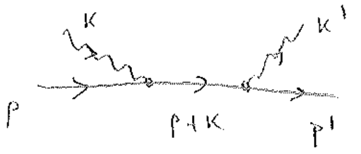
6.2)

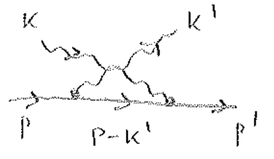
Diffusion Compton : $e^-(p) + \gamma(k) \to e^-(p') + \gamma(k')$, $p^2 = p'^2 = m^2$, $k^2 = k'^2 = 0$

(1.)



„s"

+



„u"

1. Diagrammes de Feynman
2. Cinématique
3. M_{fi}
4. $\overline{|M|}^2$
5. $d\sigma$

(2.)

$$s = (p+k)^2 = m^2 + 2pk$$
$$= (p'+k')^2 = m^2 + 2p'k'$$

$$t = (p-p')^2 = 2m^2 - 2pp'$$
$$= (k-k')^2 = -2kk'$$

$$u = (p-k')^2 = m^2 - 2pk'$$
$$= (p'-k)^2 = m^2 - 2p'k$$

$$s+t+u = \sum m_i^2 = 2m^2$$

$$s_1 = s - m^2 = 2pk = 2p'k'$$
$$u_1 = u - m^2 = -2pk' = -2p'k$$
$$s_1 + t + u_1 = 0$$

(3.)

$$M_{fi} = \bar{u}(p')\left[ie\,\not{\epsilon}'\,\frac{i}{\not{p}+\not{k}-m}\,ie\,\not{\epsilon} + ie\,\not{\epsilon}\,\frac{i}{\not{p}-\not{k}'-m}\,ie\,\not{\epsilon}' \right] u(p)$$

$$= -ie^2\,\epsilon_\nu(k',\lambda')\,\epsilon_\mu(k,\lambda)\,\bar{u}(p')\left[\frac{\gamma^\nu(\not{p}+\not{k}+m)\gamma^\mu}{s_1} + \frac{\gamma^\mu(\not{p}-\not{k}'+m)\gamma^\nu}{u_1} \right] u(p)$$

$$= -ie^2\,\epsilon'_\nu\,\epsilon_\mu\,\bar{u}(p')\left[\frac{\gamma^\nu\not{k}\gamma^\mu - \gamma^\nu\gamma^\mu(\not{p}-m) + 2p^\mu\gamma^\nu}{s_1} + \frac{\gamma^\mu(-\not{k}')\gamma^\nu - \gamma^\mu\gamma^\nu(\not{p}-m) + 2p^\nu\gamma^\mu}{u_1} \right] u(p)$$

Eqn. de Dirac : $(\not{p}-m)\,u(p) = 0$

$\Rightarrow$

$$M_{fi} = -ie^2\,\epsilon'_\nu\,\epsilon_\mu\,\bar{u}(p')\left[\frac{\gamma^\nu\not{k}\gamma^\mu + 2p^\mu\gamma^\nu}{s_1} + \frac{-\gamma^\mu\not{k}'\gamma^\nu + 2p^\nu\gamma^\mu}{u_1} \right] u(p)$$

$$M^* = M^\dagger = \bar{M} = ie^2\,\epsilon'_\alpha\,\epsilon_\beta\,\bar{u}(p')\left[\frac{\gamma^\alpha\not{k}\gamma^\beta + 2p^\beta\gamma^\alpha}{s_1} + \frac{-\gamma^\beta\not{k}'\gamma^\alpha + 2p^\alpha\gamma^\beta}{u_1} \right] u(p)$$

(4.)

$$\overline{|M_{fi}|}^2 = \frac{1}{2}\sum_\lambda \frac{1}{2}\sum_s \sum_{\lambda'}\sum_{s'} |M|^2 \quad ; \quad \sum_\lambda \epsilon_\mu\epsilon_\beta = -g_{\mu\beta} \quad ; \quad \sum_s u(p,s)\bar{u}(p,s) = \not{p}+m$$

$$= \frac{e^4}{4}\underbrace{\sum_\lambda \epsilon'_\nu\epsilon'_\alpha}_{-g_{\nu\alpha}}\underbrace{\sum_{\lambda'}\epsilon_\mu\epsilon_\beta}_{-g_{\mu\beta}}\sum_{s,s'}\bar{u}(p')\left[\frac{2p^\mu\gamma^\nu + \gamma^\nu\not{k}\gamma^\mu}{s_1} + \frac{2p^\nu\gamma^\mu - \gamma^\mu\not{k}'\gamma^\nu}{u_1} \right] u(p)\bar{u}(p)$$
$$\left[\frac{2p^\beta\gamma^\alpha + \gamma^\alpha\not{k}\gamma^\beta}{s_1} + \frac{2p^\alpha\gamma^\beta - \gamma^\beta\not{k}'\gamma^\alpha}{u_1} \right] u(p')$$

