(4.1)

(i) $s+t+u = \underbrace{m_1^2+m_2^2+2P_1P_2}_{s} + \underbrace{m_1^2+m_3^2-2P_1P_3}_{t} + \underbrace{m_1^2+m_4^2-2P_1P_4}_{u}$

$$= m_1^2+m_2^2+m_3^2+m_4^2 + 2P_1\underbrace{(P_1+P_2-P_3-P_4)}_{=0}$$

(ii) Lab.: $P_2=(m_2,\vec{0})$, $P_i=(E_i^L , \vec{p}_i^{\,L})$ pour $i=1,3,4$

Projections avec P_2 :

$E_1^L = \frac{1}{m_2} P_2\cdot P_1 = \frac{1}{2m_2}(s-m_1^2-m_2^2)$ avec $s=(P_1+P_2)^2 = m_1^2+m_2^2+2P_1\cdot P_2$

$E_3^L = \frac{1}{m_2} P_2\cdot P_3 = \frac{1}{m_2} P_2\cdot(P_1+P_2-P_4)$ car $P_1+P_2=P_3+P_4$

$= \frac{1}{2m_2}(s-m_1^2-m_2^2+2m_2^2+t-m_2^2-m_4^2)$ $t=(P_1-P_3)^2=(P_2-P_4)^2=m_2^2+m_4^2-2P_2P_4$

$= \frac{1}{2m_2}(s+t-m_1^2-m_4^2) = \frac{1}{2m_2}(-u+m_2^2+m_3^2)$ car $s+t=-u+\sum_{i=1}^{4} m_i^2$

$E_4^L = \frac{1}{m_2} P_2\cdot P_4 = \frac{1}{2m_2}(-t+m_2^2+m_4^2)$

$\Rightarrow \vec{p}_1^{\,L\,2} = E_1^{L\,2}-m_1^2 = \frac{1}{4m_2^2}\left[s^2-2s(m_1^2+m_2^2)+m_1^4+m_2^2+2m_1^2m_2^2-4m_1^2m_2^2\right]$

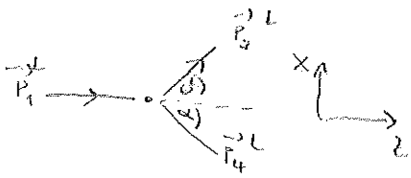
$= \frac{1}{4m_2^2}\lambda(s,m_1^2,m_2^2)$

avec $\lambda(a,b,c)=a^2+b^2+c^2-2ab-2ac-2bc$

$\vec{p}_3^{\,L\,2} = E_3^{L\,2}-m_3^2 = \frac{1}{4m_2^2}\lambda(u,m_2^2,m_3^2)$

$\vec{p}_4^{\,L\,2} = E_4^{L\,2}-m_4^2 = \frac{1}{4m_2^2}\lambda(t,m_2^2,m_4^2)$

Choix des coordonnées :



$\vec{p}_1^{\,L} = |\vec{p}_1^{\,L}|(0,0,1)$

$\vec{p}_3^{\,L} = |\vec{p}_3^{\,L}|(\sin\theta^L,0,\cos\theta^L)$, $\vec{p}_4^{\,L} = |\vec{p}_4^{\,L}|(-\sin\alpha,0,\cos\alpha)$

$\vec{p}_1^{\,L} = \vec{p}_3^{\,L}+\vec{p}_4^{\,L}$

(iii)

$$t = (p_1 - p_3)^2 = m_1^2 + m_3^2 - 2\, p_1 p_3$$

$$= m_1^2 + m_3^2 - 2 E_1^L E_3^L + 2\, |\vec{p}_1^{\,L}|\, |\vec{p}_3^{\,L}| \cos\theta^L$$

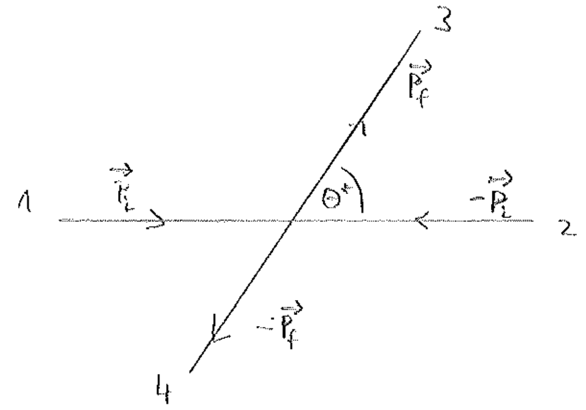
$$\Rightarrow \quad \cos\theta^L = \frac{t - m_1^2 - m_3^2 + 2 E_1^L E_3^L}{2\, |\vec{p}_1^{\,L}|\, |\vec{p}_3^{\,L}|}$$

$$= \frac{t - m_1^2 - m_3^2 + \frac{2}{4m_2^2} (s - m_1^2 - m_2^2)(-u + m_2^2 + m_3^2)}{\frac{2}{4m_2^2} \sqrt{\lambda(s, m_1^2, m_2^2)} \sqrt{\lambda(u, m_2^2, m_3^2)}}$$

$$= \frac{2 m_2^2 (t - m_1^2 - m_3^2) - (s - m_1^2 - m_2^2)(u - m_2^2 - m_3^2)}{\sqrt{\lambda(s, m_1^2, m_2^2)} \sqrt{\lambda(u, m_2^2, m_3^2)}}$$

