# LO Feynman calculation

128

## Charged Higgs Production.

$$g_c(k) + b_i(p) \longrightarrow H^-(k') + t(p')$$

~~voir~~ cinématique.

$m_b$ négligé (dans la cinématique)

$$s = (k+p)^2 = 2kp$$
$$= (k'+p')^2 = m_t^2 + m_H^2 + 2k'p'$$

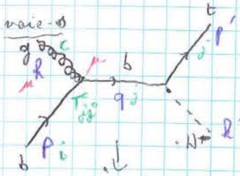
$$t = (k-p')^2 = m_t^2 - 2kp'$$
$$= (p-k')^2 = m_H^2 - 2k'p$$

$$u = (p-p')^2 = m_t^2 - 2pp'$$
$$= (k-k')^2 = m_H^2 - 2kk'$$

$$V_{qq'H} = \frac{g}{2\sqrt{2}\, m_W} \underbrace{\left(A(1+\gamma^5) + B(1-\gamma^5)\right)}_{(A+B)+(A-B)\gamma^5}$$

diagrammes

voie-s



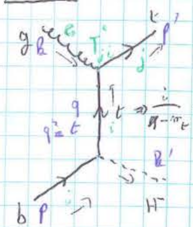
$$-i\mathcal{M}_{fi} = \bar{u}(p') \cdot 1 \times \frac{ig}{2\sqrt{2}\, m_W}\left((A+B)+(A-B)\gamma^5\right) \times \frac{i\not{q}}{\underset{=s}{q^2}} \cdot \varepsilon_\mu(k) \times ig_s T^c_{ji} \gamma^\mu u(p)$$

$$= \frac{-igg_s}{2\sqrt{2}\, m_W} \times \frac{1}{s}\, \varepsilon_\mu(k)\, \bar{u}(p')\, T^c_{ji}\, \gamma^\mu \not{q}\left[A+B + (A-B)\gamma^5\right] u(p)$$

$$\frac{i}{\not{q}} = \frac{i\not{q}}{q^2}$$

$$q = p + k = p' + k'$$

voie-t



$$-i\mathcal{M}_{fi} = \bar{u}(p')\, \varepsilon_\mu(k)\, ig_s T^c_{ji} \gamma^\mu \frac{i(\not{q}+m_t)}{q^2 - m_t^2}\left[(A+B)+(A-B)\gamma^5\right] \frac{ig}{2\sqrt{2}\, m_W} \times 1 \times u(p)$$

$$= \frac{-igg_s}{2\sqrt{2}\, m_W} \times \frac{1}{t - m_t^2}\, \varepsilon_\mu(k)\, \bar{u}\, T_{ji}\, \gamma^\mu (\not{q}+m_t)\left[(A+B)+(A-B)\gamma^5\right] u(p)$$

$$t \Rightarrow \frac{i}{\not{q} - m_t} = i\frac{\not{q}+m_t}{q^2 - m_t^2}$$

$q^2 = t$

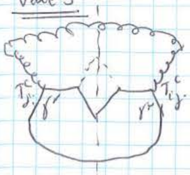
~~q = p+k~~

$$p = q + k' \qquad p' = k + q \Rightarrow q = p' - k = p - k'$$

$$|\overline{\mathcal{M}}_{fi}|^2 = |\mathcal{M}^s_{fi}|^2 + |\mathcal{M}^t_{fi}|^2 + |2\,\mathrm{Re}\, \mathcal{M}^s_{fi} \mathcal{M}^{t*}_{fi}|$$

129

Voie s



spin b, spin g, color b, color g

$$\sum_{c,i,j} T^c_{ij} T^c_{ji} = \sum_c \mathrm{Tr}(T^c T^c) = \sum_c \frac{1}{2}\delta_{cc} = \frac{8}{2} = 4$$

$$|\overline{\mathcal{M}}_{fi}|^2 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{8} \sum_{\substack{s,s'\\ \lambda,\lambda'\\ i,j,c}} \underbrace{\varepsilon_\mu(k)\varepsilon^*_\nu(k)}_{-g_{\mu\nu}}\, T^c_{ji} T^c_{ij}\, \bar{u}(p')\gamma^\mu \not{q}\left((A+B)+(A-B)\gamma^5\right) u(p)\bar{u}(p) \left[(A+B)-(A-B)\gamma^5\right]\not{q}\gamma^\nu u(p') \times \frac{g^2 g_s^2}{8 m_W^2} \times \frac{1}{s^2}$$