$$= \frac{e^4}{4}\sum_{s'}\bar{u}(p')\left[\frac{2p^\mu\gamma^\nu + \gamma^\nu\not{k}\gamma^\mu}{s_1} + \frac{2p^\nu\gamma^\mu - \gamma^\mu\not{k}'\gamma^\nu}{u_1} \right](\not{p}+m)\underbrace{\left[\frac{2p^\mu\gamma^\nu + \gamma^\mu\not{k}\gamma^\nu}{s_1} + \frac{2p^\nu\gamma^\mu - \gamma^\nu\not{k}'\gamma^\mu}{u_1} \right]}_{\text{Indices de Lorentz en bas}} u(p')$$

$$\Rightarrow \overline{|M_{fi}|^2} = \frac{e^4}{4}\left[\frac{1}{s_1^2}\,Tr_{11} + \frac{1}{s_1u_1}\underbrace{(Tr_{12}+Tr_{21})}_{=2\,\mathrm{Re}\,Tr_{12}} + \frac{1}{u_1^2}\,Tr_{22}\right]$$

$$M = M_1+M_2 \Rightarrow |M|^2 = |M_1|^2 + \underbrace{(M_1M_2^* + M_1^*M_2)}_{2\,\mathrm{Re}(M_1M_2^*)} + |M_2|^2$$

avec

$$Tr_{11} \equiv Tr\left[(\not k'+m)(2p^\mu\gamma^\nu + \gamma^\nu \not k\gamma^\mu)(\not k+m)(2p_\mu\gamma_\nu + \gamma_\mu\not k\gamma_\nu)\right]$$

$$Tr_{12} \equiv Tr\left[(\not k'+m)(2p^\mu\gamma^\nu + \gamma^\nu\not k\gamma^\mu)(\not k+m)(2p_\nu\gamma_\mu - \gamma_\nu\not k'\gamma_\mu)\right]$$

$$Tr_{21} \equiv Tr\left[(\not k'+m)(2p^\nu\gamma^\mu - \gamma^\mu\not k'\gamma^\nu)(\not k+m)(2p_\mu\gamma_\nu + \gamma_\mu\not k\gamma_\nu)\right]$$

$$Tr_{22} \equiv Tr\left[(\not k'+m)(2p^\nu\gamma^\mu - \gamma^\mu\not k'\gamma^\nu)(\not k+m)(2p_\nu\gamma_\mu - \gamma_\nu\not k'\gamma_\mu)\right] = Tr_{11}(k \leftrightarrow -k')$$
$$= Tr_{11}(s_1 \leftrightarrow u_1)$$

Besoin de $\underline{Tr_{11}, Tr_{12}}$

On trouve (voir en bas) : $Tr_{11} = -8s_1u_1 + 16m^2s_1 + 32m^4$

$Tr_{12} = Tr_{21} = 8m^2(s_1+u_1) + 32m^4$

$Tr_{22} = Tr_{11}(s_1 \leftrightarrow u_1)$

$$\Rightarrow \overline{|M|}^2 = \frac{e^4}{4}\left[\frac{1}{s_1^2}\left\{-8s_1u_1 + 16m^2s_1 + 32m^4\right\} + \frac{1}{u_1^2}\left\{-8s_1u_1 + 16m^2u_1 + 32m^4\right\}\right.$$
$$\left. + \frac{1}{s_1u_1}\left\{16m^2s_1 + 16m^2u_1 + 64m^4\right\}\right]$$

$$= 2e^4\left[-\frac{u_1}{s_1} + \frac{2m^2}{s_1} + \frac{4m^4}{s_1^2} - \frac{s_1}{u_1} + \frac{2m^2}{u_1} + \frac{4m^4}{u_1^2} + \frac{2m^2}{u_1} + \frac{2m^2}{s_1} + \frac{8m^4}{s_1u_1}\right]$$

$$= 32\pi^2\alpha^2\left[-\frac{u_1}{s_1} - \frac{s_1}{u_1} + 4\left(\frac{m^2}{s_1} + \frac{m^2}{u_1}\right) + 4\left(\frac{m^2}{s_1} + \frac{m^2}{u_1}\right)^2\right]$$

5.