(4.2)

$1 + 2 \longrightarrow 3 + 4$



$$P_1 = \left(E_1^* = \sqrt{\vec{p}_i^{\,2} + m_1^2}\ ,\ \vec{p}_i \right)$$
$$P_2 = \left(E_2^* = \sqrt{\vec{p}_i^{\,2} + m_2^2}\ ,\ -\vec{p}_i \right)$$
$$P_3 = \left(E_3^* = \sqrt{\vec{p}_f^{\,2} + m_3^2}\ ,\ \vec{p}_f \right)$$
$$P_4 = \left(E_4^* = \sqrt{\vec{p}_f^{\,2} + m_4^2}\ ,\ -\vec{p}_f \right)$$

(i) Considérons, par ex.,

$$\left.\begin{aligned} &\underbrace{(P_1 + P_2)(P_1 - P_2)} = P_1^2 - P_2^2 = m_1^2 - m_2^2 \\ &= \underbrace{(E_1^* + E_2^*)}_{\sqrt{s}}(E_1^* - E_2^*) = \sqrt{s}\,\left(2E_1^* - \sqrt{s}\right) \end{aligned}\right\} \Rightarrow \underline{\underline{E_1^* = \frac{1}{2\sqrt{s}}\left(s + m_1^2 - m_2^2\right)}}$$

De même : $E_2^* = \frac{1}{2\sqrt{s}}\left(s + m_2^2 - m_1^2\right)$ et $E_3^* = \frac{1}{2\sqrt{s}}\left(s + m_3^2 - m_4^2\right)$

$\quad 4 \qquad\qquad 4 \quad 3$

$$\vec{p}_i^{\,2} = E_1^{*2} - m_1^2 \quad \left(= E_2^{*2} - m_2^2\right)$$

$$= \frac{1}{4s}\Big[\underbrace{s^2 - 2s\left(m_1^2 + m_2^2\right) + \left(m_1^2 - m_2^2\right)^2}_{\lambda(s, m_1^2, m_2^2)} \Big] = \underline{\underline{\frac{1}{4s}\,\lambda(s, m_1^2, m_2^2)}}$$

$$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$$
$$= a^2 - 2a(b+c) + (b-c)^2 \longrightarrow a^2 \quad \text{pour } a \gg b,\ a \gg c$$

De même : $\vec{p}_f^{\,2} = E_3^{*2} - m_3^2 = E_4^{*2} - m_4^2 = \frac{1}{4s}\,\lambda(s, m_3^2, m_4^2)$

<u>Comportement asymptotique ($s \gg m_i^2$) :</u>

$$E_1^* \simeq E_2^* \simeq E_3^* \simeq E_4^* \simeq |\vec{p}_i| \simeq |\vec{p}_f| \simeq \frac{\sqrt{s}}{2}$$

(ii)

$$t = (p_1 - p_3)^2 = m_1^2 + m_3^2 - 2E_1^* E_3^* + 2|\vec{p}_i||\vec{p}_f| \cos\Theta^* \qquad \rightsquigarrow \cos\Theta^* [s, t, m_i^2]$$

$$u = (p_1 - p_4)^2 = m_1^2 + m_4^2 - 2E_1^* E_4^* - 2|\vec{p}_i||\vec{p}_f| \cos\Theta^* \qquad \rightsquigarrow \cos\Theta^* [s, u, m_i^2]$$

$$\Rightarrow t - u = m_3^2 - m_4^2 + \underbrace{2E_1^* (E_4^* - E_3^*)}_{} + 4|\vec{p}_i||\vec{p}_f| \cos\Theta^* \qquad \rightsquigarrow \cos\Theta^* [s, t, u, m_i^2]$$

$$= \frac{2E_1^* (E_4^{*2} - E_3^{*2})}{\sqrt{s}} = \frac{2E_1^*}{\sqrt{s}} (p_4^2 - p_3^2) = \frac{2E_1^*}{\sqrt{s}} (m_4^2 - m_3^2)$$

$$= (m_3^2 - m_4^2)\left(1 - \frac{2E_1^*}{\sqrt{s}}\right) + 4 p_i p_f \cos\Theta^*$$

