Electroweak interactions of quarks

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PART 1 : Hadron decay, history of flavour mixing

PART 2 : Oscillations and CP Violation

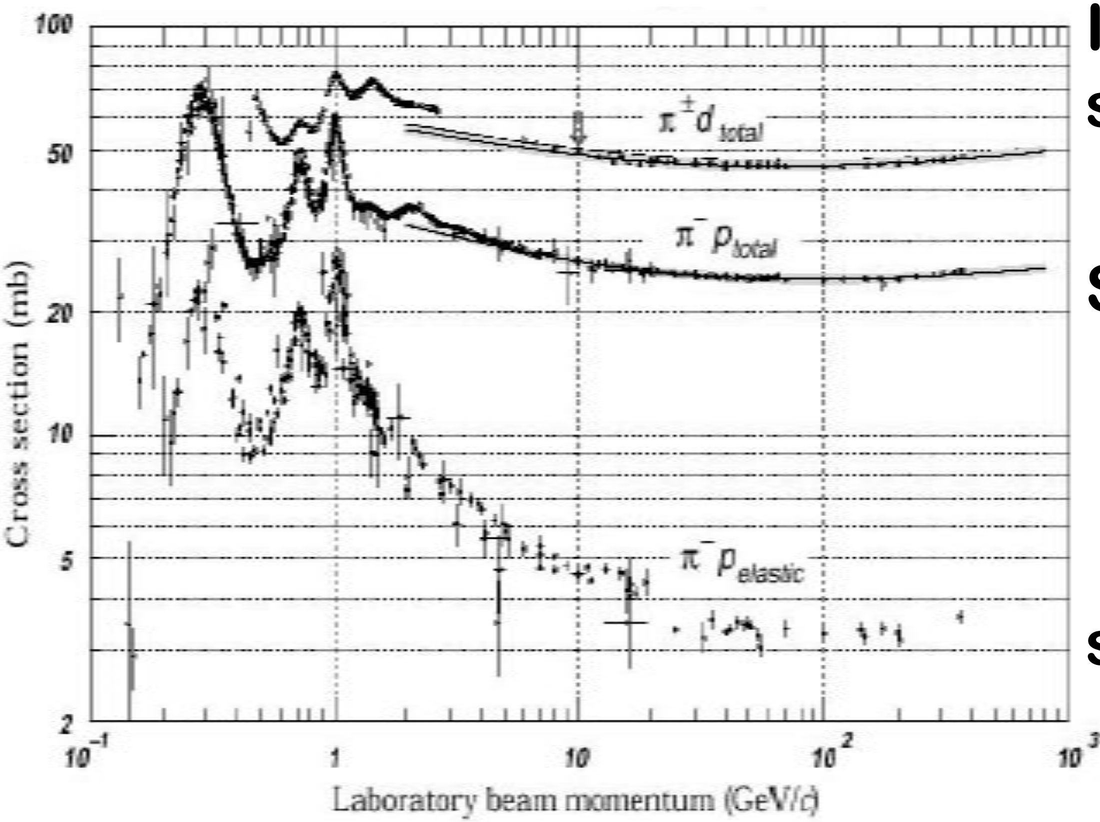
PART 3 : Top quark and electroweak physics

PART 1

Hadron decays
History of flavour mixing

Hadron spectroscopy

1950/1960 : lots of « hadronic states » observed resonances in inelastic diffusion of nucleons or pions (pp, pπ, np,…)



Interpreted as bound states of strong force

Several particles

- almost same mass
- same spin
- same behavior wrt. strong interaction
- different electric charge

Multiplet structure associated with SU(2) group symmetry

Strange hadrons

A few particles does not fit into this scheme :

Name	Mass (MeV)	Decay	Lifetime
Λ	1116	$N\pi$	1.10^{-10}s
Ξ^0, Ξ^-,	1320	$\Lambda\pi$	3.10^{-10}s
Σ^+,Σ^-	1190	$N\pi$	8.10^{-11}s
Σ^0	1200	$\gamma\Lambda$	1.10^{-20}s (EM)

Decay time characteristic from weak interaction

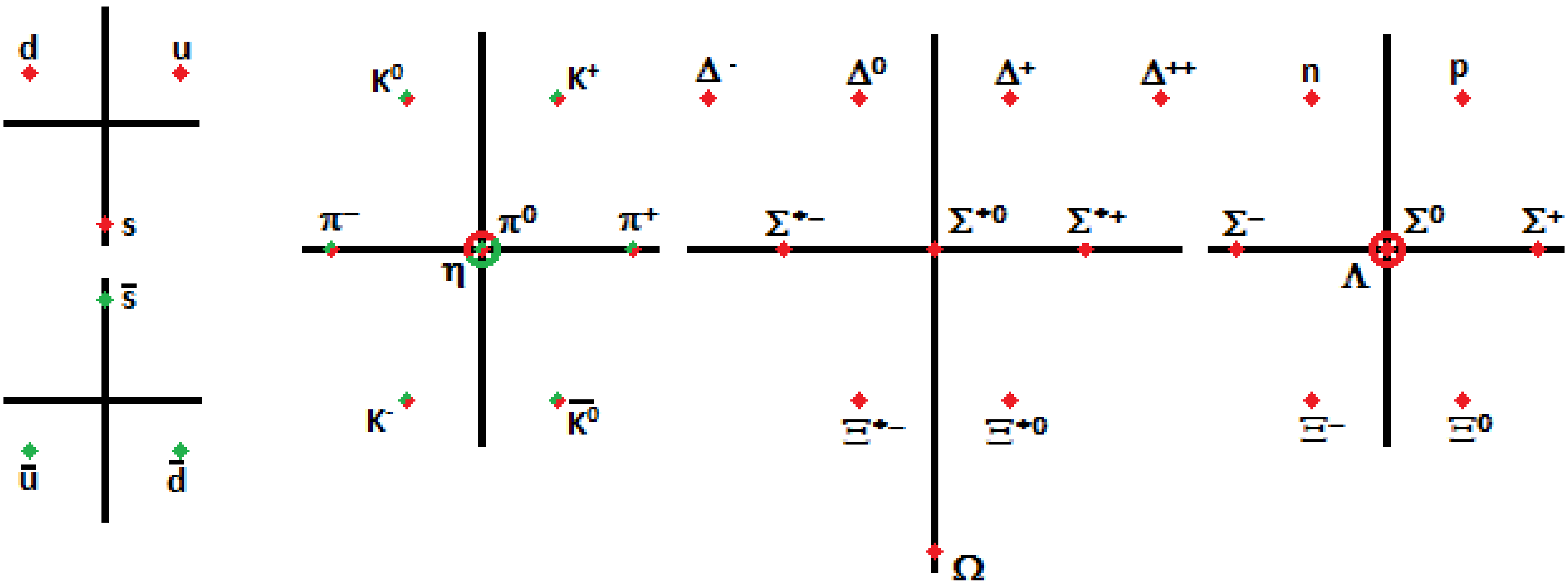
Particles stable wrt. strong/EM coupling

Conserved quantum number : strangeness

Quark model

Gell-Mann (Nobel 69) and Zweig, 1963 :

hadrons are build from 3 quarks u, d and s.



