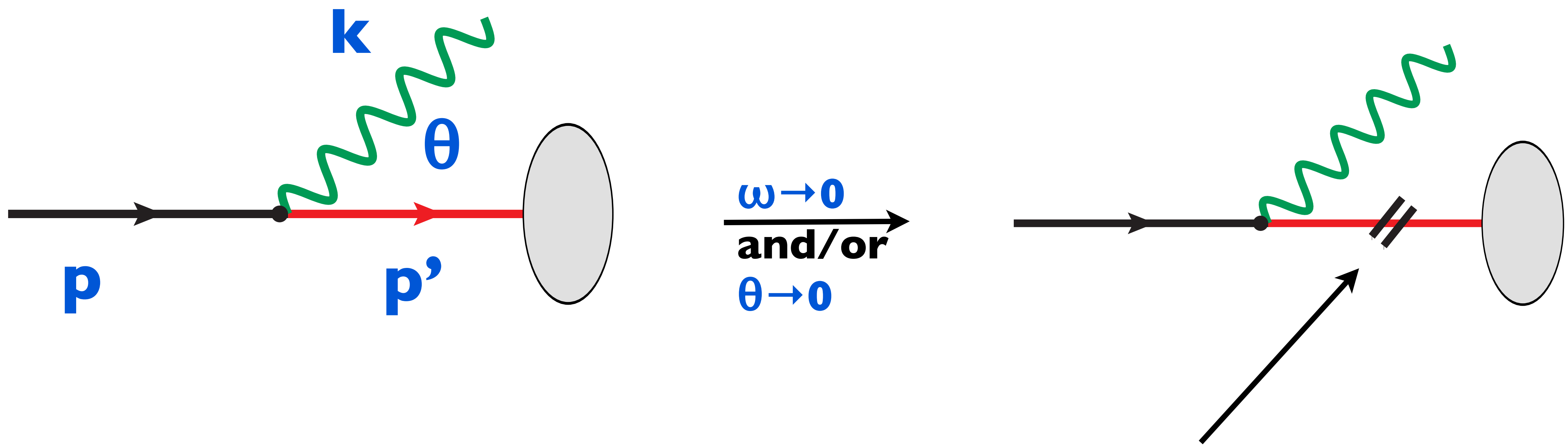
$$\frac{i}{\not{p}' - m} = \frac{i(\not{p}' + m)}{p'^2 - m^2} \sim \frac{1}{p'^2 - m^2} = \frac{1}{p^2 - 2p\cdot k + k^2 - m^2} = \frac{-1}{2p \cdot k} = \frac{-1}{E\omega(1 - \beta\cos\theta)}$$

We see that the propagator is **divergent/large** for

a) **ω→0** (soft divergence)

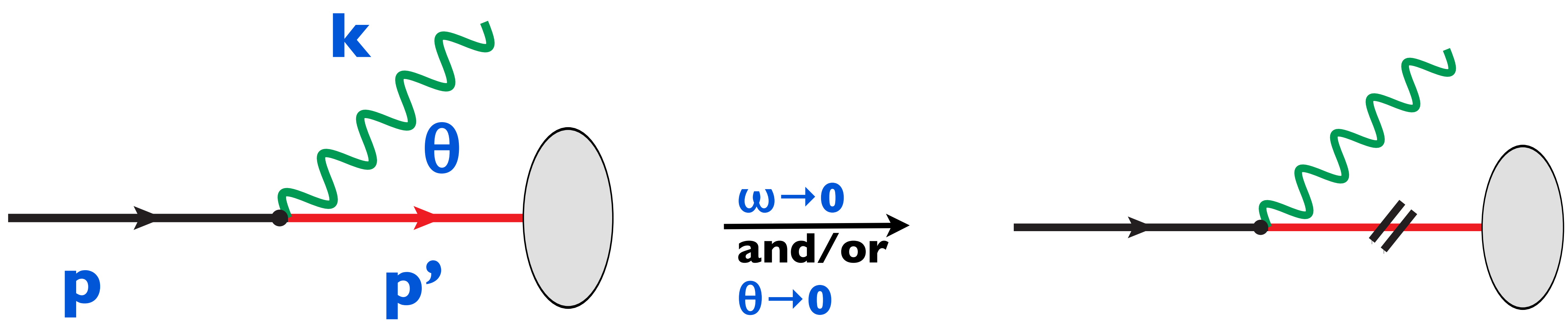
b) **θ→0** (collinear 'divergence' regulated by the mass)

Where do IR divergences come from?



The cut indicates that propagator is on-shell, travels a long distance before interaction with the blob

Where do IR divergences come from?



Discussion:

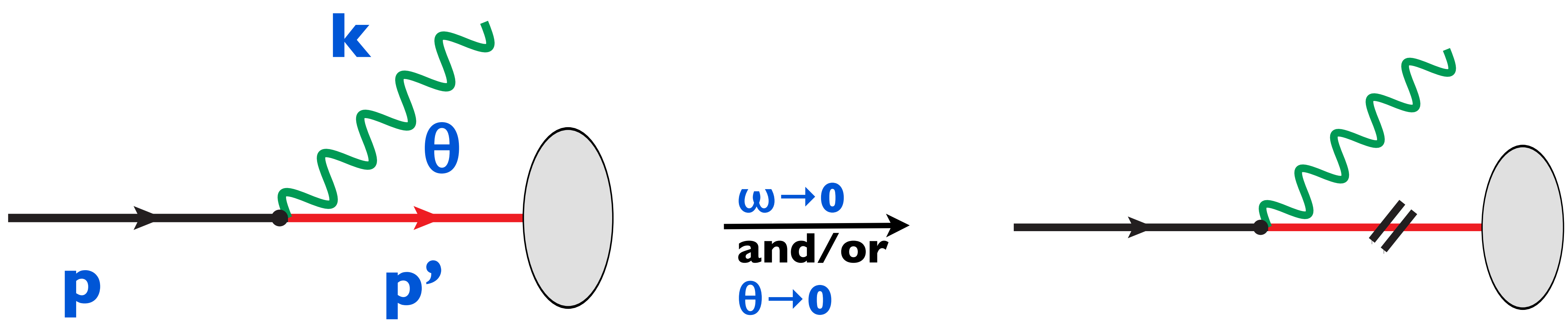
- In the soft/collinear regions the cross section factorizes into two independent pieces which are convoluted (the phase space factorizes as well):
a) the probability for emitting a soft/collinear photon
b) the cross section for the interaction of the on-shell red line with the blob

- The soft part cancels once virtual corrections are included'

- The dominant leading logarithmic contributions to the amplitude come from the collinear regions. The collinear emission probability can be interpreted as finding an electron with a momentum fraction **z** inside a parent electron:

$$\Gamma_{ee}(z,\mu^2) = \frac{\alpha}{2\pi} P^{(0)}_{ee}(z) \ln \mu^2/m^2$$

Not yet ready with the discussion



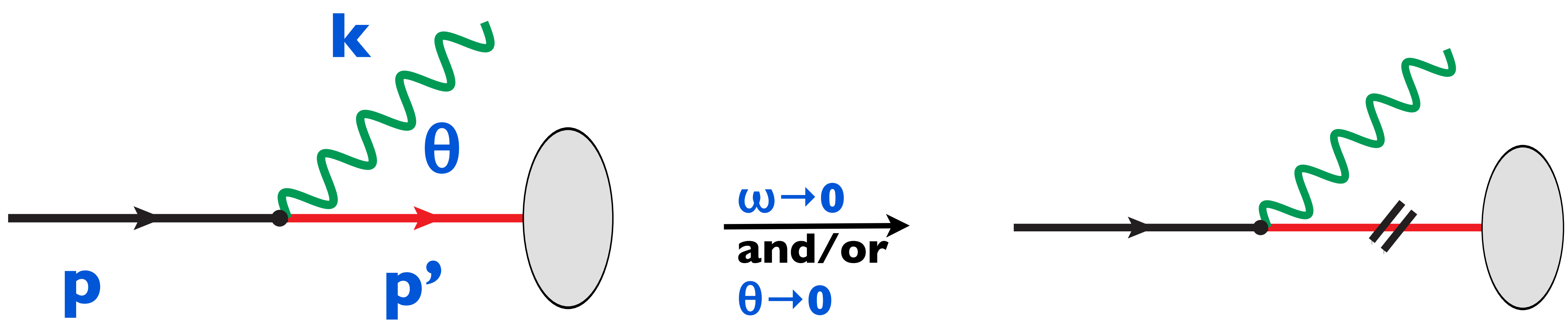
- The collinear logarithms [alpha Pee ln mu2/m2] can in principle be kept in fixed order perturbation theory since the fine structure constant is small.