$$= \frac{m_3^2 - m_4^2}{\sqrt{s}} (\underbrace{\sqrt{s} - 2E_1^*}_{E_2^* - E_1^*}) + 4 p_i p_f \cos\Theta^* \qquad , \sqrt{s} = E_1^* + E_2^*$$

$$= \frac{1}{s}(m_3^2 - m_4^2)(\underbrace{E_2^{*2} - E_1^{*2}}_{= p_2^2 - p_1^2}) + 4 p_i p_f \cos\Theta^*$$

$$= -\frac{1}{s}(m_1^2 - m_2^2)(m_3^2 - m_4^2) + 4 p_i p_f \cos\Theta^*$$

$$\Rightarrow \quad \cos\Theta^* = \frac{s(t-u) + (m_1^2 - m_2^2)(m_3^2 - m_4^2)}{4s \; p_i p_f} \qquad ; \; p_i = \frac{\sqrt{\lambda_i}}{2\sqrt{s}} , \; p_f = \frac{\sqrt{\lambda_f}}{2\sqrt{s}}$$

$$= \frac{s(t-u) + (m_1^2 - m_2^2)(m_3^2 - m_4^2)}{\sqrt{\lambda(s, m_1^2, m_2^2)} \sqrt{\lambda(s, m_3^2, m_4^2)}} \qquad (*)$$

(iii) $s+t+u = \sum m_i^2 \quad \Rightarrow \quad s(t-u) = s(2t+s-\sum m_i^2)$

$$= 2st + s^2 - s\sum m_i^2$$

$$(*) \Rightarrow 2t = \frac{1}{s}\sqrt{\lambda_i}\sqrt{\lambda_f}\cos\theta^* + \sum m_i^2 - s - \frac{1}{s}(m_1^2-m_2^2)(m_3^2-m_4^2)$$

$\theta^* = 0 \to t_{min}$

$\theta^* = \pi \to t_{max}$ avec $t_{max} \leq t \leq t_{min}$

$$(|t_{min}| \leq t \leq |t_{max}|)$$

<u>Valeur asymptotique de t_{min} :</u> $(s \gg m_i^2)$

$m_i \neq m_i'$: $t_{min} = -\frac{1}{s}(m_1^2-m_3^2)(m_2^2-m_4^2) + O\left(\frac{m_i^4}{s^2}\right)$

par développement de $\frac{1}{s}\sqrt{\lambda(s,m_1^2,m_2^2)}\sqrt{\lambda(s,m_3^2,m_4^2)}$ jusqu'à $O(1/s)$

$m_2 = m_4$: $t_{min} = -\frac{m_2^2}{s^2}(m_1^2-m_3^2)^2 + O\left(\frac{m_i^6}{s^3}\right)$

développement jusqu'à $O(1/s^2)$ nécessaire car $O(1/s) = 0$

(iv)

$$(E_1^* + E_2^*)^2 \overset{CMS}{=} s \overset{Lab}{=} (P_1^L + P_2^L)^2 = m_1^2 + m_2^2 + 2 P_1^L P_2^L \quad , P_2^L = \left(\begin{smallmatrix} m_2 \\ \vec{0} \end{smallmatrix}\right)$$

$$= m_1^2 + m_2^2 + 2 m_2 E_1^L$$

$$\Rightarrow \boxed{E_1^L = \frac{(E_1^* + E_2^*)^2 - m_1^2 - m_2^2}{2 m_2}}$$

$$\vec{P}_1^{L\,2} \overset{(4.1)}{=} \frac{1}{4 m_2^2} \lambda_i$$

$$\vec{P}_i^{*\,2} \overset{(4.2)}{=} \frac{1}{4s} \lambda_i$$

$$\Rightarrow \frac{|\vec{P}_1^L|}{|\vec{P}_i^*|} = \frac{\sqrt{s}}{m_2}$$

$$\Rightarrow \boxed{|\vec{P}_1^L| = \frac{E_1^* + E_2^*}{m_2} P_i^* \quad , \quad P_i^* = \frac{m_2}{\sqrt{m_1^2 + m_2^2 + 2 m_2 E_1^L}} |\vec{P}_1^L|}$$

$$\frac{m_2^2}{s} = \frac{\vec{P}_i^{*\,2}}{\vec{P}_1^{L\,2}} = \frac{E_1^{*\,2} - m_1^2}{E_1^{L\,2} - m_1^2} \Rightarrow E_1^{*\,2} = \frac{m_1^2 s + m_2^2 (E_1^{L\,2} - m_1^2)}{s}$$

$$\Rightarrow E_1^{*\,2} = \frac{m_1^2 (m_1^2 + \cancel{m_2^2} + 2 m_2 E_1^L) + m_2^2 E_1^{L\,2} - \cancel{m_1^2 m_2^2}}{m_1^2 + m_2^2 + 2 m_2 E_1^L} = \frac{(m_1^2 + m_2 E_1^L)^2}{s}$$

$$\Rightarrow \boxed{E_1^* = \frac{m_1^2 + m_2 E_1^L}{\sqrt{m_1^2 + m_2^2 + 2 m_2 E_1^L}}}$$