Strangeness : content in strange quarks

Only weak interaction can induce change of flavour

$s \rightarrow Wu$ or ~~$s \rightarrow Zd$~~ eg. $\frac{BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{BR(K^+ \rightarrow \pi^0 \mu^+ \nu)} < 10^{-5}$

No Flavor Changing Neutral Currents (**FCNC**) in SM

Electroweak lagrangian

$$
\begin{aligned}
\mathcal{L}_{GSW} = & \sum_{\ell=e,\mu,\tau}\left[i\bar{\psi}_\ell\gamma^\mu\partial_\mu\psi_\ell + i\bar{\psi}_{\nu_\ell}\gamma^\mu\partial_\mu\psi_{\nu_\ell}\right] - \frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial^\mu W^{-\nu} - \partial^\nu W^{-\mu}) \\
& -\frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) + \frac{1}{2}\partial_\mu h \partial^\mu h \\
& -\sum_\ell \frac{\lambda_\ell v}{\sqrt{2}}\bar{\psi}_\ell\psi_\ell - \left(\frac{gv}{2}\right)^2 W_\mu^+ W^{-\mu} - \frac{1}{2}\left(\frac{gv}{2\cos\theta_W}\right)^2 Z_\mu Z^\mu - \frac{1}{2}\left(-2m^2\right)^2 h^2 \\
& -g_{em}\sum_\ell \bar{\psi}_\ell\gamma^\mu\psi_\ell A_\mu + \frac{g}{2\sqrt{2}}\sum_\ell\left[\bar{\psi}_{\nu_\ell}\gamma^\mu(1-\gamma^5)\psi_\ell W_\mu^+ + \bar{\psi}_\ell\gamma^\mu(1-\gamma^5)\psi_{\nu_\ell}W_\mu^-\right] \\
& +\frac{g}{4\cos\theta_W}\sum_\ell\left[\bar{\psi}_{\nu_\ell}\gamma^\mu(1-\gamma^5)\psi_{\nu_\ell}Z_\mu - \bar{\psi}_\ell\gamma^\mu(1-4\sin^2\theta_W-\gamma^5)\psi_\ell Z_\mu\right] - \sum_\ell\frac{\lambda_\ell}{\sqrt{2}}\bar{\psi}_\ell\psi_\ell h \\
& +ig_{em}\left[\partial_\mu A_\nu W^{-\mu}W^{+\nu} + \partial_\mu W_\nu^+ W^{-\nu}A^\mu + \partial_\mu W_\nu^- W^{+\mu}A^\nu - \partial_\mu A_\nu W^{-\nu}W^{+\mu}\right. \\
& \qquad \left. -\partial_\mu W_\nu^+ W^{-\mu}A^\nu - \partial_\mu W_\nu^- W^{+\nu}A^\mu\right] \\
& +ig\cos\theta_W\left[\partial_\mu Z_\nu W^{-\mu}W^{+\nu} + \partial_\mu W_\nu^+ W^{-\nu}Z^\mu + \partial_\mu W_\nu^- W^{+\mu}Z^\nu - \partial_\mu Z_\nu W^{-\nu}W^{+\mu}\right. \\
& \qquad \left. -\partial_\mu W_\nu^+ W^{-\mu}Z^\nu - \partial_\mu W_\nu^- W^{+\nu}Z^\mu\right] + \frac{g^2 v}{2}W_\mu^+ W^{-\mu}h + \frac{g^2 v}{4\cos^2\theta_W}Z_\mu Z^\mu h - \lambda v h^3 \\
& +g_{em}^2\left[W_\nu^+ W^{-\mu}A_\nu A^\mu - W_\mu^+ W^{-\mu}A_\nu A^\nu\right] + g^2\cos^2\theta_W\left[W_\nu^+ W^{-\mu}Z_\nu Z^\mu - W_\mu^+ W^{-\mu}Z_\nu Z^\nu\right] \\
& +g^2\cos\theta_W\sin\theta_W\left[2W_\mu^+ W^{-\mu}Z_\nu A^\nu - W_\mu^+ W^{-\nu}A_\nu Z^\mu - W_\mu^+ W^{-\nu}A^\mu Z_\nu\right] \\
& +\frac{g^2}{2}\left[W_\mu^- W^{-\mu}W_\nu^+ W^{+\nu} - W_\mu^- W^{+\mu}W_\nu^- W^{+\nu}\right] + \frac{g^2}{4}W_\mu^+ W^{-\mu}h^2 + \frac{g^2}{8\cos^2\theta_W}Z_\mu Z^\mu h^2 - \frac{\lambda}{4}h^4
\end{aligned}
$$

OK for leptons

What happens for quarks ?

$$
g_{em} = g sin\theta_W \quad , \quad v^2 = \frac{-m^2}{\lambda} \quad , \quad m^2 < 0 \quad , \quad \lambda > 0
$$

$SU(2)_L$ and quarks

$SU(2)_L$ symetry :

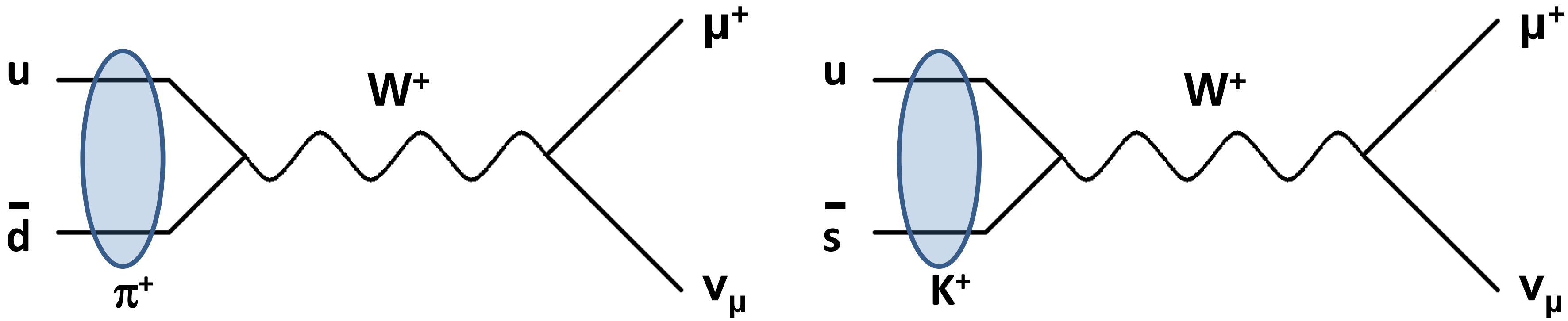
doublets of left handed fermions with $\Delta Q=1$:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \text{ and } s_L \quad \text{or} \quad \begin{pmatrix} u \\ s \end{pmatrix}_L \text{ and } d_L$$

singlets for right handed fermions : u_R, d_R, s_R

But both Wud and Wus vertices happen !