- Conversely, they can be **resummed to all orders** by introducing parton distributions inside an electron (with an electron parton and a photon parton inside the electron).

- The collinear terms can then be absorbed into the PDFs which can be evolved with (inhomogeneous) DGLAP equations and convoluted with 'partonic processes'

- This "QED structure functions method" is really easy (!) and has been widely used at LEP and HERA to calculate the QED radiative corrections in the leading log approximation (which reproduces the exact result to 1% ... 2% accuracy)

...more to say...



- There is a fundamental difference between QED and QCD!

- The results for QED are reliable in fixed order perturbation theory or we can subtract the long distance terms and introduce QED structure functions as a technique to resum large collinear logs to all orders using the renormalization group

- On the other hand, a quark propagating a long distance will hadronize!

  The results with an almost on-shell free quark propagator are certainly not reliable and we are FORCED to subtract these long distance pieces from the cross section (and to replace them by experimentally determined PDFs)

Mass factorization

Let's calculate a cross section/structure function in the QCD improved parton model at NLO!

It is given by a convolution of the partonic cross section (already renormalized) with PDFs:

$$\sigma = \sum_i \tilde{f}_i \otimes \tilde{\sigma}_i$$

independent of **μ=μR** depends on **μ=μR**

The partonic cross section still contains collinear singularities (poles in 1/eps in dim. reg.).

The collinear pieces factorize into universal factorization constants and hard scattering cross sections. This is the crucial mass factorization relation.

$$\tilde{\sigma}_i = \sum_l \Gamma_{li} \otimes \hat{\sigma}_l$$

factorization constants containing the divergence

finite (IR-safe) hard scattering cross sections

Mass factorization

The factorization constants have to be determined once and for all. They can then be used in all kinds of process to do the mass factorization (analogue: renormalization constants)

Using dim. reg. with **d=4+ε** dimensions

$$\Gamma_{ij}(z,\mu_F^2,\mu^2) = \delta_{ij}\delta(1-z) + \Gamma_{ij}^{(1)}(z,\mu_F^2,\mu^2)$$

$$\Gamma_{ij}^{(1)} = \frac{\alpha_s(\mu^2)}{2\pi}\left[P_{ij}(z)\frac{2}{\epsilon} + F_{ij}\right]$$

μ=μR is the scale of dim. reg.
μF is the factorization scale

$$F_{ij}^{\overline{\text{MS}}} = P_{ij}(z)(\gamma_E - \ln(4\pi) + \ln\frac{\mu_F^2}{\mu^2})$$

We can now express the cross section as convolution of renormalized PDFs with hard scattering cross sections:

$$\sigma = \sum_i \sum_l \tilde{f}_i \otimes \Gamma_{li} \otimes \hat{\sigma}_i = \sum_l f_l \otimes \hat{\sigma}_l \quad \text{with} \quad f_l = \sum_i \Gamma_{li} \otimes \tilde{f}_i$$

The desired end result:

renormalized PDFs

Construction of the hard scattering cross sections

$$\tilde{\sigma}_i = \sum_l \Gamma_{li} \otimes \hat{\sigma}_l$$

The mass factorization relation can be inverted in a **perturbative approach**

$$\Gamma_{ij} = \delta_{ij} + \Gamma_{ij}^{(1)} + \ldots$$

$$\tilde{\sigma} = \tilde{\sigma}^{(0)} + \tilde{\sigma}^{(1)} + \ldots$$

$$\hat{\sigma} = \hat{\sigma}^{(0)} + \hat{\sigma}^{(1)} + \ldots$$

Entering the perturbative expressions into the mass factorization formula and comparing the lhs with the rhs we find iteratively (exercise!):

$$\hat{\sigma}_i^{(0)} = \tilde{\sigma}_i^{(0)}$$

$$\hat{\sigma}_i^{(1)} = \tilde{\sigma}_i^{(1)} - \Gamma_{li}^{(1)} \otimes \tilde{\sigma}_l^{(0)} = \tilde{\sigma}_i^{(1)} - \Gamma_{li}^{(1)} \otimes \hat{\sigma}_l^{(0)}$$

Wilson Coefficients

In DIS the hard scattering cross sections are called Wilson coefficients:

$$x^{-1}F_2 = q \otimes C_{2,q} + g \otimes C_{2,g}$$
$$x^{-1}F_L = q \otimes C_{L,q} + g \otimes C_{L,g}$$
$$F_1 = q \otimes C_{1,q} + g \otimes C_{1,g}$$

The Wilson coefficients have a perturbative expansion in the strong coupling constant:

$$C_{i,q}(z, Q^2/\mu^2) = C_{i,q}^{(0)}(z) + C_{i,q}^{(1)}(z, Q^2/\mu^2) + \ldots$$
$$C_{i,g}(z, Q^2/\mu^2) = C_{i,g}^{(0)}(z) + C_{i,g}^{(1)}(z, Q^2/\mu^2) + \ldots$$

We recover the leading order parton model results:

$$C_{2,q}^{(0)}(z, Q^2, Q^2/\mu^2) = e_q^2\delta(1-z), C_{2,g}^{(0)} = 0, C_{L,q}^{(0)} = 0, C_{L,g}^{(0)} = 0$$
$$F_2(x, Q^2) = x\sum_q e_q^2(q+\bar{q})(x, Q^2) = 2xF_1(x, Q^2), F_L(x, Q^2) = 0$$

Higher order Wilson coefficient functions

Reference	Boson	SFs	Order	Coefficients	Scheme	Comments
BBDM'78 [18]	NC,CC$^{\pm}$	F_2, F_L, F_3	α_S^1	C_2	$\overline{\mathrm{MS}}$	$C^{(1)}_{3,+}(x) = C^{(1)}_{3,-}(x)$
AEM'78 [70]	NC,CC$^{\pm}$	F_2	α_S^1	C_2	$\overline{\mathrm{MS}}$	—
FP'82 [62]	NC,CC$^{\pm}$	F_2	α_S^1	C_2	$\overline{\mathrm{MS}}$	—
GMMPS'91 [71]	NC,CC^{+}	F_L	α_S^2	$C_{L,q}(x)$, $C_{L,g}(x)$	$\overline{\mathrm{MS}}$	$C_{L,g}$ corrected in [72]
NZ'91 [73]	NC,CC$^{\pm}$	F_2	α_S^2	$C_{2,q}(x)$	$\overline{\mathrm{MS}}$	first calc.
ZN'91 [72]	NC,CC^{+}	F_2, F_L	α_S^2	$C_{2,g}(x), C_{L,g}(x)$	$\overline{\mathrm{MS}}$	first calc.
ZN'92 [74]	NC,CC^{+}	F_2	α_S^2	C_2	$\overline{\mathrm{MS}}$	—
NV'00 [75]	NC,CC^{+}	F_2, F_L	α_S^2	C_2^{NS}	$\overline{\mathrm{MS}}$	x-space param.
NV'00 [76]	NC,CC^{+}	F_2	α_S^2	C_2^{S}	$\overline{\mathrm{MS}}$	x-space param.
ZN'92 [77]	NC,CC^{+}	F_3	α_S^2	$C^{(2)}_{3,-}(x)$	$\overline{\mathrm{MS}}$	first calc.
MV'00 [78]	NC,CC^{+}	F_2, F_L, F_3	α_S^2	—	$\overline{\mathrm{MS}}$	all N, confirms [72, 73, 77]
MRV'08 [79]	CC^{-}	F_2, F_L, F_3	α_S^2	$\delta C^{(2)}_{2,L,3}(x)$	$\overline{\mathrm{MS}}$	x-space param., $\delta C^{(2)}_L$ new
MVV'09 [80]	CC^{+}	F_3	α_S^2	$C^{(2)}_{3,-}(x)$	$\overline{\mathrm{MS}}$	x-space param.
VVM'05 [81]	NC,CC^{+}	F_2, F_L	α_S^3	C_2, C_L	$\overline{\mathrm{MS}}$	x-space calc. and param.
MVV'02 [82]	NC,CC^{+}	F_2	α_S^3	C_2^{NS}	$\overline{\mathrm{MS}}$	x-space param.
MVV'05 [83]	NC,CC^{+}	F_L	α_S^3	C_L^{NS}	$\overline{\mathrm{MS}}$	x-space param.
MR'07 [84]	CC^{-}	F_2, F_L, F_3	α_S^3	—	$\overline{\mathrm{MS}}$	N-space, fixed $N \leq 10$
MRV'08 [79]	CC^{-}	F_2, F_L, F_3	α_S^3	$\delta C^{(2)}_{2,L,3}(N)$	$\overline{\mathrm{MS}}$	N-space, first 5 moments
MVV'09 [80]	CC^{+}	F_3	α_S^3	—	$\overline{\mathrm{MS}}$	x-space calc.