Alternativement :

$$E_2^* = \frac{1}{2\sqrt{s}} (s - m_1^2 + m_2^2) \quad \text{et} \quad s = m_1^2 + m_2^2 + 2 m_2 E_1^L$$

$$\Rightarrow \boxed{E_2^* = \frac{m_2^2 + m_2 E_1^L}{\sqrt{m_1^2 + m_2^2 + 2 m_2 E_1^L}}}$$

(4.3)

$$d\sigma = \frac{1}{F} |M|^2 \, d\phi$$

Flux : $\quad F := \underbrace{2E_1 \, 2E_2}_{\text{Normalisation}} \underbrace{|\vec{V}_1 - \vec{V}_2|}_{\text{vitesse relative}}$

$$\vec{V} = \frac{\vec{P}}{E} \Rightarrow F = 4 E_1 E_2 \left| \frac{\vec{P}_1}{E_1} - \frac{\vec{P}_2}{E_2} \right| = 4 \left| E_2 \vec{P}_1 - E_1 \vec{P}_2 \right|$$

$$\overset{CMS}{=} 4 \left| E_2^* \vec{P}_1^* + E_1^* \vec{P}_1^* \right| = 4 P_i^* (E_a^* + E_b^*) = 4 P_i^* \sqrt{s}$$

Avec $P_i^* = \dfrac{\sqrt{\lambda_i}}{2\sqrt{s}}$

$$F = 2 \sqrt{\underbrace{s^2 + m_1^4 + m_2^4 - 2 s m_1^2 - 2 s m_2^2 - 2 m_1^2 m_2^2}_{(\underbrace{s - m_1^2 - m_2^2}_{(2 P_1 \cdot P_2)^2})^2 - 4 m_1^2 m_2^2}}$$

$$= 4 \sqrt{(P_1 \cdot P_2)^2 - m_1^2 m_2^2}$$ invariant (L.I.) valable dans tous les systèmes inertiels

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Alternativement : $\quad P_1 \cdot P_2 \overset{CMS}{=} E_1^* E_2^* - \vec{P}_1^* \cdot \vec{P}_2^* = E_1^* E_2^* + P_i^{*2}$

$$(P_1 \cdot P_2)^2 = E_1^{*2} E_2^{*2} + 2 P_i^{*2} E_1^* E_2^* + P_i^{*4}$$

$$= (m_1^2 + P_i^{*2})(m_2^2 + P_i^{*2}) + 2 P_i^{*2} E_1^* E_2^* + P_i^{*4}$$

$$\Rightarrow F = 4 \sqrt{(P_1 \cdot P_2)^2 - m_1^2 m_2^2} \overset{CMS}{=} 4 \sqrt{P_i^{*2} (\underbrace{m_1^2 + m_2^2 + 2 E_1^* E_2^* + 2 P_i^{*2}}_{= s = m_1^2 + m_2^2 + 2 P_1 P_2})}$$

$$= 4 P_i^* \sqrt{s}$$

$a + b \to 1 \dots + n$: $$d\Phi_n = (2\pi)^4 \, \delta^{(4)}\Big(\underbrace{P_a + P_b}_{\text{état init.}} - \underbrace{\sum_{i=1}^{n} P_i}_{\text{état final}}\Big) \underbrace{\prod_{k=1}^{n} \frac{d^3 P_k}{(2\pi)^3 2E_k}}_{\text{espace de phase}}$$

$d\Phi_n$ ↑ espace de phase

$(2\pi)^4$ ↑ Fourier, Norm

Conservation de 4-impulsion

espace de phase ≙ ※ États finaux des particules dans un volume unitaire ($V=1$)

Rappel :

$$\int \frac{d^3p}{2E} \overset{!}{=} \int d^4p \; \delta(p^2 - m^2)\, \Theta(p^0) =: \int d^4p \; \delta_+(p^2 - m^2)$$

$$= \int dp^0 \, d^3p \; \delta(p^{0\,2} - \vec{p}^{\,2} - m^2)\, \Theta(p^0)$$

$$= \int \underline{dp^0} \, d^3p \left[\frac{1}{\underbrace{|2E(\vec{p})|}_{>0}} \delta(p^0 - E(\vec{p})) + \frac{1}{|2E(\vec{p})|} \cancel{\delta(p^0 + \underbrace{E(\vec{p})}_{>0})} \right] \Theta(p^0)$$