Cours : $\dfrac{d\sigma}{dt} = \dfrac{1}{16\pi\,\lambda(s,m^2,0)}\,\overline{|M|}^2$, $\lambda(s,m^2,0) = s_1^2$

$$\Rightarrow \boxed{\frac{d\sigma}{dt} = \frac{2\pi\alpha^2}{s_1^2}\left[\ \dots\ \right]}$$

avec: $s_1 = \text{const}$

$s_1 + u_1 + t = 0 \Rightarrow u_1 = -s_1 - t$

$$\mathrm{Tr}_{11} = \mathrm{Tr}\left[\not p' \left(2p^\mu \gamma^\nu + \gamma^\nu \not k \gamma^\mu\right) \not k \left(2p_\mu \gamma_\nu + \gamma_\mu \not k \gamma_\nu\right)\right]$$

$$+ m^2 \, \mathrm{Tr}\left[\left(2p^\mu \gamma^\nu + \gamma^\nu \not k \gamma^\mu\right)\left(2p_\mu \gamma_\nu + \gamma_\mu \not k \gamma_\nu\right)\right]$$

$$= 4m^2 \, \mathrm{Tr}\left[\not p' \overbrace{\gamma^\nu \not p \gamma_\nu}^{-2\not p}\right] + 2\mathrm{Tr}\left[\not p' \overbrace{\gamma^\nu \underbrace{\not p \not p}_{m^2} \not k \gamma_\nu}^{-2m^2 \not k}\right] + 2\mathrm{Tr}\left[\not p' \overbrace{\gamma^\nu \not k \underbrace{\not p \not p}_{m^2} \gamma_\nu}^{-2m^2 \not k}\right]$$

$$+ \mathrm{Tr}\left[\not p' \gamma^\nu \not k \gamma^\mu \not p \gamma_\mu \not k \gamma_\nu\right]$$

$$+ m^2 \, \mathrm{Tr}\left[4m^2 \cdot 4 + 2\gamma^\nu \not p \not k \gamma_\nu + 2\gamma^\nu \not k \not p \gamma_\nu + \gamma^\nu \underbrace{\not k \; 4 \; \not k}_{=0} \gamma_\nu\right]$$

$$\mathrm{Tr}(\not a \not b) = 4ab$$

Es gilt: $\underbrace{\gamma^\nu \not p}_{-\not p \gamma^\nu + 2p^\nu} \gamma_\nu = -4\not p + 2\not p = -2\not p$

$\not k \not p = -\not p \not k + 2kp$

$\leadsto \not k \not p \not k = 2kp \not k$, da $k^2 = 0$

$$= -8m^2 \, \mathrm{Tr}\overbrace{\left[\not p' \not p\right]}^{4pp'} - 8m^2 \, \mathrm{Tr}\overbrace{\left[\not p' \not k\right]}^{4p'k} + \mathrm{Tr}\left[\not p' \gamma^\nu \underbrace{\not k (-2\not p) \not k}_{-4kp \not k} \gamma_\nu\right]$$

$$+ m^2 \left(64m^2 + 16 \, \mathrm{Tr}\overbrace{(\not p \not k)}^{4pk}\right)$$

$$= -32m^2 \left(pp' + p'k\right) - 4kp \, \overbrace{\mathrm{Tr}\left[\not p' \underbrace{\gamma^\nu \not k \gamma_\nu}_{-2\not k}\right]}^{-8p'k}$$

$$+ m^2 \left(64m^2 + 64pk\right)$$

$$= -32m^2 \left(p' \underbrace{(p+k)}_{=p'+k'}\right) + 32 \, pk \, p'k + \underbrace{64m^2 (m^2 + pk)}_{\left[32m^2 \left((p+k)^2 + m^2\right)\right]}$$

$pk = \frac{s_1}{2} = p'k'$

$p'k = -\frac{u_1}{2}$

$$= -32m^4 - \overbrace{32m^2 p'k'}^{16m^2 s_1} + \overbrace{32 \, pk \, p'k}^{-8s_1u_1} + \overbrace{64pk}^{32s_1} m^2 + 64m^4$$

$$\underline{\underline{= 32m^4 + 16m^2 s_1 - 8s_1u_1}}$$

✓

$$\mathrm{Tr}_{12} = \mathrm{Tr}\left[\not{p}' (2p^\mu \gamma^\nu + \gamma^\nu \not{k} \gamma^\mu) \not{p} (2p_\nu \gamma_\mu - \gamma_\nu \not{k}' \gamma_\mu) \right]$$