Table 2.1: Massless higher order Wilson coefficient functions in the literature. 'NC' corresponds to neutral current DIS with γ and Z exchange while 'CC$^{\pm}$' stands for charged current DIS with $W^+ \pm W^-$ exchange.

XII. Parton evolution

Scale dependence

$$W^{\mu\nu} = \sum_i \int_0^1 \frac{d\xi}{\xi} f_i(\xi, \mu_F^2, \mu_R^2) \hat{w}_i^{\mu\nu}(\xi, Q^2, \mu_F^2/Q^2, \mu_R^2/Q^2, \alpha_s(\mu_R^2)) + \mathcal{O}(Q^2/\Lambda^2)$$

- The PDFs depend on the **factorization scale μ_F** and the **renormalization scale μ_R**.
- Beyond LO the PDFs depend on the **factorization/renormalization scheme**.
- In **DIS** usually the scale choice **$\mu_F=\mu_R=Q$** is made
- The hard part is also scheme dependent (beyond LO) and one has to use the <u>same</u> scheme as for the PDFs and alphas.
- **Physical observables do not depend on the scales and schemes** up to missing higher orders in the perturbation series.

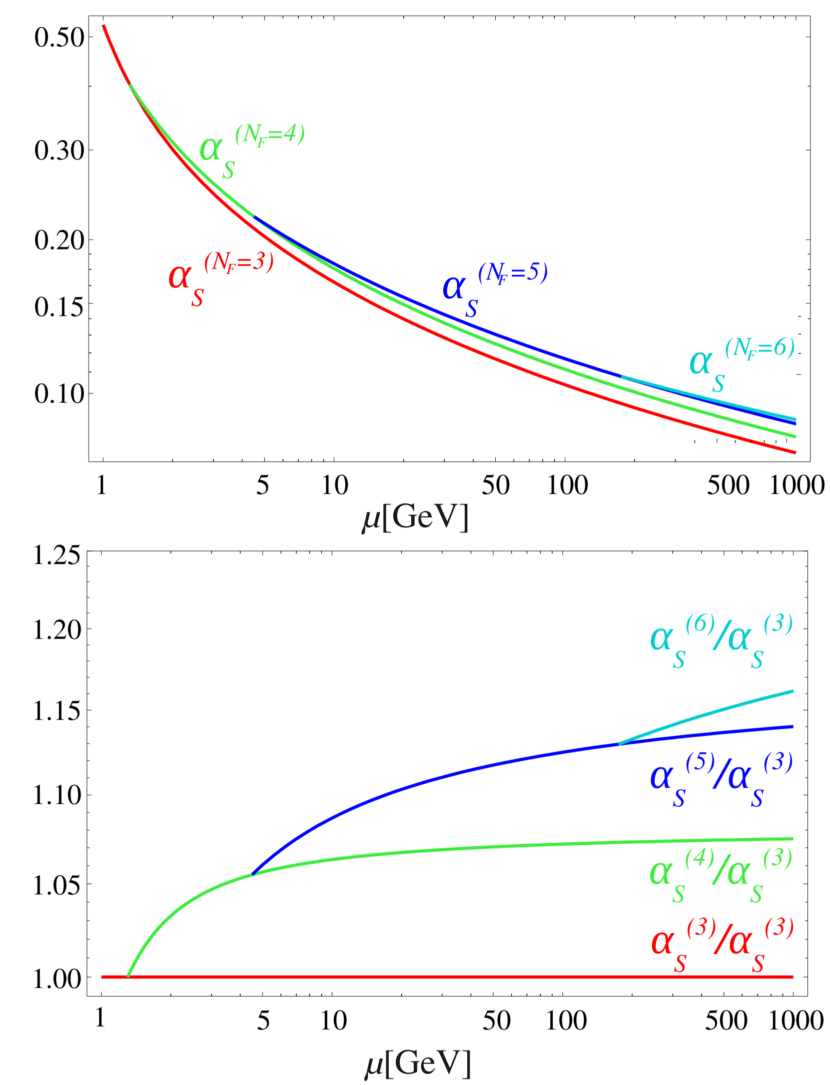
Scale dependence of a_s

The scale dependence of the strong coupling constant is also governed by a renormalization group equation:

$$\frac{\partial a_s}{\partial \ln \mu^2} = \beta[a_s(\mu^2)] = -(\beta_0 a_s^2 + \beta_1 a_s^3 + \beta_2 a_s^4 + \ldots)$$

$$a_s := \frac{\alpha_s}{4\pi} \quad , \quad \beta_0 = 11 - \frac{2}{3} n_f \quad , \quad \beta_1 = 102 - \frac{38}{3} n_f \quad \ldots$$

The QCD beta-function is negative at small **a_s** which leads to asymptotic freedom:



Scale dependence of PDFs

We had introduced renormalized dressed PDFs $f_l(x,\mu_F^2)$:

$$f_l(x,\mu_F^2) = \tilde{f}_i(y,\mu^2) \otimes \Gamma_{li}(z,\mu_F^2,\mu^2)$$

The dependence of $f_l(x,\mu_F^2)$ is necessitated by the fact that the physical cross section has to be independent of μ_F

depends on μ (see above)

$$\Gamma_{ij}(z,\mu_F^2,\mu^2) = \delta_{ij}\delta(1-z) + \Gamma_{ij}^{(1)}(z,\mu_F^2,\mu^2)$$

$$\Gamma_{ij}^{(1)} = \frac{\alpha_s(\mu^2)}{2\pi}\left[P_{ij}(z)\frac{2}{\epsilon} + F_{ij}\right]$$

$$F_{ij}^{\overline{\text{MS}}} = P_{ij}(z)(\gamma_E - \ln(4\pi) + \ln\frac{\mu_F^2}{\mu^2})$$

We can then calculate the logarithmic derivative w.r.t. μ_F:

$$\frac{\partial f_i(x,\mu_F^2)}{\partial \ln \mu_F^2} = \frac{\alpha_s(\mu^2)}{2\pi} P_{ij}(z) \otimes \tilde{f}_j(y,\mu^2)$$

$$\underset{\text{all LL}}{\overset{\mathcal{O}(\alpha_s)}{=}} \frac{\alpha_s(\mu_F^2)}{2\pi} P_{ij}(z) \otimes f_j(y,\mu_F^2)$$

See F. E. Paige, QCD and Event Simulation, TASI lecture 1989

Scale dependence

The scale dependence of the PDFs is governed by a coupled system of integro-differential equations, the **DGLAP** (Dokshitzer, Gribov, Lipatov, Altarelli, Parisi) **evolution equations**:

$$\frac{\partial f_i}{\partial \ln \mu^2} = P_{ij}(x,\mu^2) \otimes f_j(x,\mu^2)$$