Eg. Leptonic pion and kaon decays :



Flavour mixing

strong interaction eigenstates (mass eigenstates)

may be different from

weak interaction eigenstates

Some mixing of d and s : $\begin{pmatrix} d_C \\ s_C \end{pmatrix} = U \begin{pmatrix} d \\ s \end{pmatrix}$

universality of weak interaction : conserve the overall coupling : $U^{+}U=1$

U is 2x2 rotation matrix , 1 parameter

$$U = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}$$

θ_C is the Cabibbo angle (1963)

Back to lagrangian

1 doublet $\begin{pmatrix} u \\ d_C \end{pmatrix}_L$ **and 4 singlets** $u_R, d_{CR}, s_{CR}, s_{CL}$

Charged currents :

$$L_{CC} = \frac{ig}{2\sqrt{2}} \bar{\psi}_{\boldsymbol{u}} \gamma^{\mu}(1-\gamma^5)\psi_{\boldsymbol{d_C}} W_{\mu}^{+} + h.c.$$

$$= \frac{ig\cos\theta_C}{2\sqrt{2}} \bar{\psi}_{\boldsymbol{u}} \gamma^{\mu}(1-\gamma^5)\psi_{\boldsymbol{d}} W_{\mu}^{+} + \frac{ig\sin\theta_C}{2\sqrt{2}} \bar{\psi}_{\boldsymbol{u}} \gamma^{\mu}(1-\gamma^5)\psi_{\boldsymbol{s}} W_{\mu}^{+} + h.c.$$

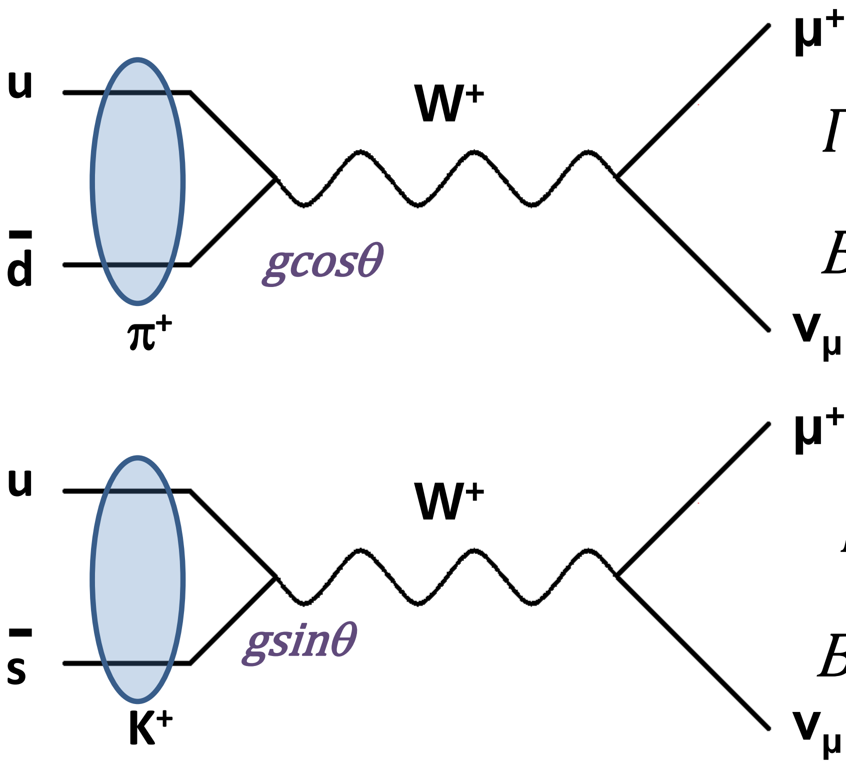
Vertices :

Wud : $-\frac{ig\cos\theta_C}{2\sqrt{2}}\gamma^{\mu}(1-\gamma^5)$ **Wus :** $-\frac{ig\sin\theta_C}{2\sqrt{2}}\gamma^{\mu}(1-\gamma^5)$

Naive estimation of θ_c

From pion and kaon lifetimes

$$\tau_\pi = 2.603 \times 10^{-8} s \qquad \tau_K = 1.23 \times 10^{-8} s$$



$\Gamma_{\pi\to\mu\nu} \propto$ $g^4 cos^2\theta_C$ $\dfrac{\left(m_\pi^2 - m_\mu^2\right)^2}{m_\pi^3}$

$BR \sim 100\%$

$\Gamma_{K\to\mu\nu} \propto$ $g^4 sin^2\theta_C$ $\dfrac{\left(m_K^2 - m_\mu^2\right)^2}{m_K^3}$

$BR \sim 63\%$

Feynman amplitude | **Phase space**

$$tan^2\theta_C = 0.63 \frac{\tau_\pi}{\tau_K} \frac{m_K^3}{m_\pi^3} \frac{\left(m_\pi^2 - m_\mu^2\right)^2}{\left(m_K^2 - m_\mu^2\right)^2} \Longrightarrow \begin{cases} sin\theta_C = 0.265 \\ cos\theta_C = 0.964 \end{cases}$$

FCNC troubles

Neutral currents (d and s quark only)

$$L_{NC} = \frac{ig}{2\cos\theta_W}\left(\bar{\psi}_{d_C}\gamma^\mu(\widehat{c_V} - \widehat{c_A}\gamma^5)\psi_{d_C}Z_\mu + \bar{\psi}_{s_C}\gamma^\mu(\widehat{c_V} - \widehat{c_A}\gamma^5)\psi_{s_C}Z_\mu\right)$$

$$\widehat{c_V} = \hat{T}^3 - 2\sin^2\theta_W\hat{Q} \;:\; \widehat{c_V}\psi_{d_C} = \left(-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W\right)\psi_{d_C} \;;\; \widehat{c_V}\psi_{s_C} = \left(\frac{2}{3}\sin^2\theta_W\right)\psi_{s_C}$$

$$\widehat{c_A} = \hat{T}^3 \;:\; \widehat{c_A}\psi_{d_C} = -\frac{1}{2}\psi_{d_C} \;;\; \widehat{c_V}\psi_{s_C} = 0$$