$$\text{avec} \quad E(\vec{p}) = \sqrt{m^2 + \vec{p}^{\,2}} > 0$$

$$= \int \frac{d^3p}{2E(\vec{p})} \qquad E \equiv E(\vec{p})$$

$1 + 2 \to 3 + 4$:

$$\int d\Phi_2 = \int (2\pi)^4 \, \delta^{(4)}(P_1 + P_2 - P_3 - P_4) \frac{d^3P_3}{(2\pi)^3 2E_3} \frac{d^3P_4}{(2\pi)^3 2E_4}$$

$$= \frac{1}{4\pi^2} \int \frac{d^3P_3}{2E_3} \int \underline{d^4P_4} \; \delta_+(P_4^2 - m_4^2)\, \delta^{(4)}(P_1 + P_2 - P_3 - P_4)$$

$$= \frac{1}{4\pi^2} \int \frac{d^3P_3}{2E_3} \; \delta\big((P_1 + P_2 - P_3)^2 - m_4^2\big)\, \Theta(E_1 + E_2 - E_3)$$

$$\overset{\text{CMS}}{=} \frac{1}{4\pi^2} \int \frac{d^3P_3^*}{2E_3^*} \; \delta\big(s - 2\sqrt{s}\, E_3^* + m_3^2 - m_4^2\big)\, \Theta(\sqrt{s} - E_3^*)$$

$$d^3P_3^* = |\vec{P}_3^*|^2 \, d|\vec{P}_3^*| \, d\Omega^* \quad ; \quad E_3^* = \sqrt{m_3^2 + \vec{P}_3^{*2}} \Rightarrow |\vec{P}_3^*| \, d|\vec{P}_3^*| = E_3^* \, dE_3^* \, \Theta(E_3^*)$$

$$= |\vec{P}_3^*| \, E_3^* \, dE_3^* \, \Theta(E_3^*) \, d\Omega^*$$

$$= \frac{1}{4\pi^2} \frac{1}{2} \int dE_3^* \, d\Omega^* \; \Theta(E_3^*) \, |\vec{P}_3^*| \, \frac{1}{2\sqrt{s}} \, \delta\Big(E_3^* - \underbrace{\frac{s + m_3^2 - m_4^2}{2\sqrt{s}}}_{\uparrow \text{ voir Ex. 4.2}}\Big)\, \Theta(\sqrt{s} - E_3^*)$$

$$P_f^* \equiv |\vec{P}_3^*| \Big|_{E_3^* = \frac{s + m_3^2 - m_4^2}{2\sqrt{s}}}$$

$$= \frac{1}{4\pi^2} \frac{1}{4\sqrt{s}} P_f^* \int d\Omega^* \qquad \text{avec} \quad P_f^* = \frac{1}{2\sqrt{s}} \sqrt{\lambda_f} = \frac{1}{2\sqrt{s}} \sqrt{\lambda(s, m_3^2, m_4^2)}$$

(4.4)

$$P = (M, \vec{0}) \quad , \quad \vec{P} = \vec{0} = \vec{P}_1 + \vec{P}_2 + \vec{P}_3$$

$$M^2 \gg m_i^2 \simeq 0 \qquad \text{avec} \quad P_i^2 = m_i^2$$

$$d\Phi_3 = (2\pi)^4 \, \delta^{(4)}\Big(P - \sum_{f=1}^{3} P_f\Big) \prod_{f=1}^{3} \frac{d^3 P_f}{(2\pi)^3 2E_f}$$

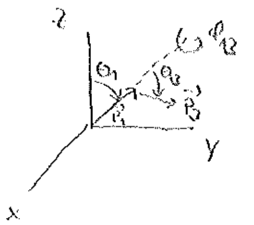
$$R_3 \equiv \int \frac{d^3P_1}{2E_1} \frac{d^3P_2}{2E_2} \frac{d^3P_3}{2E_3} \, \delta^{(4)}\Big(P - \sum_{f=1}^{3} P_f\Big) \quad , \quad \Phi_3 = \frac{1}{(2\pi)^5} R_3 \quad , \text{ intégration sur } P_2 \text{ (choix)}$$

$$= \int d^4P_1 \, d^4P_3 \; \delta_+(\underbrace{P_1^2 - m_1^2}_{\to 0}) \, \delta_+(\underbrace{P_3^2 - m_3^2}_{\to 0}) \, \delta_+(\underbrace{(P - P_1 - P_3)^2 - m_2^2}_{\to 0})$$