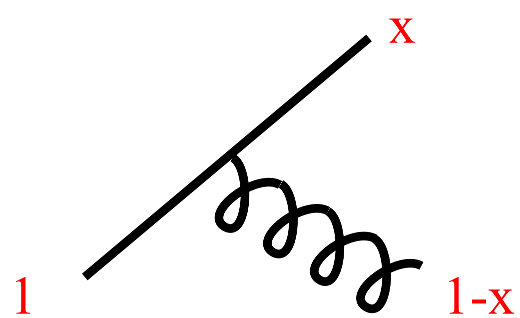
Here, a summation over all parton species **'j'** is understood. **Pij** are the **QCD splitting functions** which are known perturbatively up to NNLO (3-loop) and $\otimes$ stands for the **Mellin convolution** of two functions/distributions with support in [0,1]:

$$\begin{aligned}(f \otimes g)(x) &= \int_0^1 dy \int_0^1 dz\; \delta(x-yz) f(y) g(z) \\ &= \int_0^1 \frac{dy}{y}\; \theta(0 \leq x/y \leq 1) f(y) g(x/y) \\ &= \int_x^1 \frac{dy}{y}\; f(y) g(x/y) = \int_x^1 \frac{dy}{y}\; g(y) f(x/y)\end{aligned}$$

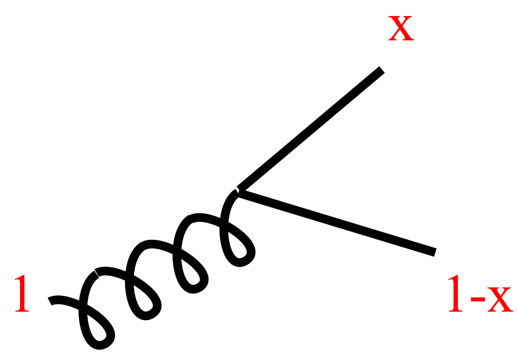
The leading order Splitting Functions

Read backwards

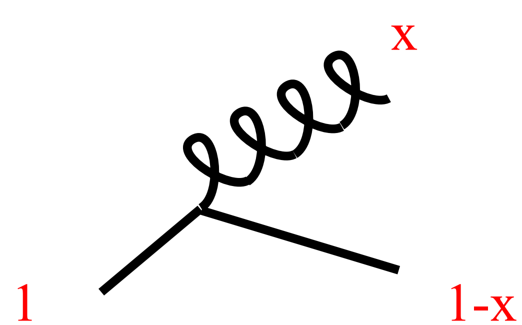
Note singularities



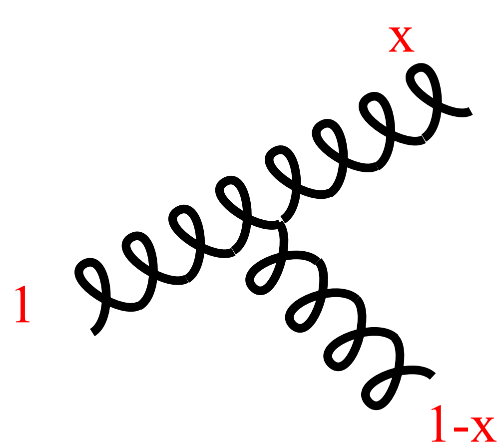
$$P_{qq}^{(1)}(x) = C_F \left[\frac{1+x^2}{1-x}\right]_+$$



$$P_{qg}^{(1)}(x) = T_F \left[(1-x)^2 + x^2\right]$$



$$P_{gq}^{(1)}(x) = C_F \left[\frac{(1-x)^2+1}{x}\right]$$



$$P_{gg}^{(1)}(x) = 2C_F \left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right]$$

$$+ \left[\frac{11}{6}C_A - \frac{2}{3}T_F N_F\right] \ \delta(1-x)$$

Homework

Definition of the Plus prescription:

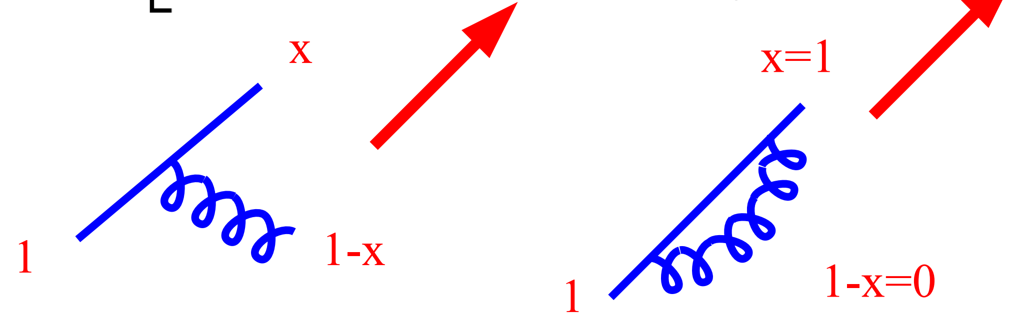
$$\int_0^1 dx \, \frac{f(x)}{(1-x)_+} = \int_0^1 dx \, \frac{f(x)-f(1)}{(1-x)}$$

Compute:

$$\int_a^1 dx \, \frac{f(x)}{(1-x)_+} = ???$$

Verify:

$$P_{qq}^{(1)}(x) = C_F \left[\frac{1+x^2}{1-x}\right]_+ \equiv C_F \left[(1+x^2)\left[\frac{1}{1-x}\right]_+ + \frac{3}{2}\,\delta(1-x)\right]$$

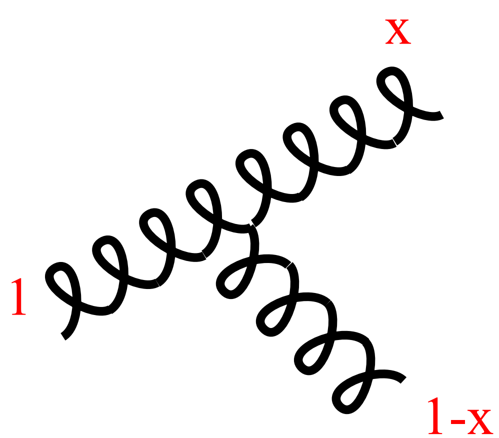


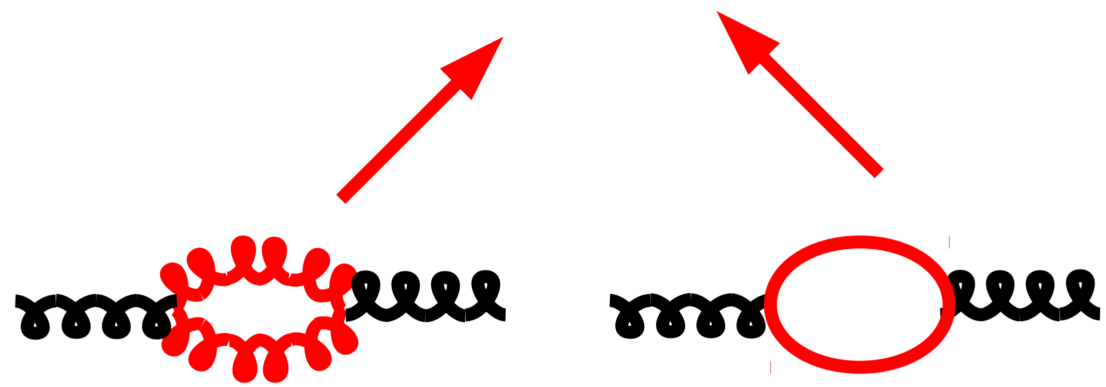
Homework

Observe

$$P_{gg}^{(1)}(x) = 2C_F\left[\frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x)\right]$$

$$+\left[\frac{11}{6}C_A - \frac{2}{3}T_F\,N_F\right]\;\delta(1-x)$$



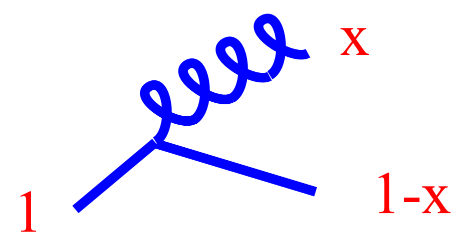


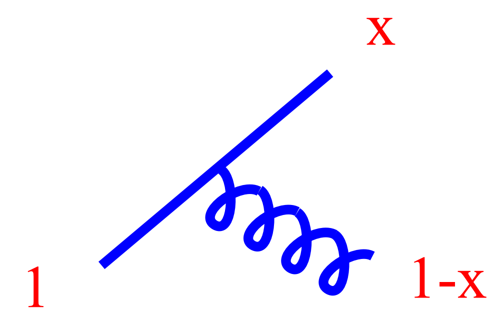
Homework: Symmetries and Limits

Verify the following relation among the regular parts (from the real graphs)