$$|\overline{\mathcal{M}}_{fi}|^2 = \frac{-1}{24} \frac{g^2 g_s^2}{8 m_W^2} \times \frac{1}{s^2}\, g_{\mu\nu}\, \mathrm{Tr}\left[(\not{p}'+m_t)\gamma^\mu \not{q}\, \not{p} \underbrace{\left[(A+B)-(A-B)\gamma^5\right]^2}\, \not{q}\gamma^\nu\right]$$

$$A^2 + 2AB + B^2 + A^2 + B^2 - 2AB - 2(A+B)(A-B)\gamma^5 = S + P\gamma^5 \qquad S = (A^2+B^2) \times 2 \qquad P = -2(A+B)(A-B)$$

4 Traces :

résultat identique

$$g_{\mu\nu}\, p'_\rho q_\alpha p_\sigma q_\beta \underbrace{\mathrm{Tr}\left[\gamma^\rho \gamma^\mu \gamma^\alpha \gamma^\sigma \gamma^\beta \gamma^\nu\right]}_{\substack{= \mathrm{Tr}[\gamma^\nu\gamma^\rho\gamma_\nu\gamma^\alpha\gamma^\sigma\gamma^\beta] \\ = 2\mathrm{Tr}[\gamma^\rho\gamma^\alpha\gamma^\sigma\gamma^\beta]}} S = -8 p'_\rho q_\alpha p_\sigma q_\beta \left[g^{\rho\alpha}g^{\sigma\beta} - g^{\rho\sigma}g^{\alpha\beta} + g^{\rho\beta}g^{\alpha\sigma}\right] S$$

$$= -16(A^2+B^2)\left[2(p'\cdot q)(p\cdot q) - q^2(p\cdot p')\right].$$

$$g_{\mu\nu}\, p'_\rho q_\alpha p_\sigma q_\beta\, \mathrm{Tr}\left[\gamma^\rho\gamma^\mu\gamma^\alpha\gamma^\sigma\gamma^\beta\gamma^\nu\gamma^5\right] P = 2 p'_\rho \underbrace{q_\alpha p_\sigma q_\beta}_{\text{sym}}\, \mathrm{Tr}\left[\gamma^\rho\gamma^\alpha\gamma^\sigma\gamma^\beta\gamma^5\right] P = 0.$$

$\propto \varepsilon^{\rho\alpha\sigma\beta}$ antisym.

$$g_{\mu\nu}\, q_\alpha p_\sigma q_\beta\, \mathrm{Tr}\left[\gamma^\mu\gamma^\alpha\gamma^\sigma\gamma^\beta\gamma^\nu\right] = 0 \; ; \; g_{\mu\nu}\, q_\alpha p_\sigma q_\beta\, \mathrm{Tr}\left[\gamma^\mu\gamma^\alpha\gamma^\sigma\gamma^\beta\gamma^\nu\gamma^5\right] = 0.$$

Cinématique :

$$p'q = p'(p+k) = p'p + p'k = \tfrac{1}{2}(m_t^2 - u + m_t^2 - t)$$
$$p'p = \tfrac{1}{2}(m_t^2 - u)$$
$$q^2 = s$$
$$pq = \underset{0}{p^2} + pk = \tfrac{1}{2}s$$

$$2(p'q)(pq) - q^2(pp') = \frac{s}{2}\left[m_t^2 - u + m_t^2 - t\right] - \frac{s}{2}\left[m_t^2 - u\right]$$
$$= \frac{s}{2}(m_t^2 - t)$$

$$|\overline{\mathcal{M}}_{fi}|^2 = \frac{1}{3} \frac{g^2 g_s^2}{8 m_W^2}(A^2+B^2)\frac{m_t^2 - t}{s}$$

# LO Feynman calculation (2)

130

**Voie t**


$q = p' - k' = p - k'$

$$\overline{|\mathcal{M}^2_{t}|} = -\frac{1}{24}\,\frac{g^2 g_s}{8 m_W^2}\,\frac{1}{(t-m_t^2)^2}\, g_{\mu\nu}\,\mathrm{Tr}\left[(\not p' + m_t)\gamma^\mu(\not q + m_t)\not p\,(S + P\gamma^5)(\not q + m_t)\gamma^\nu\right]$$

- Traces en $m_t$ et $m_t^3$ : nbre impair de $\gamma^\mu$ avec/sans $\gamma^5 \to 0$.
- Traces en $\gamma^5$ : • $4\gamma$ et $\gamma^5$ : antisymétrique et $q_\alpha q_\beta$ symétrique $\to 0$.
  • $2\gamma$ et $\gamma^5 = 0$.