$$= \int |\vec{P}_1|^2 \, \underline{d|\vec{P}_1|} \; |\vec{P}_3|^2 \, \underline{d|\vec{P}_3|} \, d\Omega_1 d\Omega_3 \, dE_1 dE_3 \; \delta_+(E_1^2 - \vec{P}_1^{\,2}) \, \delta_+(E_3^2 - \vec{P}_3^{\,2})$$
$$\cdot \, \delta_+\Big(M^2 - 2M(E_1 + E_3) \underbrace{- m_2^2 + m_1^2 + m_3^2}_{\to 0} + 2E_1E_3 - 2|\vec{P}_1||\vec{P}_3| \, \underline{\cos\theta_{13}} \Big)$$

$$|\vec{P}|^2 \, d|\vec{P}| = \frac{1}{2} |\vec{P}| \, d|\vec{P}|^2$$

$$= \frac{1}{8} \int dE_1 \, dE_3 \, d\Omega_1 d\Omega_3 \; \delta\Big(\frac{M^2 - 2M(E_1+E_3)}{2E_1E_3} + 1 - \cos\theta_{13} \Big) \, \Theta(M - E_1 - E_3) \Theta(E_1) \Theta(E_3)$$



En général, la direction d'une particule sortante, par ex. $\vec{P}_1$ par rapport à une axe z quelconque, n'est pas distinguée (pour un état initial non-polarisé)

$\Rightarrow \; |M_{fi}|^2$ indép. de θ_1, φ_1

$$\Rightarrow \int d\Omega_1 = 4\pi$$

$$\int d\Omega_3 = \int_0^{2\pi} d\varphi_{13} \int_{-1}^{+1} d\cos\theta_{13} = 2\pi \, d\cos\theta_{13}$$

$$\Rightarrow \quad R_3 = \pi^2 \int dE_1 \, dE_3 \, \underline{d\cos\theta_{13}} \; \delta\Big(\underbrace{\frac{M^2 - 2M(E_1+E_3)}{2E_1E_3} + 1}_{= \cos\theta_{13}} - \cos\theta_{13} \Big) \, \Theta(M - E_1 - E_3) \Theta(E_1) \Theta(E_3)$$

$$= \pi^2 \int dE_1 \, dE_3 \; \Theta(M - E_1 - E_3) \; \Theta\big(\underbrace{-1 \le \cos\theta_{13} \le 1}_{\to \text{ bords pour } E_3} \big) \; \Theta(E_1) \Theta(E_3)$$

Bords d'intégration:

$$-1 \le 1 + \frac{M^2 - 2M(E_1+E_3)}{2E_1E_3} \le 1$$

$$\Leftrightarrow \quad \frac{M^2 - 2M(E_1+E_3)}{2E_1E_3} \ge -2 \quad \text{et} \quad \frac{M^2 - 2M(E_1+E_3)}{2E_1E_3} \le 0$$

$$\Leftrightarrow \quad M^2 - 2ME_1 - 2ME_3 \ge -4E_1E_3 \quad \text{et} \quad M^2 - 2ME_1 \le 2ME_3$$

$$\Leftrightarrow \quad E_3 \le \frac{M^2 - 2ME_1}{2M - 4E_1} = \frac{M}{2} \quad \text{et} \quad E_3 \ge \frac{M}{2} - E_1$$

$$\Rightarrow \quad R_3 = \pi^2 \int_0^{\infty} dE_1 \int_{M/2-E_1}^{M/2} dE_3 \quad \Theta(M-E_1-E_3)\,\Theta(E_3)$$

$$E_3 \geq 0 \iff \frac{M}{2} - E_1 \geq 0 \iff \boxed{E_1 \leq \frac{M}{2}}$$

$$M - E_1 - E_3 \geq 0 \iff M - E_1 - E_3^{max} \geq 0 \iff M - M/2 \geq E_1$$

$$\iff \boxed{E_1 \leq \frac{M}{2}}$$

$$\Rightarrow \quad R_3 = \pi^2 \int_0^{M/2} dE_1 \int_{M/2-E_1}^{M/2} dE_3$$

ou numériquement $\quad R_3 = \pi^2 \int_0^{\infty \text{ ou } M} dE_1 \int_0^{M} dE_3 \quad \Theta(M-E_1-E_3)\,\Theta(-1 \leq \cos\bar{\theta}_{13} \leq 1)$

$$\Rightarrow \quad \phi_3 = \frac{1}{32\pi^3} \int_0^{M/2} dE_1 \int_{\frac{M}{2}-E_1}^{M/2} dE_3$$

Important pour calculer la désintégration β d'un muon,

dès que $|M_{fi}|^2$ connu.