For the regular part show:

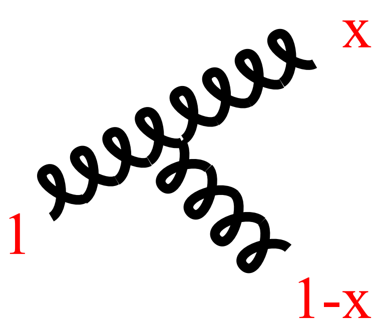
$$P_{gq}^{(1)}(x) = P_{qq}^{(1)}(1-x)$$





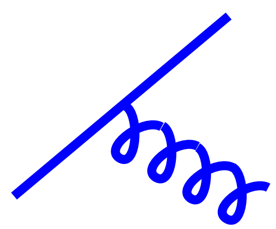
For the regular part show:

$$P_{gg}^{(1)}(x) = P_{gg}^{(1)}(1-x)$$

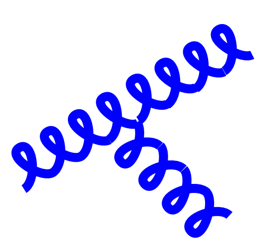


Verify, in the soft limit:

$$P_{qq}^{(1)}(x) \xrightarrow[x\to 1]{} 2C_F \frac{1}{(1-x)_+}$$



$$P_{gg}^{(1)}(x) \xrightarrow[x\to 1]{} 2C_F \frac{1}{(1-x)_+}$$



Homework: Conservation rules

Verify conservation of momentum fraction

$$\int_0^1 dx\, x\, \left[P_{qq}(x) + P_{gq}(x)\right] = 0$$

$$\int_0^1 dx\, x\, \left[P_{qg}(x) + P_{gg}(x)\right] = 0$$

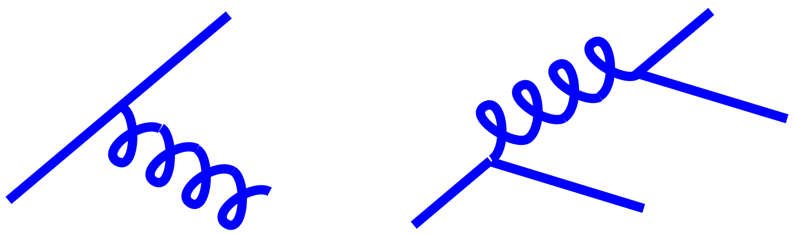
Verify conservation of fermion number

$$\int_0^1 dx \quad \left[P_{qq}(x) - P_{q\bar{q}}(x)\right] = 0$$

Homework: Using the real to guess the virtual

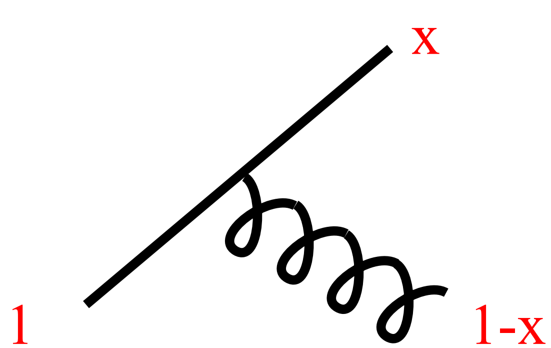
Use conservation of fermion number to compute
the delta function term in P(q←q)

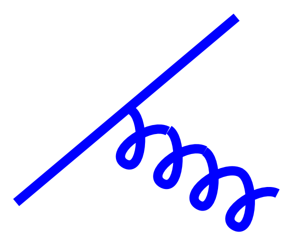
$$\int_0^1 dx \quad [P_{qq}(x) - P_{q\bar{q}}(x)] = 0$$

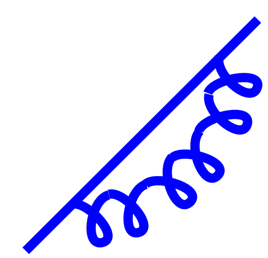


This term only starts at NNLO

$$P_{qq}^{(1)}(x) = C_F \left[\frac{1+x^2}{1-x}\right]_+ \equiv C_F \left[(1+x^2)\left[\frac{1}{1-x}\right]_+ + \frac{3}{2}\,\delta(1-x)\right]$$





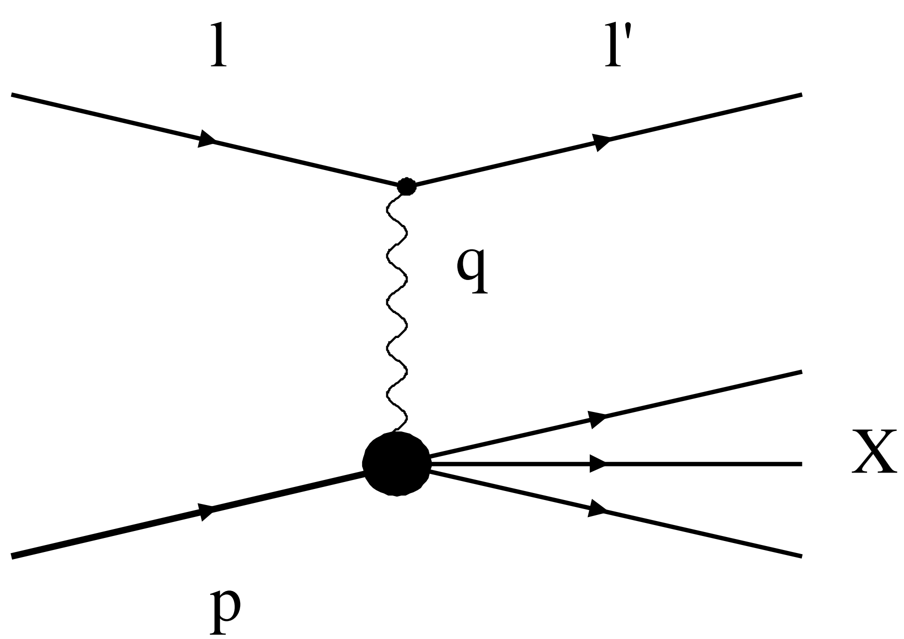


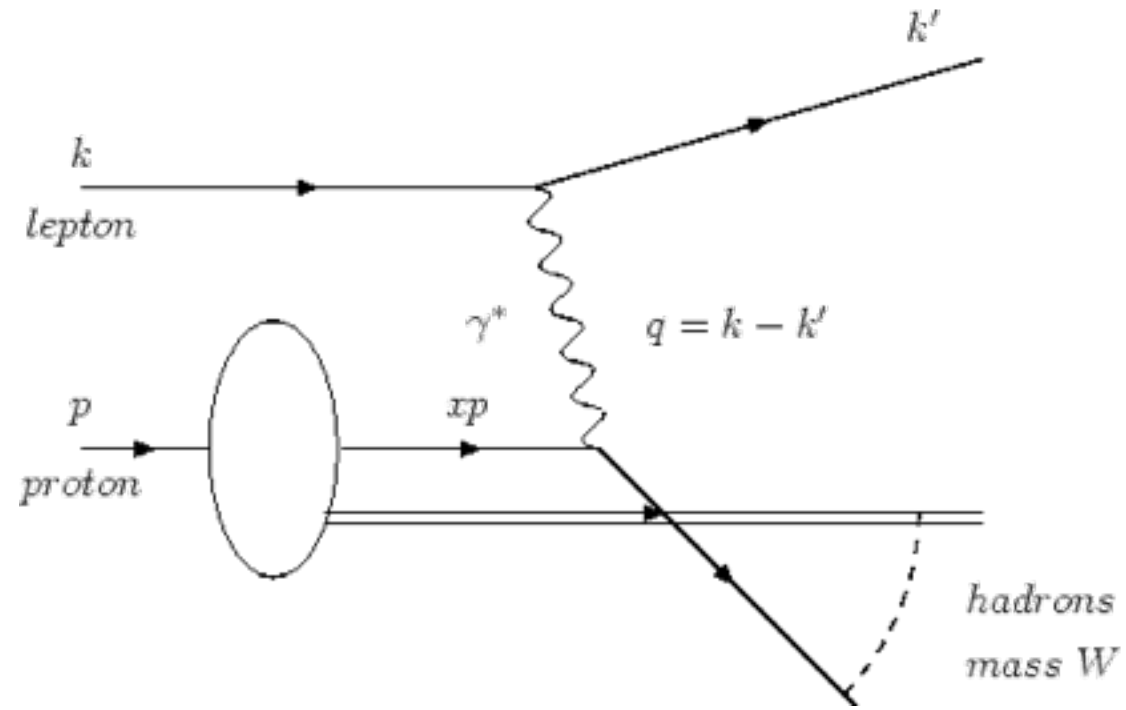
Powerful tool: Since we know real and virtual must balance, we can use to our advantage!!!