$$+ m^2\, \mathrm{Tr}\left[(2p^\mu \gamma^\nu + \gamma^\nu \not{k} \gamma^\mu)(2p_\nu \gamma_\mu - \gamma_\nu \not{k}' \gamma_\mu) \right]$$

$$= \mathrm{Tr}\Big[\not{p}'\, 4 \overbrace{\not{p}\not{p}}^{m^2}\not{p} \quad - \not{p}'\, 2 \overbrace{\gamma^\nu \not{p}\, \gamma_\nu}^{-2\not{p}} \not{k}' \not{p} \quad + 2\not{p}' \not{p} \not{k}\, \overbrace{\gamma^\mu \not{p}\, \gamma_\mu}^{-2\not{p}}$$

$$- \not{p}' \gamma^\nu \not{k} \gamma^\mu \not{p}\, \gamma_\nu \not{k}' \gamma_\mu \Big]$$

$$+ m^2\, \mathrm{Tr}\left[4m^2 \quad - 8 \not{k}' \not{p} \quad + 8 \not{p} \not{k} \quad - \gamma^\nu \not{k} \gamma^\mu \gamma_\nu \not{k}' \gamma_\mu \right]$$

$$= 4m^2\, 4p'p \quad + \mathrm{Tr}\Big[4\not{p}' \overbrace{\not{p}\, \not{k}' \not{p}}^{-m^2\not{k}' + 2pk'\not{p}} - 4\not{p}' \overbrace{\not{p}\, \not{k}\, \not{p}}^{-m^2\not{k} + 2pk\not{p}} \Big] - \mathrm{Tr}\Big[\not{p}' \overbrace{\gamma^\nu \not{k} \gamma^\mu \not{p}\, \gamma_\nu}^{-2\not{p}\gamma^\mu\not{k}} \not{k}' \gamma_\mu \Big]$$

$$+ m^2 \left(16m^2 - 32 k'p + 32 pk \right) \quad - m^2\, \mathrm{Tr}\Big[\underbrace{\gamma^\nu \not{k} \gamma^\mu \gamma_\nu}_{4k^\mu} \not{k}' \gamma_\mu \Big]$$

$$\underbrace{\not{a} \not{b}}_{-\not{b}\not{a} + 2ab} \not{a} = -\not{b}\, a^2 + 2ab\, \not{a}$$

$$= 16m^2 p'p \quad - 16m^2 p'k' \quad + 8pk'\, 4pp' \quad + 16m^2 p'k \quad - 8pk\, 4pp'$$

$$+ m^2 (16m^2 - 32k'p + 32pk)$$

$$+ 2\mathrm{Tr}\Big[\not{p}' \not{p}\, \underbrace{\gamma^\mu \not{k} \not{k}' \gamma_\mu}_{4kk'} \Big] \quad - 4m^2\, \underbrace{\mathrm{Tr}[\not{k}' \not{k}]}_{4k'k}$$

$$\gamma^\mu \not{a} \not{b} \gamma_\mu = 4ab$$
$$\gamma^\mu \not{a} \not{b} \not{c} \gamma_\mu = -2 \not{c} \not{b} \not{a}$$
$$\gamma^\mu \gamma^\alpha \gamma^\beta \gamma_\mu = 4 g^{\alpha\beta}$$

$$pk = p'k' = \frac{s_1}{2} \;;\; p'k = \frac{-u_1}{2} = pk'$$

$$p'p = p(p+k-k') = m^2 + pk - pk' = m^2 + \frac{s_1+u_1}{2}$$

$$kk' = \frac{s_1+u_1}{2}$$

$$= 16m^2 \left(m^2 + \frac{s_1+u_1}{2} - \frac{s_1}{2} \right) + 4pp' (8pk' - 8pk) + 16m^2 \frac{-u_1}{2}$$

$$+ m^2 (16m^2 + 16u_1 + 16s_1)$$

$$+ 8kk'\, 4pp' \quad - 4m^2\, 4kk'$$

$$= \underline{16m^4} + 4pp'\left(\cancel{-4u_1 - 4s_1}\right) + \underline{16m^4} + 16m^2(u_1+s_1) + \cancel{4(s_1+u_1)\overbrace{\left(4m^2 + 2(s_1+u_1)\right)}^{4pp'}}$$
$$- 8m^2 (s_1+u_1)$$

$$= \underline{\underline{32m^4 + 8m^2 (s_1+u_1)}} \quad \checkmark$$