(4,4) <u>Avec la récursion :</u>

$$d\Phi_2 = (2\pi)^4\, \delta^{(4)}(p_1+p_2-p_3-p_4)\, \frac{d^3p_3}{(2\pi)^3 2E_3}\, \frac{d^3p_4}{(2\pi)^3 2E_4}$$

$$= \frac{1}{4\pi^2}\, \delta^{(4)}(\overbrace{p_1+p_2}^{P_T}-p_3-p_4)\, d^4p_3\, \delta_+(p_3^2-m_3^2)\, \frac{d^3p_4}{2E_4}$$

$$= \frac{1}{4\pi^2}\, \delta_+((P_T-p_4)^2-m_3^2)\, \frac{d^3p_4}{2E_4}$$

$$\delta_+((P_T-p_4)^2-m_3^2)$$

$$= \delta(s-2P_Tp_4+m_4^2-m_3^2)\,\Theta(E_T-E_4)$$

$$= \delta(s+m_4^2-m_3^2-2E_TE_4+2\underbrace{\vec{P}_T\cdot\vec{p}_4}_{=|\vec{P}_T||\vec{p}_4|\cos\theta_4})$$

$$d^3p_4 = |\vec{p}_4|^2\, d|\vec{p}_4|\, 2\pi\, d\cos\theta_4$$

$$= \frac{1}{2\pi}\, \overbrace{|\vec{p}_4|^{\cancel{2}}\, d|\vec{p}_4|}^{E_4\,dE_4}\, \frac{1}{2E_4}\, \frac{1}{2|\vec{P}_T|\cancel{|\vec{p}_4|}}\, \Theta(E_T-E_4)\, \Theta(-1\le\cos\theta_4\le 1) \quad ; \quad \cos\theta_4 = \frac{s+m_4^2-m_3^2-2E_TE_4}{2|\vec{P}_T||\vec{p}_4|}$$

$$\vec{p}_4^{\,2} = E_4^2-m_4^2 \quad ; \quad |\vec{p}_4|\,d|\vec{p}_4| = E_4\,dE_4$$

$$= \frac{1}{8\pi|\vec{P}_T|}\, dE_4\, \Theta(E_T-E_4)\, \Theta(|\cos\theta_4|\le 1)$$

$$m_4 \le E_4 \le E_1+E_2$$

$$d\Phi_3(P_T) = \frac{d^3 p_1}{(2\pi)^3 2E_1} \, d\Phi_2(P_T - P_1)$$

$$= \frac{\overbrace{|\vec{P}_1|^2 \, d|\vec{P}_1| \, d\Omega_1}^{|\vec{P}_1| E_1 dE_1}}{(2\pi)^3 2E_1} \quad \frac{1}{8\pi |\vec{P}_T - \vec{P}_1|} \, dE_3 \quad \Theta(\overbrace{E_T - E_1 - E_3}^{E_2}) \; \Theta(|\cos\theta_3| \leq 1)$$

$$\cos\theta_3 = -\frac{\bar{s} + m_3^2 - m_2^2 - 2(E_T - E_1)E_3}{2|\vec{P}_T - \vec{P}_1||\vec{P}_3|}$$

$$\bar{s} = (P_T - P_1)^2$$

$$= \frac{4\pi \, |\vec{P}_1| \, dE_1}{16\pi^3 \cdot 8\pi \, |\vec{P}_T - \vec{P}_1|} \, dE_3 \quad \Theta(\ldots) \; \Theta(|\cos\theta_3| \leq 1)$$

$$= \frac{1}{32\pi^3} \frac{|\vec{P}_1|}{\underbrace{|\vec{P}_T - \vec{P}_1|}_{\vec{P}_2 + \vec{P}_3}} \, dE_1 \, dE_3 \quad \Theta(E_T - E_1 - E_3) \; \Theta(|\cos\theta_3| \leq 1)$$

système repos du muon : $\vec{P}_T = 0$, $E_T = M$

$$= \frac{1}{32\pi^3} \, dE_1 \, dE_3 \quad \Theta(M - E_1 - E_3) \; \Theta(|\cos\theta_3| \leq 1)$$

$$, \quad \underset{\substack{m_i \to 0 \\ \vec{P}_T = 0 \\ P_T^2 = M^2 \\ E_T = M}}{\cos\theta_3} = -\frac{M^2 - 2ME_1 - 2ME_3 + 2E_1E_3}{2E_1E_3}$$

(4.5)

a) $SS^\dagger = 1 \Rightarrow \delta_{fi} = \langle f | SS^\dagger | i \rangle = \sum_n \langle f|S|n\rangle\langle n|S^\dagger|i\rangle$

$$= \sum_n \langle f|S|n\rangle (\langle i|S|n\rangle)^* = \sum_n S_{fn} S_{in}^* \qquad (1)$$

$\uparrow$ y compris les intégrations sur l'espace de phase

Avec : $S_{fi} = \delta_{fi} + i(2\pi)^4 \delta^{(4)}(P_f - P_i)\, T_{fi}$ (appelé $\tilde{M}_{fi}$ en cours)

$$(1) \Rightarrow \delta_{fi} = \sum_n \left[\delta_{fn} + i(2\pi)^4 \delta^{(4)}(P_f - P_n) T_{fn}\right]\left[\delta_{in} - i(2\pi)^4 \delta^{(4)}(P_i - P_n) T_{in}^*\right]$$