The End of Lecture 2

Quark masses in the parton model

Massless Parton Model





- In factorization ansatz: relate 4-momenta of partons to 4-momenta of hadrons

$$\mathrm{d}\sigma[P] = \int_0^1 dx\, f(x) \mathrm{d}\hat{\sigma}[\hat{p} = xP] \quad \text{with} \quad P^2 = \hat{p}^2 = 0$$

- In dynamics: massless parton propagators
- In kinematics: massless partons in phase space
- At higher orders: calculate in n dimensions, renormalization and mass factorization in $\overline{\mathrm{MS}}$
- However, not fully massless (would be unphysical!)
  Example: No contribution from $\gamma^\star c \to c$ to DIS structure functions at scale $Q < m_c$ (overestimation).

LO parton kinematics

Proton momentum: $$P_\mu = (P_0, P_x, P_y, P_z)$$

Light cone coordinates: $$P_\mu = (P^+, P^-, \vec{P}_T)$$

$$P^\pm = (P_0 \pm P_z)/\sqrt{2}$$

Useful if strongly ordered: $$P_\mu \simeq (P^+, M^2/(2P^+), \vec{0}_T)$$

$$P^+ \gg P^-, \vec{P}_T$$

Parton momentum: $$\hat{p}_\mu \simeq (\xi P^+, m^2/(2\xi P^+, \vec{0}_T)$$

LO parton kinematics

Momentum conservation: $\delta^{(4)}(q + \hat{p}_i - \hat{p}_j) \to \delta(\xi - \chi)$

$$M^2 = m^2 = 0 \Rightarrow \chi = x$$

Nachtmann variable

$$M^2 \neq 0, m^2 = 0 \Rightarrow \chi = xR_M$$

$$M^2 \neq 0, m^2 \neq 0 \Rightarrow \chi = xR_M R_{ij}$$

Crucial observation: target mass and quark mass effects factorize!

where

$$R_M = \frac{2}{1 + \sqrt{1 + (4x^2M^2/Q^2)}} = \frac{2}{1+r} \,,$$

$$R_{ij} = \frac{Q^2 - m_i^2 + m_j^2 + \Delta(-Q^2, m_i^2, m_j^2)}{2Q^2} \,,$$

$$\Delta(a,b,c) = \sqrt{a^2 + b^2 + c^2 - 2(ab + bc + ca)} \,,$$

ACOT scheme

Theoretical basis: Factorization theorem with massive quarks

J. Collins '98

$$d\sigma[P] = \sum_i \int_0^1 d\xi f_i(\xi, \mu_F^2) d\hat{\sigma}(\gamma^* i \to cX)[\mu_R^2, \mu_F^2/Q^2, m^2/Q^2]|_{\hat{p}^+=\xi P^+} + \mathcal{O}(\Lambda^2/Q^2)$$

Sum over all possible subprocesses

Mass term contained in the hard scattering coefficient

massive hard scattering cross section; IR safe

massive partonic cross section; not yet IR safe

collinear subtraction term, mass fact. with massive regulator

$$d\hat{\sigma}[m] = d\tilde{\sigma}[m] - d\sigma^{\text{sub}}$$

Target mass corrections

Operator product expansion

$$\int d^4x\, e^{iq\cdot x}\langle N|T(J^\mu(x)J^\nu(0))|N\rangle$$

$$= \sum_k \left(-g^{\mu\nu}q^{\mu_1}q^{\mu_2} + g^{\mu\mu_1}q^\nu q^{\mu_2} + q^\mu q^{\mu_1}g^{\nu\mu_2} + g^{\mu\mu_1}g^{\nu\mu_2}Q^2\right)$$

$$\times q^{\mu_3}\cdots q^{\mu_{2k}}\frac{2^{2k}}{Q^{4k}}\underbrace{A_{2k}\Pi_{\mu_1\cdots\mu_{2k}}}_{\langle N|\mathcal{O}_{\mu_1\cdots\mu_{2k}}|N\rangle}$$

local operators

Georgi, Politzer (1976)

$$\Pi_{\mu_1\cdots\mu_{2k}} = p_{\mu_1}\cdots p_{\mu_{2k}} - (g_{\mu_i\mu_j}\ \text{terms})$$

$$= \sum_{j=0}^{k}(-1)^j\frac{(2k-j)!}{2^j(2k)^j}g\cdots g\ p\cdots p$$

traceless, symmetric
rank-$2k$ tensor

OPE

- n-th Cornwall-Norton moment of F_2 structure function

$$M_2^n(Q^2) = \int dx\; x^{n-2}\; F_2(x, Q^2)$$

$$= \sum_{j=0}^{\infty} \left(\frac{M^2}{Q^2}\right)^j \frac{(n+j)!}{j!(n-2)!} \frac{A_{n+2j}}{(n+2j)(n+2j-1)}$$

"quark distribution function"

$$\Rightarrow \quad A_n = \int_0^1 dy\; y^n\; F(y) \qquad\qquad F(y) \equiv \frac{F_2(y)}{y^2}$$

TMC

- take inverse Mellin transform (+ tedious manipulations)

  ⇒ target mass corrected structure function

$$F_2^{\mathrm{GP}}(x,Q^2) = \frac{x^2}{r^3}F(\xi) \; + \; 6\frac{M^2}{Q^2}\frac{x^3}{r^4}\int_\xi^1 d\xi' \; F(\xi')$$

$$+ \; 12\frac{M^4}{Q^4}\frac{x^4}{r^5}\int_\xi^1 d\xi' \; \int_{\xi'}^1 d\xi'' \; F(\xi'')$$

Nachtmann variable

$$\eta = \quad \xi = \frac{2x}{1+r} \qquad\qquad r = \sqrt{1+4x^2M^2/Q^2}$$

... similarly for other structure functions F_1, F_L

TMC: Master formula

$$
\begin{aligned}
F_1^{\mathrm{TMC}}(x,Q^2) &= \frac{x}{\eta r} F_1^{(0)}(\eta,Q^2) + \frac{M^2 x^2}{Q^2 r^2} h_2(\eta,Q^2) + \frac{2M^4 x^3}{Q^4 r^3} g_2(\eta,Q^2)\ , \\
F_2^{\mathrm{TMC}}(x,Q^2) &= \frac{x^2}{\eta^2 r^3} F_2^{(0)}(\eta,Q^2) + \frac{6M^2 x^3}{Q^2 r^4} h_2(\eta,Q^2) + \frac{12M^4 x^4}{Q^4 r^5} g_2(\eta,Q^2)\ , \\
F_3^{\mathrm{TMC}}(x,Q^2) &= \frac{x}{\eta r^2} F_3^{(0)}(\eta,Q^2) + \frac{2M^2 x^2}{Q^2 r^3} h_3(\eta,Q^2) + 0\ ,
\end{aligned}
$$