4 Traces non nulles :

$$g_{\mu\nu}\, p'_\rho q_\alpha p_\sigma q_\beta\, S\, \mathrm{Tr}\Big[\underbrace{\gamma^\rho\gamma^\mu\gamma^\alpha\gamma^\sigma\gamma^\beta\gamma^\nu}_{-2\ \text{contr.}}\Big] = -2\, p'_\rho q_\alpha p_\sigma q_\beta\, S\,\mathrm{Tr}[\gamma^\rho\gamma^\alpha\gamma^\sigma\gamma^\beta]\, S$$
$$= -16\big(2(p\cdot q)(p'\cdot q) - q^2(p\cdot p')\big)(A^2+B^2)$$

$$g_{\mu\nu}\, m_t^2 S\, p'_\rho p_\sigma\, \mathrm{Tr}[\gamma^\rho\gamma^\mu\gamma^\sigma\gamma^\nu] = -2 m_t^2 S\, p'_\rho p_\sigma\, \mathrm{Tr}[\gamma^\rho\gamma^\sigma] = -8\, p'p\, m_t^2 S$$
$$= -16(A^2+B^2)\, p'p\, m_t^2$$

$$\left.\begin{array}{l} g_{\mu\nu}\, m_t^2 S\, q_\alpha p_\sigma\, \mathrm{Tr}[\gamma^\mu\gamma^\alpha\gamma^\sigma\gamma^\nu] \\ g_{\mu\nu}\, m_t^2 S\, q_\alpha p_\sigma\, \mathrm{Tr}[\underbrace{\gamma^\mu\gamma^\sigma\gamma^\alpha\gamma^\nu}_{+4}] \end{array}\right\}\ 32\, S m_t^2 (p\cdot q) = 64 (A^2+B^2) m_t^2\, p\cdot q.$$

$$-16(A^2+B^2)\left[2(p\cdot q)(p'q) - q^2(pp') + p'p\, m_t^2 - 4 pq\, m_t^2\right]$$

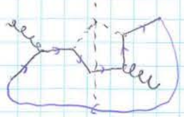
$$\left.\begin{array}{l} p'p = \frac{1}{2}(m_t^2 - u) = \frac{1}{2}(s+t-m_H^2) \\ q^2 = t \\ p\cdot q = p^2 - pk' = \frac{1}{2}(t - m_H^2) \\ p'q = p'^2 - p'k = m_t^2 + \frac{1}{2}(t-m_t^2) \\ \quad = \frac{t+m_t^2}{2} \end{array}\right\}$$

$$\frac{1}{2}\Big[(t-m_H^2)(t+m_t^2) - t(s+t-m_H^2) + m_t^2(s+t-m_H^2) - 4m_t^2(t-m_H^2)\Big]$$
$$= \frac{1}{2}\Big[\cancel{t^2} + \cancel{t m_t^2} - \cancel{t m_H^2} \boxed{- m_H^2 m_t^2} - ts - \cancel{t^2} + \cancel{t m_H^2} + s m_t^2 + t m_t^2$$
$$\boxed{- m_t^2 m_H^2}\ \cancel{- 4 m_t^2 t}\ \boxed{+ 4 m_t^2 m_H^2}\Big]$$
$$= \frac{1}{2}\Big[s(m_t^2 - t) + 2m_t^2(m_H^2 - t)\Big].$$

$$\overline{|\mathcal{M}_{t}|}^2 = \frac{1}{3}\,\frac{g_s^2 g^2}{8 m_W^2}\,(A^2+B^2)\,\frac{s(m_t^2-t) + 2m_t^2(m_H^2-t)}{(t-m_t^2)^2}$$

131

**Interférence**



$+\ cc \quad \left|2\,\mathrm{Re}\big[\mathcal{M}_s \mathcal{M}_t^*\big]\right|^2 = \overline{(I)}$.