Introducing mass eigenstates :

$$L_{NC} = \frac{ig}{2\cos\theta_W}\left(\bar{\psi}_{d_C}\gamma^\mu c_Z^1\psi_{d_C}Z_\mu + \bar{\psi}_{s_C}\gamma^\mu c_Z^2\psi_{s_C}Z_\mu\right)$$

$$= \frac{ig}{2\cos\theta_W}\left(\bar{\psi}_d\gamma^\mu(\cos^2\theta_C c_Z^1 + \sin^2\theta_C c_Z^2)\psi_d Z_\mu + \bar{\psi}_s\gamma^\mu(\sin^2\theta_C c_Z^1 + \cos^2\theta_C c_Z^2)\psi_s Z_\mu\right)$$

$$+ \frac{ig\cos\theta_C\sin\theta_C}{2\cos\theta_W}\left(\bar{\psi}_d\gamma^\mu(c_Z^1 - c_Z^2)\psi_s Z_\mu + \bar{\psi}_s\gamma^\mu(c_Z^1 - c_Z^2)\psi_d Z_\mu\right)$$

FCNC inducing term

GIM and charm

Natural solution proposed by Glashow, Iliopoulos, Maiani in 1970(GIM mechanism)

Add a 4th quark to restore the symmetry : charm

2 $SU(2)_L$ doublets + right singlets

$$\begin{pmatrix} u \\ d_C \end{pmatrix}_L, \begin{pmatrix} c \\ s_C \end{pmatrix}_L, u_R, d_{CR}, c_R, s_{CR}$$

Then the coupling to the Z becomes :

$$c_Z^1 = c_Z^2 = -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W - \frac{1}{2}\gamma^5$$

And the FCNC terms cancel out :

$$\frac{ig\cos\theta_C\sin\theta_C}{2\cos\theta_W}\left(\bar{\psi}_d\gamma^\mu\underbrace{(c_Z^1 - c_Z^2)}_{=0}\psi_s Z_\mu + \bar{\psi}_s\gamma^\mu\underbrace{(c_Z^1 - c_Z^2)}_{=0}\psi_d Z_\mu\right)$$

Top and bottom

Generalization to 6 quarks : $\binom{u}{d}, \binom{c}{s}, \binom{t}{b}$

Complex 3x3 unitary matrix :

Kobayashi & Maskawa in 1973 (Nobel in 2008)

Cabibbo-Kobayashi-Maskawa or CKM Matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad , \qquad \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Lepton vertex $: -\frac{ig}{2\sqrt{2}}\gamma^{\mu}(1-\gamma^5)$

$W^+ q_u \bar{q}_d$ vertex $: -\frac{ig\boldsymbol{V_{q_u q_d}}}{2\sqrt{2}}\gamma^{\mu}(1-\gamma^5)$

$W^- \bar{q}_u q_d$ vertex $: -\frac{ig\boldsymbol{V^*_{q_u q_d}}}{2\sqrt{2}}\gamma^{\mu}(1-\gamma^5)$

CKM matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix}$$

First approximation : Diagonal matrix $V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

no family change : Wud, Wcs and Wtb vertices

Second approximation : Block matrix $V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & 0 \\ V_{cd} & V_{cs} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

submatrix is almost the Cabibbo matrix

$$V_{ud} \approx V_{cs} \approx \cos\theta_C \quad \text{and} \quad V_{us} \approx V_{cd} \approx \sin\theta_C$$

Top quark only decays to bottom quark

Charm quark mostly decays to strange quark

Bottom and Strange decays are CKM suppressed

PART 2

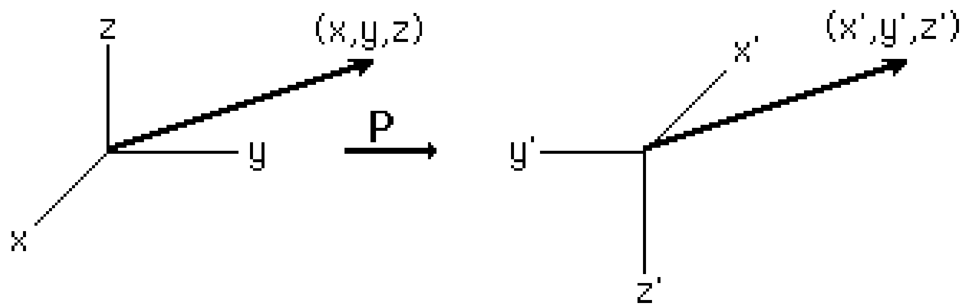
Oscillations and CP Violation

Discrete symmetries

3 discrete symmetries, such as $\hat{S}^2=1$

Affects : coordinates, operators, particles fields

$\hat{P}$ = Parity : space coordinates reversal : $\vec{x} \rightarrow -\vec{x}$



$\hat{C}$ = Charge conjugaison : particle to antiparticle transformation (i.e. inversion of all conserved charges, lepton and baryon numbers)

eg. $e^- \overset{\hat{C}}{\leftrightarrow} e^+, u \overset{\hat{C}}{\leftrightarrow} \bar{u}, \pi^-(d\bar{u}) \overset{\hat{C}}{\leftrightarrow} \pi^+(u\bar{d}), K^0(d\bar{s}) \overset{\hat{C}}{\leftrightarrow} \overline{K^0}(s\bar{d})$

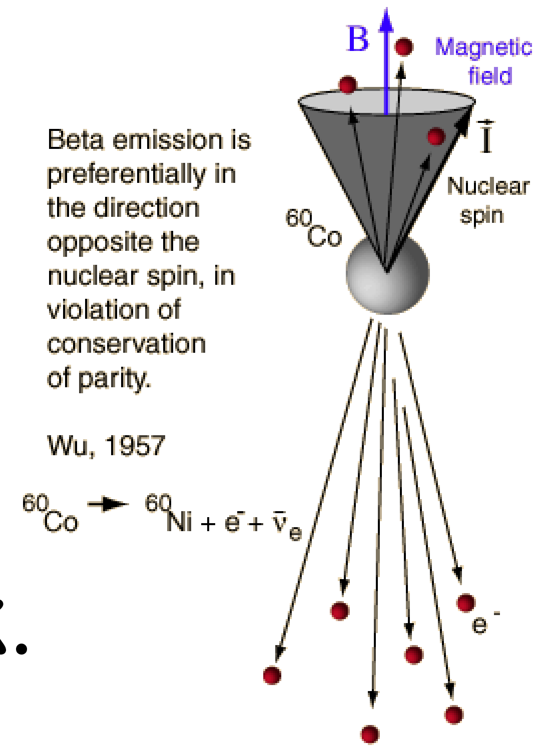
$\hat{T}$ = Time : time coordinate reversal : $t \rightarrow -t$

Parity

Weak interaction is not invariant under parity :

maximum violation of parity (C.S.Wu experiment on ^{60}Co beta decay)

Strong and EM interaction are OK.