- Modular, easy to use!
- Resums leading twist TMC to all orders in $(M^2/Q^2)^n$
- Input: standard structure functions in the parton model with M=0
  - any order in alpha_s
  - can include quark masses

TMC important at large x and small Q^2

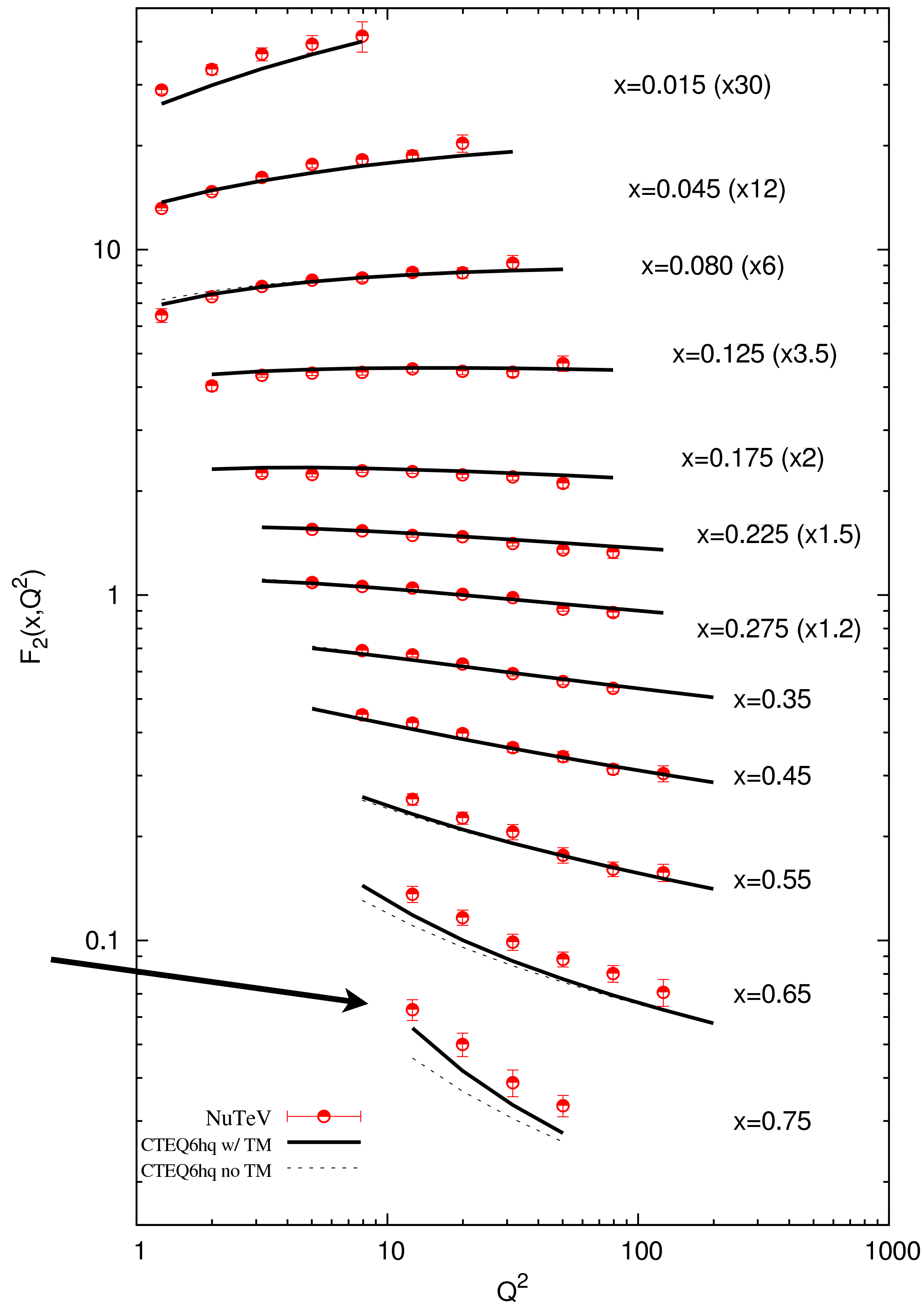


Figure 9. Comparison of the F_2 structure function, with and without target mass corrections, and NuTeV data [64]. The base PDF set is CTEQ6HQ [7].

QCD studies with neutrinos

Flavor separation of PDFs

NC charged lepton DIS: 2 structure functions (γ-exchange)

$$F_2^{\gamma}(x) \sim \frac{1}{9}[4(u+\bar{u}+c+\bar{c})+d+\bar{d}+s+\bar{s}](x)$$

$$F_2^{\gamma}(x) = 2xF_1^{\gamma}(x)$$

CC Neutrino DIS: 6 additional structure functions $F_{1,2,3}^{W+}$, $F_{1,2,3}^{W-}$

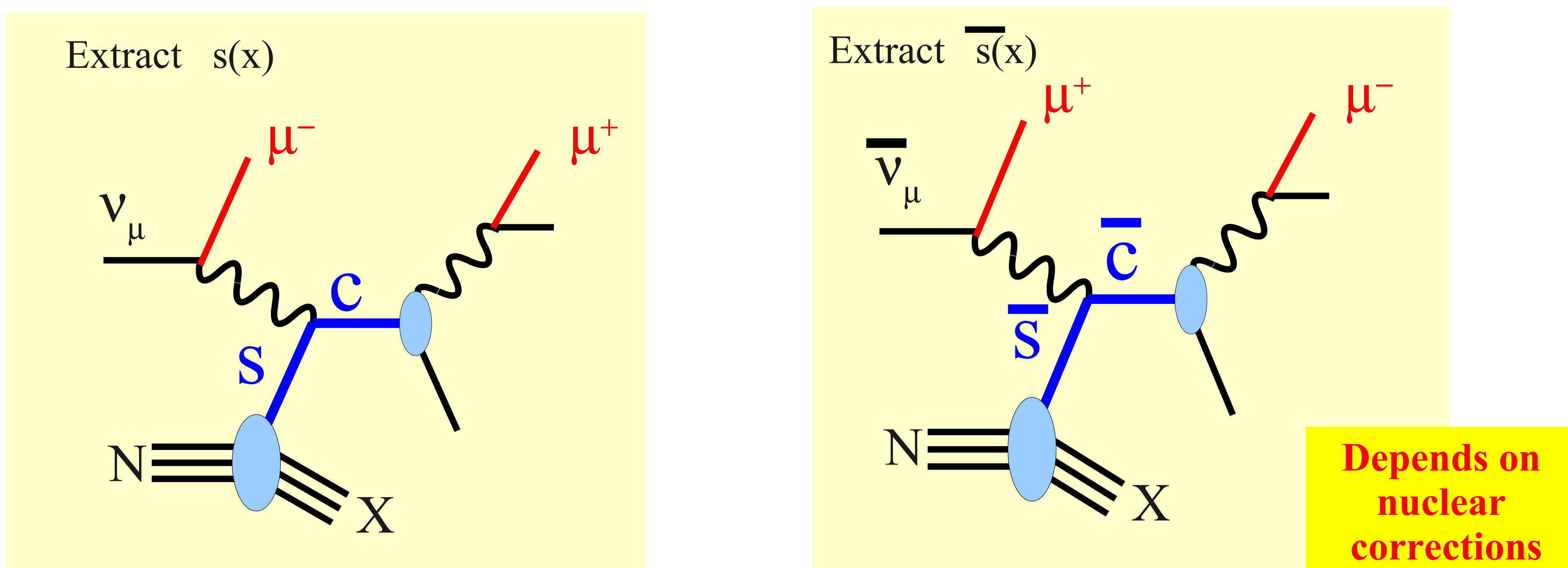
$$F_2^{W^+} \sim [d+s+\bar{u}+\bar{c}] \qquad F_3^{W^+} \sim 2[d+s-\bar{u}-\bar{c}]$$

$$F_2^{W^-} \sim [\bar{d}+\bar{s}+u+c] \qquad F_3^{W^-} \sim 2[u+c-\bar{d}-\bar{s}]$$

Useful/needed to disentangle different quark parton flavors in a global analysis of proton or nuclear PDFs