$\frac{1}{24} \to \frac{1}{12}$.

$$\overline{|I|} = -\frac{1}{12}\,\frac{g^2 g_s^2}{8 m_W^2}\,\frac{1}{s(t-m_t^2)}\, g_{\mu\nu}\,\mathrm{Tr}\left[(\not p' + m_t)\,\not q_s \gamma^\mu \not p\,[S + P\gamma^5](\not q_t + m_t)\gamma^\nu\right]$$

- Traces en $m_t \to = 0$ (nbre impair de $\gamma$)

$$\to g_{\mu\nu}\, S\, \mathrm{Tr}\left[\not p'\, \not q_s \gamma^\mu \not p\, \not q_t\, \gamma^\nu\right] = g_{\mu\nu}\, p'_\rho q_{s\alpha} p_\sigma q_{t\beta}\, \mathrm{Tr}[\gamma^\rho\gamma^\alpha\gamma^\mu\gamma^\sigma\gamma^\beta\gamma^\nu]\, S$$
$$= p'_\rho q_{s\alpha} p_\sigma q_{t\beta}\, \mathrm{Tr}\Big[\gamma^\rho\gamma^\alpha\underbrace{\gamma^\mu\gamma^\sigma\gamma^\beta\gamma_\mu}_{4 g^{\sigma\beta}}\Big]\, S$$
$$= 4(p\cdot q_t)\, p'_\rho q_{s\alpha}\, \mathrm{Tr}\Big[\underbrace{\gamma^\rho\gamma^\alpha}_{4 g^{\rho\alpha}}\Big]\, S.$$
$$= 16 (p\cdot q_t)(p'\cdot q_s)\, S.$$
$$= 32(A^2+B^2)(p\cdot q_t)(p'\cdot q_s).$$

- de même avec $\gamma^5 \quad \propto \mathrm{Tr}[\gamma^\rho\gamma^\alpha\gamma^\mu\gamma^\sigma\gamma^\beta\gamma_\mu\gamma^5] \propto \mathrm{Tr}[\gamma^\rho\gamma^\alpha\gamma^5] = 0$.

$$\to g_{\mu\nu}\, S\, m_t^2\, \mathrm{Tr}[\not q_s \gamma^\mu \not p\, \gamma^\nu] = q_{s\alpha} p_\rho\, S m_t^2\, \mathrm{Tr}\Big[\underbrace{\gamma^\alpha\underbrace{\gamma^\mu\gamma^\rho\gamma_\mu}_{-2\gamma^\rho}}_{-8 g^{\alpha\rho}}\Big].$$
$$= -8\, S m_t^2 (q_s\cdot p)$$
$$= -16(A^2+B^2)\, q_s\cdot p\, m_t^2$$

$$-16(A^2+B^2)\left[(q_s\cdot p) m_t^2 - (p\cdot q_t)(p'\cdot q_s)\right].$$

$$\left.\begin{array}{l} q_s\cdot p = \frac{s}{2} \\ p\cdot q_t = \frac{1}{2}(t - m_H^2) \\ p'\cdot q_s = \frac{1}{2}(m_t^2 - t - u + m_t^2) \\ \quad = \frac{1}{2}(m_t^2 - m_H^2 + s) \end{array}\right\}\ \frac{1}{2}\Big[s m_t^2 + (t-m_H^2)(m_H^2 - m_t^2 - s)\Big].$$

$$\overline{|I|} = +\frac{2}{3}\,\frac{g^2 g_s^2}{8 m_W^2}\,\frac{(A^2+B^2)}{s(t-m_t^2)}\,\big(s m_t^2 + (t-m_H^2)(m_H^2 - m_t^2 - s)\big).$$

# LO Feynman calculation (end)

132

$$\overline{|\mathcal{M}_{fi}|} = \frac{1}{3} g_s^2 \frac{G_F}{\sqrt{2}} (A^2+B^2)\left[\frac{m_t^2-t}{s} + \frac{s(m_t^2-t)+2m_t^2(m_H^2-t)}{(t-m_t^2)^2} + 2\,\frac{s m_t^2+(t-m_H^2)(m_H^2-m_t^2-s)}{s(t-m_t^2)}\right]$$

$$d\sigma = \frac{1}{F}\overline{|\mathcal{M}_{fi}|}^2\, dPS^{(2)} = \frac{1}{16\pi}\frac{1}{\underbrace{\lambda(s,0,0)}_{=s}}|\mathcal{M}_{fi}|^2\, dt$$

$$\frac{d\sigma}{dt} = \frac{1}{12}\alpha_s \frac{G_F}{\sqrt{2}}\left[A^2+B^2\right]\frac{M}{s}$$

Section efficace Partonique

$s \to \hat{s}$, $t \to \hat{t}$.