Parity does not change the nature of particles parity eigenstates : $\hat{P}|p\rangle = \eta|p\rangle$

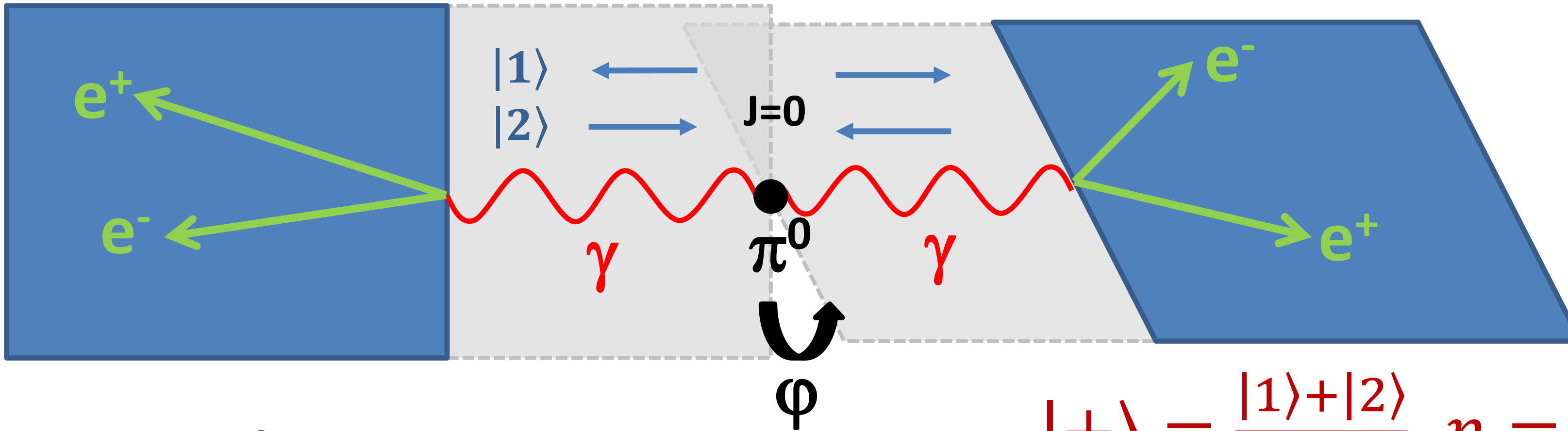
η : Intrinsic partity since $\hat{P}^2=1$, $\eta=\pm1$

Under $\hat{P}$: $\vec{E} \to -\vec{E}$, vector; $\vec{B} \to \vec{B}$, pseudovector

4 -potential : $(\phi, \vec{A}) \to (\phi, -\vec{A})$ so : $\eta_{photon} = -1$

Strong and EM interaction conserves parity.

Parity of the pion



$$|+\rangle = \frac{|1\rangle + |2\rangle}{\sqrt{2}}, \eta = 1$$

$$|-\rangle = \frac{|1\rangle - |2\rangle}{\sqrt{2}}, \eta = -1$$

EM decay : $\pi^0 \rightarrow \gamma\gamma$, conserves parity

Parity eigenstates for photon pair :

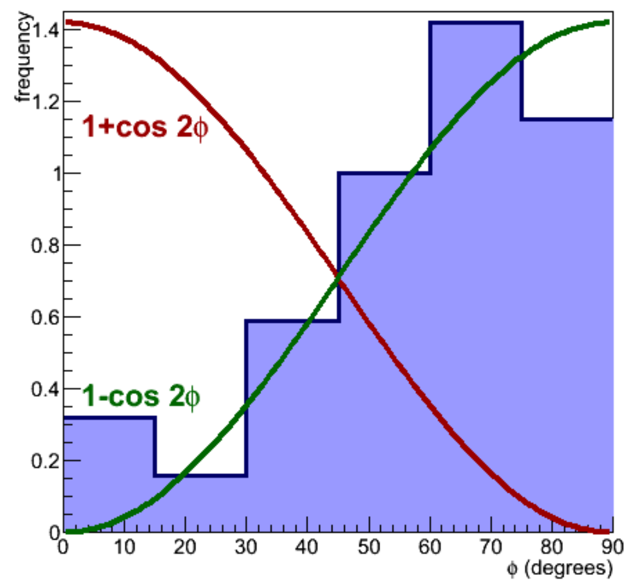
Pions can only decay in one of these states

Measure angular distribution of e^+e^- pairs :