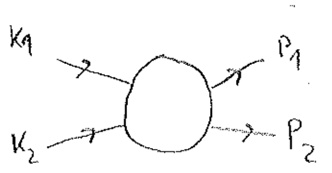
$$= \delta_{fi} + i(2\pi)^4 \delta^{(4)}(P_f - P_i)\left[T_{fi} - T_{if}^*\right] - i^2 (2\pi)^8 \sum_n \underbrace{\delta^{(4)}(P_f - P_n)\delta^{(4)}(P_i - P_n)}_{\delta^{(4)}(P_f - P_n)\delta^{(4)}(P_i - P_f)} T_{fn} T_{in}^*$$

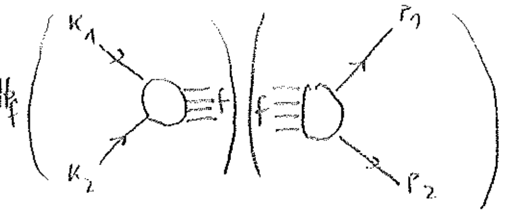
$$\Rightarrow \underline{T_{fi} - T_{if}^* = i(2\pi)^4 \sum_n \delta^{(4)}(P_f - P_n)\, T_{fn} T_{in}^*} \qquad (2)$$

$$\Rightarrow \underbrace{\langle f | T - T^\dagger | i \rangle}_{} = i(2\pi)^4 \sum_n \delta^{(4)}(P_f - P_n) \langle f|T|n\rangle\langle n|T^\dagger|i\rangle$$

$= 2i\, \mathrm{Im} \langle f|T|i\rangle$

la partie absorptive de T_{fi}

2 Im 

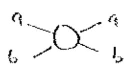
$= \sum_f \int d\Phi_f$ 

b) $i = f$; $i \equiv a + b$

$$\sum_n \delta^{(4)}(P_i - P_n) \equiv \sum{}' \int \delta^{(4)}(P_i - P_n) \prod_{k=1}^{n} \left(\frac{V}{(2\pi)^3} d^3 p_k\right)$$

$\uparrow$ Spins, sortes de particules dans n

$$(2) \Rightarrow \mathrm{Im}\, T_{ii} = \frac{1}{2}(2\pi)^4 \sum{}' \int \delta^{(4)}(P_i - P_n)\, |T_{ni}|^2 \prod_{k=1}^{n} \left(\frac{V}{(2\pi)^3} d^3 p_k\right)$$

[$T_{ii} = \langle i|T|i\rangle \equiv \langle ab|T|ab\rangle$ 

, $P_a = P_a'$, $P_b = P_b'$]

Exprimé par M_{fi} : $T_{fi} = i \left(\prod_i \frac{1}{\sqrt{2E_i V}} \right) \left(\prod_f \frac{1}{\sqrt{2E_f V}} \right) M_{fi}$

$$\cancel{\left(\frac{1}{\sqrt{2E_a V}\sqrt{2E_b V}} \right)^2} \operatorname{Im} M_{ii} = \frac{1}{2} (2\pi)^4 \underbrace{\sum' \int \delta^{(4)}(P_i - P_n) \, |M_{ni}|^2 \cancel{\left(\frac{1}{\sqrt{2E_a V}\sqrt{2E_b V}} \right)^2} \prod_{k=1}^{n} \left(\frac{\cancel{V}}{(2\pi)^3 \cancel{V}} \frac{d^3 p_k}{2E_k} \right)}$$

au facteur de flux près, c'est la section efficace totale

$$\sigma_{tot} = \sum_n' \sigma_{ni}$$

pour tous les processus de diffusion possibles à partir d'un état initial i : $a + b \to 1 + 2 + \ldots + n$

$$\operatorname{Im} M_{ii} = \frac{1}{2} F \sigma_{tot}$$

$$\overset{\text{Ex. 4.3}}{=} 2 p_i^* \sqrt{s} \; \sigma_{tot} \quad , \quad p_i^* = \frac{\sqrt{\lambda_i}}{2\sqrt{s}}$$

$$\Rightarrow \quad \boxed{\operatorname{Im} M_{ii} = \sqrt{\lambda(s, m_a^2, m_b^2)} \; \sigma_{tot}}$$ "Théorème optique"

Pour une dérivation en TQC : Peskin, Schröder Sec. 7.3
et application aux diagrammes de Feynman

$$\boxed{\operatorname{Im} M(k_1 k_2 \to k_1 k_2) = \sqrt{\lambda_i} \; \sigma_{tot}(k_1 k_2 \to \text{anything})}$$

c'est un résultat non-perturbatif