Dimuon production and the strange PDF

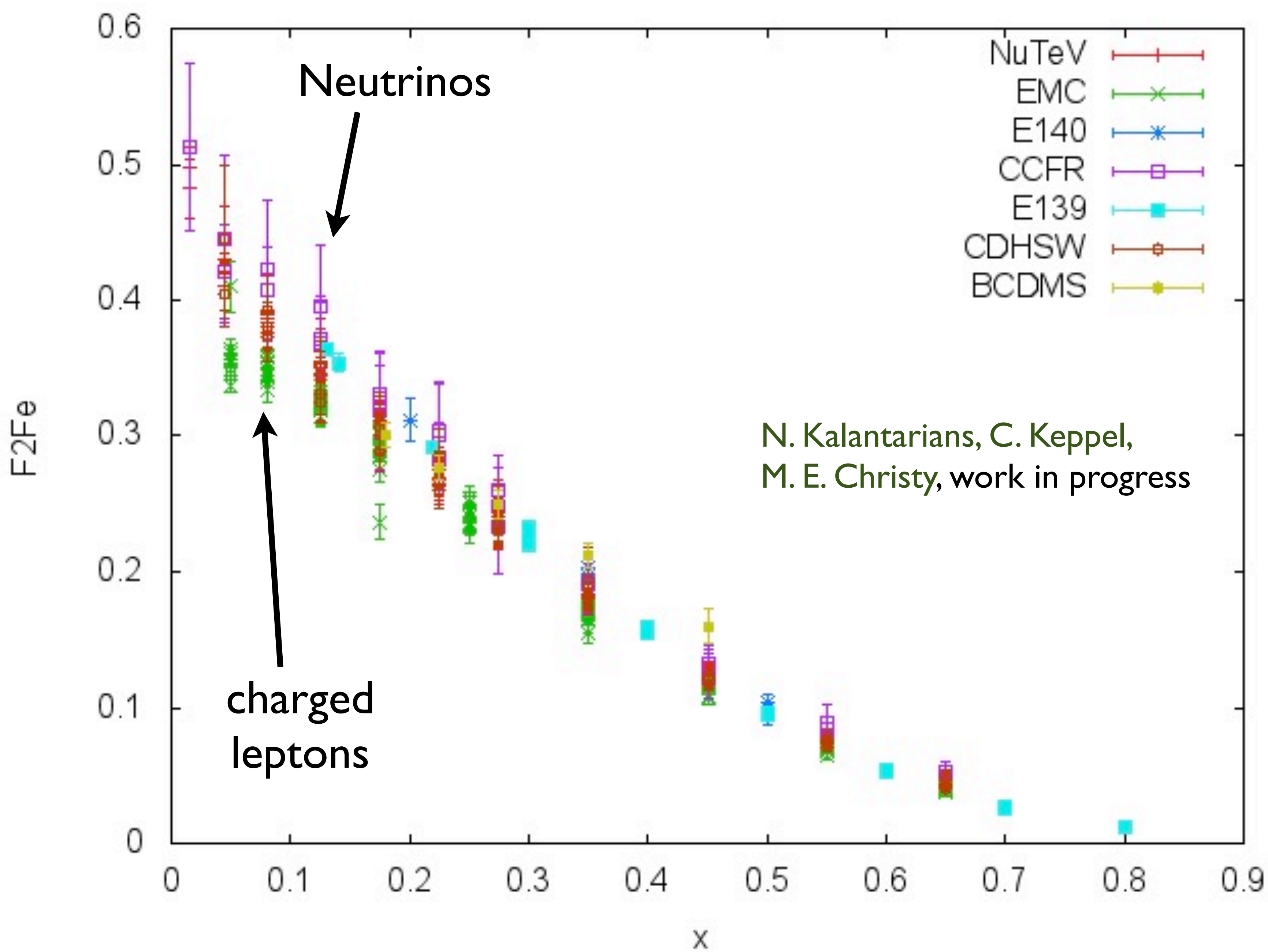
Opposite sign dimuon production in neutrino DIS: $\nu N \to \mu^+\mu^- X$



- High-statistics data from CCFR and NuTeV: **Main source** of information!
- x~[0.01,0.4]
- νFe DIS: need nuclear corrections! Problem: Final State Interactions (FSI)
- CHORUS (νPb): compatible with NuTeV, could be included
- NOMAD (νFe): data not yet published, in principle very interesting

World data on 18/5 F_2^{NC} and F_2^{CC} on iron

$$\Delta F_2 = \frac{5}{18} F_2^{CC} - F_2^{NC} \simeq \frac{x}{6}[s(x) + \bar{s}(x)]$$



Data available at Durham database;

Data brought to the same Q^2=8 GeV^2

Info on nuclear corrections in ν-Fe DIS vs l-Fe DIS:
Advantage: no deuterium

Info on strange PDF in iron:

Advantage: inclusive, no FSI

Disadvantage: difference of two large numbers

xF_3 and Isospin Violation

- xF_3 uniquely determined by neutrino-DIS

$$\boxed{\begin{aligned} \frac{1}{2}F_3^{\nu A}(x) &= d_A + s_A - \bar{u}_A - \bar{c}_A + ..., \\ \frac{1}{2}F_3^{\bar{\nu} A}(x) &= u_A + c_A - \bar{d}_A - \bar{s}_A + ... \end{aligned}}$$

- The sum is sensitive to the valence quarks

→ Nonsinglet QCD evolution, determination of $\alpha_s(Q)$

- The difference can be used to constrain isospin violation

$$\boxed{\begin{aligned} \Delta x F_3 &= xF_3^{\nu A} - xF_3^{\bar{\nu} A} = 2xs_A^+ - 2xc_A^+ + x\,\delta I^A + \mathcal{O}(\alpha_S) \\ \delta I^A &= \left(d_{p/A} - u_{n/A}\right) + \left(d_{n/A} - u_{p/A}\right) + \left(\bar{d}_{p/A} - \bar{u}_{n/A}\right)\left(\bar{d}_{n/A} - \bar{u}_{p/A}\right) \end{aligned}}$$

Hadronic Precision Observables

$$R^{\nu} = \frac{\sigma^{\nu}_{\rm NC}}{\sigma^{\nu}_{\rm CC}} \simeq g_L^2 + r g_R^2$$

$$R^{\bar\nu} = \frac{\sigma^{\bar\nu}_{\rm NC}}{\sigma^{\bar\nu}_{\rm CC}} \simeq g_L^2 + r g_R^2$$

$$r = \frac{\sigma^{\bar\nu}_{\rm CC}}{\sigma^{\nu}_{\rm CC}}$$

g_L and g_R are effective L and R νq couplings

$$g_L^2 = \rho^2\left(\frac{1}{2} - s_w^2 + \frac{5}{9}s_w^4\right)$$

$$g_R^2 = \rho^2\left(\frac{5}{9}s_w^4\right)$$

Paschos-Wolfenstein (PW):

$$R^{-} = \frac{\sigma^{\nu}_{\rm NC} - \sigma^{\bar\nu}_{\rm NC}}{\sigma^{\nu}_{\rm CC} - \sigma^{\bar\nu}_{\rm CC}} \simeq g_L^2 - g_R^2 = \rho^2\left(\frac{1}{2} - s_w^2\right)$$

QCD for PW-style analysis

$$R^{-} = \frac{\sigma^{\nu}_{\mathrm{NC}} - \sigma^{\bar{\nu}}_{\mathrm{NC}}}{\sigma^{\nu}_{\mathrm{CC}} - \sigma^{\bar{\nu}}_{\mathrm{CC}}} \simeq \frac{1}{2} - s_w^2 + \delta R^{-}_{A} + \delta R^{-}_{QCD} + \delta R^{-}_{EW}$$

non-isoscalarity of the target

QCD effects

higher order ew effects

$$\delta R^{-}_{QCD} = \delta R^{-}_{s} + \delta R^{-}_{I} + \delta R^{-}_{NLO}$$

due to strangeness asymmetry:

$$s^{-} \equiv s - \bar{s} \neq 0$$

due to isospin violation:

$$u^{p}(x) \neq d^{n}(x)$$

higher order QCD effects

see, e.g., hep-ph/0405221