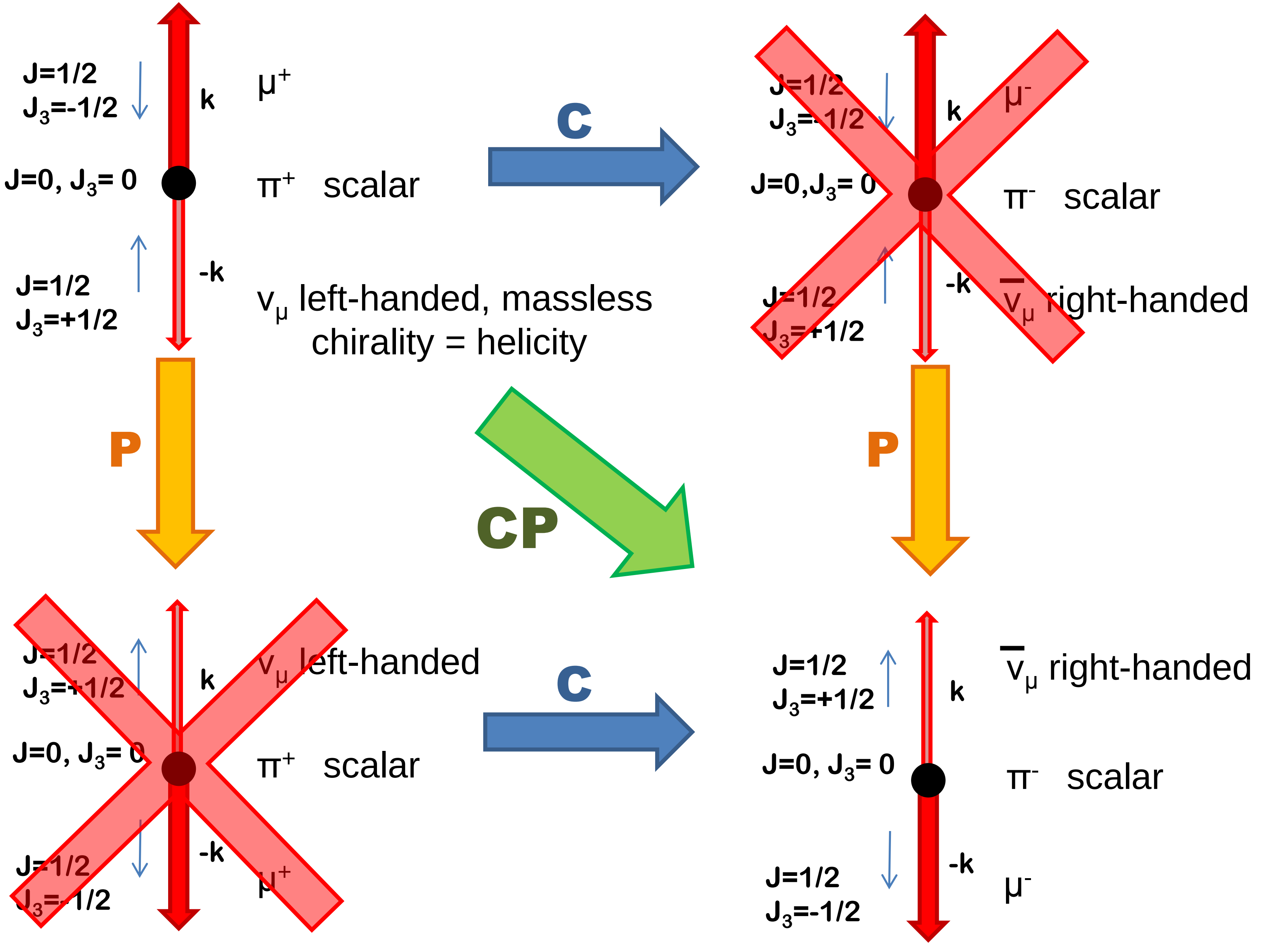
$|+\rangle : 1 + \cos 2\varphi$

$|-\rangle : 1 - \cos 2\varphi$

Experimentally : $\eta_{\pi^0} = -1$



CP symmetry : pion decay



CP symmetry : neutral kaons

Kaons are similar to pions :

→ same SU(3) octet pseudoscalar mesons

$$\hat{P}|K^0\rangle = -|K^0\rangle \text{ and } \hat{P}|\overline{K^0}\rangle = -|\overline{K^0}\rangle$$

K^0 (=u$\bar{s}$) and $\overline{K}^0$ (=$\bar{u}$s) are antiparticle of each other

$$\hat{C}|K^0\rangle = |\overline{K^0}\rangle \text{ and } \hat{C}|\overline{K^0}\rangle = |K^0\rangle$$

So: $$\hat{C}\hat{P}|K^0\rangle = -|\overline{K^0}\rangle \text{ and } \hat{C}\hat{P}|\overline{K^0}\rangle = -|K^0\rangle$$

Then CP-eigenstates are :

$$|K_1^0\rangle = \frac{|K^0\rangle - |\overline{K^0}\rangle}{\sqrt{2}}, \eta_{CP} = 1 \qquad |K_2^0\rangle = \frac{|K^0\rangle + |\overline{K^0}\rangle}{\sqrt{2}}, \eta_{CP} = -1$$

Does weak currents conserve CP ?

CP violation in Kaon decays

If CP is conserved by weak interactions then only

$|K_1^0\rangle \longrightarrow \pi\pi, \quad (\eta_{CP} = 1)$ and $|K_2^0\rangle \longrightarrow \pi\pi\pi, \quad (\eta_{CP} = -1)$

With a longer lifetime for $|K_2^0\rangle$ (more vertices)

Experimentally : $\tau_1 = 0.9 \times 10^{-10} s \qquad \tau_2 = 5.2 \times 10^{-8} s$

After $t >> \tau_1$ only the long-lived components remains but a few 2 pions decays are still observed ! Cronin & Fitch, 1964, Nobel 1980

Conclusion : the long lived component isn't a pure CP eigenstate : small CP violation !

Physical states : $|K_L^0\rangle = \frac{|K_1^0\rangle - \varepsilon |K_2^0\rangle}{\sqrt{1+\varepsilon^2}}, \quad |K_S^0\rangle = \frac{\varepsilon |K_1^0\rangle + |K_2^0\rangle}{\sqrt{1+\varepsilon^2}}$

$$\varepsilon = 2.3 \times 10^{-3}$$

Meson oscillations

Neutral pseudoscalar mesons $|P\rangle$ and $|\bar{P}\rangle$ (K^0, D^0, B^0, B_S)

Propagation/strong interaction Hamiltonian : $\hat{H}_0$

$$\hat{H}_0|P\rangle = m_P|P\rangle, \qquad \hat{H}_0|\bar{P}\rangle = m_P|\bar{P}\rangle$$ (in rest frame)

Interaction (weak) states ≠ propagation states.

Decay is allowed : effective hamiltonian $\hat{H}_W$ is not hermitian

$$\hat{H}_W = \hat{M} - i\frac{\hat{\Gamma}}{2}$$ with $\hat{M} = \frac{\hat{H}_W + \hat{H}_w^\dagger}{2}$ $\quad \hat{\Gamma} = \hat{H}_W - \hat{H}_w^\dagger$ hermitian

For eigen states : $\hat{M} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \qquad \hat{\Gamma} = \begin{pmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{pmatrix}$

And the time evolution of state 1 is :

$$|P_1(t)\rangle = e^{-im_1 t} e^{-\frac{\gamma_1 t}{2}} |P_1(0)\rangle \qquad I(t) = |\langle P_1(0)|P_1(t)\rangle|^2 = e^{-\gamma_1 t}$$

Exponential decay : $\hat{M}$ mass matrix, $\hat{\Gamma}$ decay width

Interaction eigenstates

In the basis $|P\rangle, |\bar{P}\rangle$, the 2 states have the same properties (CPT symmetry) : $M_{11} = M_{22} \equiv M_0$ and $\Gamma_{11} = \Gamma_{22} \equiv \Gamma_0$

If we assume $\widehat{M}, \widehat{\Gamma}$ to be real : $M_{12} = M_{21} \equiv \widetilde{M}$ and $\Gamma_{12} = \Gamma_{21} \equiv \widetilde{\Gamma}$

Then : $\widehat{H}_W = \begin{pmatrix} M_0 - i\frac{\Gamma_0}{2} & \widetilde{M} - i\frac{\widetilde{\Gamma}}{2} \\ \widetilde{M} - i\frac{\widetilde{\Gamma}}{2} & M_0 - i\frac{\Gamma_0}{2} \end{pmatrix}$

From perturbation theory :
$\widetilde{\Gamma} \approx -\Gamma_0$

In new basis :

$$|P_L\rangle = \frac{|P\rangle + |\bar{P}\rangle}{\sqrt{2}} \qquad |P_S\rangle = \frac{|P\rangle - |\bar{P}\rangle}{\sqrt{2}}$$

$$\widehat{H}_W = \begin{pmatrix} M_0 + \widetilde{M} - i\frac{\Gamma_0 + \widetilde{\Gamma}}{2} & 0 \\ 0 & M_0 - \widetilde{M} - i\frac{\Gamma_0 - \widetilde{\Gamma}}{2} \end{pmatrix}$$

$|P_L\rangle$: long lived, mass $M_L = M_0 + \widetilde{M}$ and width $\Gamma_L = \Gamma_0 + \widetilde{\Gamma} \ll \Gamma_0$

$|P_S\rangle$: short lived, mass $M_S = M_0 - \widetilde{M}$ and width $\Gamma_S = \Gamma_0 - \widetilde{\Gamma} \approx \Gamma_0$

Time evolution

General state : mixing of $|P_L\rangle$ and $|P_S\rangle$:

$$\left|\tilde{P}(t)\right\rangle = c_L(t)|P_L\rangle + c_S(t)|P_S\rangle$$

Coefficients c_L and c_S satisfy the Schrödinger eq. :

$$i\frac{d}{dt}\begin{pmatrix} c_L(t) \\ c_S(t) \end{pmatrix} = \begin{pmatrix} M_L - i\frac{1}{2}\Gamma_L & 0 \\ 0 & M_S - i\frac{1}{2}\Gamma_S \end{pmatrix}\begin{pmatrix} c_L(t) \\ c_S(t) \end{pmatrix}$$

Then time evolution is :

$$\left|\tilde{P}(t)\right\rangle = e^{-iM_0 t}\left(c_L(0)e^{i\tilde{M}t-\frac{\Gamma_L t}{2}}|P_L\rangle + c_S(0)e^{-i\tilde{M}t-\frac{\Gamma_S t}{2}}|P_S\rangle\right)$$

Time evolution

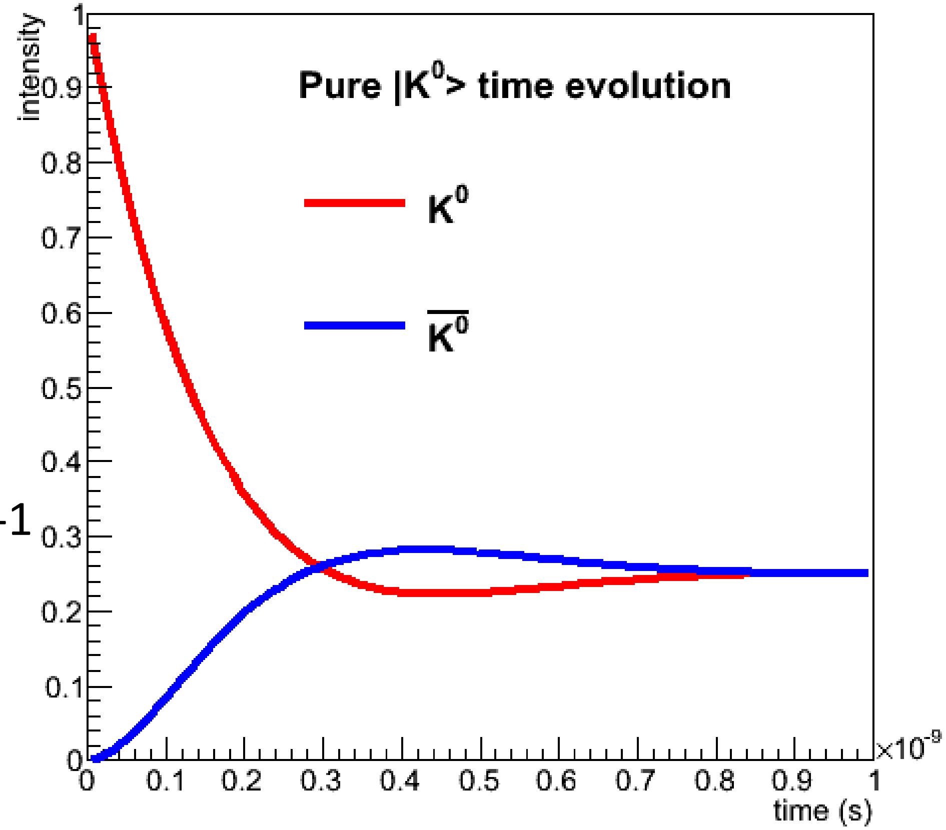
For a pure $|P\rangle$ initial state : $c_L(0) = c_S(0) = \frac{1}{\sqrt{2}}$.

And the intensity after a given time is :

$$I(t) = |\langle P|P(t)\rangle|^2 = \frac{1}{4}\left(e^{-\Gamma_L t} + e^{-\Gamma_S t} + 2e^{-\frac{\Gamma_L+\Gamma_S}{2}t}\cos(2\widetilde{M}t)\right)$$

Oscillations $|P_L\rangle \leftrightarrow |P_S\rangle$ in time, with a frequency equal to the mass difference :

$$\Delta m = M_L - M_S = 2\widetilde{M}$$



For the Kaon system :

$\Delta m = 3.48 \times 10^{-12}$ MeV = 5.29 ns^{-1}

τ_S= 89.6 ps

τ_L= 51.2 ns

CP violation

Previously we assumed $\hat{M}$ and $\hat{\Gamma}$ to be real

Interaction eigenstates = CP-eigenstates

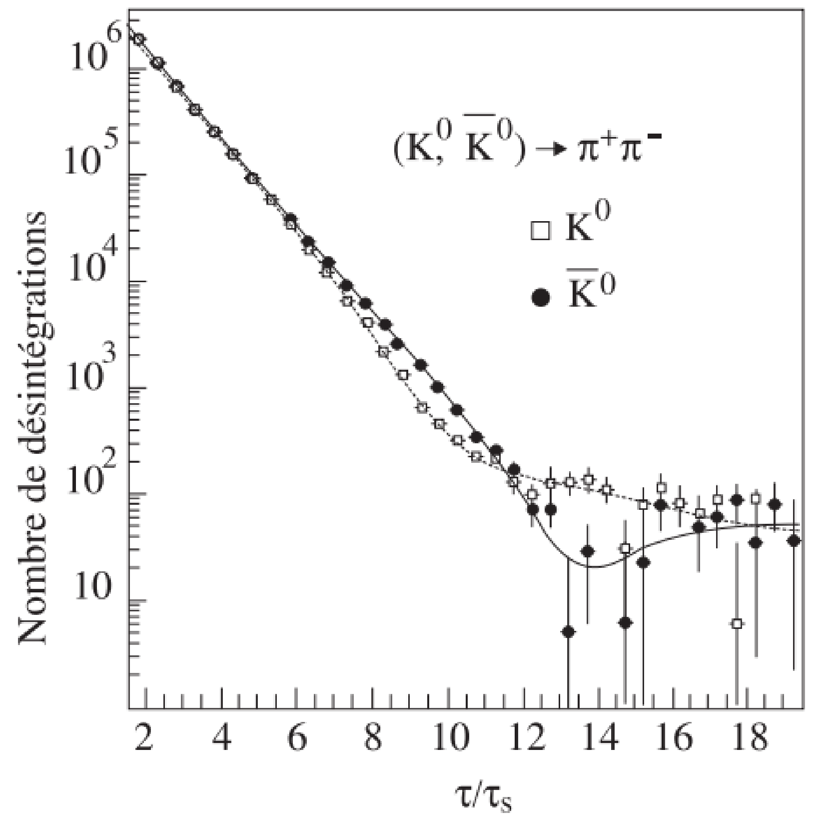
But if complex $M_{12} = M_{21}^*$ and $\Gamma_{12} = \Gamma_{21}^*$

Then the eigenstates ares :

$$|P_L\rangle = N(|P\rangle + \varepsilon|\bar{P}\rangle)$$
$$|P_S\rangle = N(|P\rangle - \varepsilon|\bar{P}\rangle)$$

$$\varepsilon = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}$$

$$N^{-2} = 1 + \sqrt{\frac{|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 + Im(\Gamma_{12}^* M_{12})}{|M_{12}|^2 + \frac{1}{4}|\Gamma_{12}|^2 - Im(\Gamma_{12}^* M_{12})}}$$



Complex mass matrix induces CP-violation

Parametrization of CKM

3x3 complex unitary matrix has 4 parameters

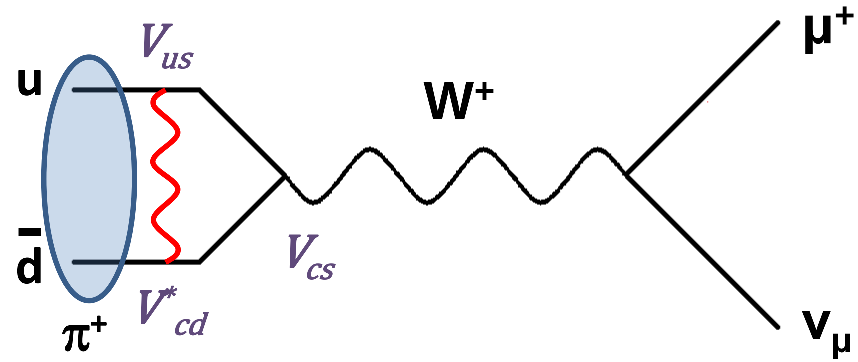
3 angles :

$$\alpha = \varphi_{13} = arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \beta = \varphi_{23} = arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \gamma = \varphi_{12} = arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

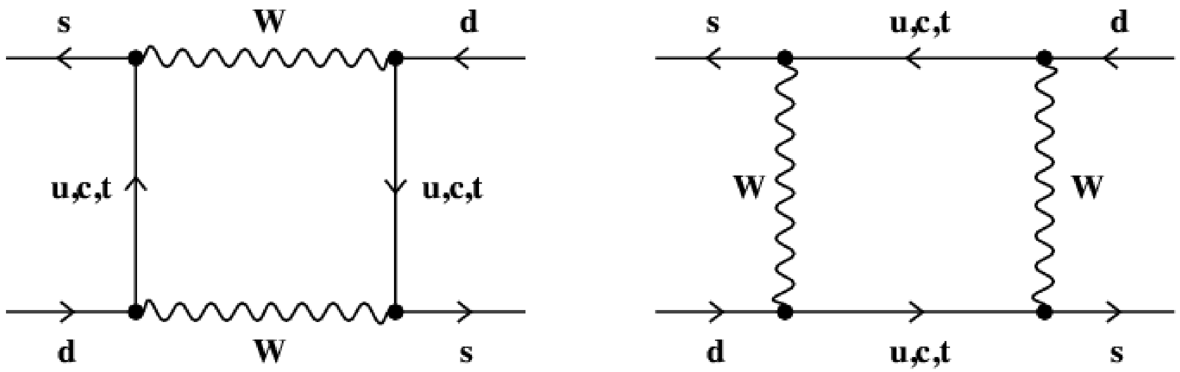
1 complex phase : δ

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{-i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{-i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{-i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{-i\delta} & c_{23}c_{13} \end{pmatrix}$$

Complex phase allows CP violation



Oscillation and boxes



Quark interpretation : box diagram

The mass matrix off-diagonal elements becomes:

$$M_{12} = C(V_{us}^{*}V_{ud}m_u + V_{cs}^{*}V_{cd}m_c + V_{ts}^{*}V_{td}m_t)^2$$

$$= C(V_{us}^{*}V_{ud}(m_u - m_c) + V_{ts}^{*}V_{td}(m_t - m_c))^2$$

Complex because of phase in CKM matrix :

CP violation :

can only happen if at least 3 families.

only happens in loop diagrams : rare decays and oscillations.

Unitarity triangles

Unitarity of CKM matrix :

$$V_{CKM}^{\dagger} V_{CKM} = 1$$

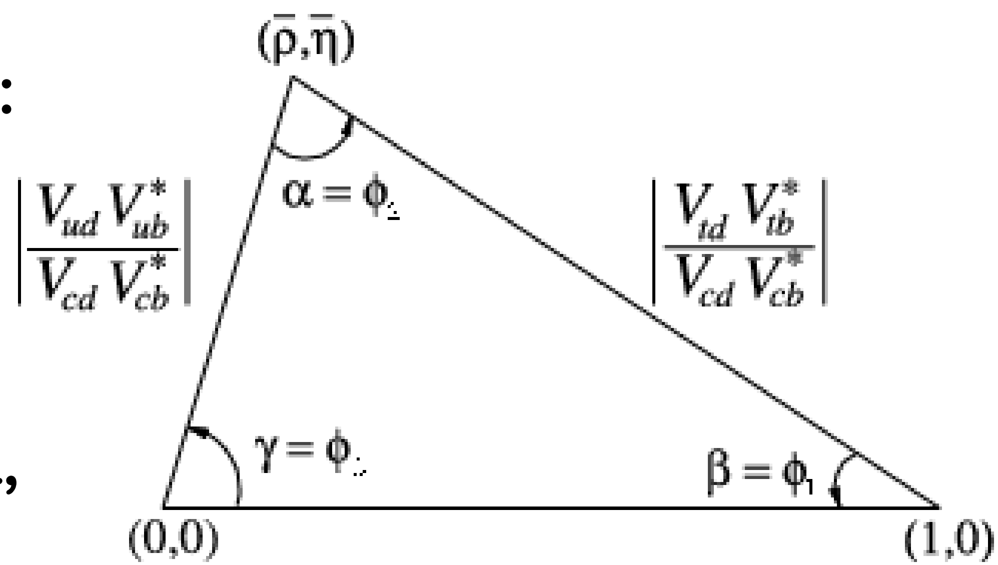
$$V_{ud}V_{ud}^* + V_{us}V_{us}^* + V_{ub}V_{ub}^* = 1$$
$$V_{cd}V_{cd}^* + V_{cs}V_{cs}^* + V_{cb}V_{cb}^* = 1$$
$$V_{td}V_{tb}^* + V_{ts}V_{ts}^* + V_{tb}V_{tb}^* = 1$$

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0$$
$$\boldsymbol{V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0}$$
$$V_{us}V_{ud}^* + V_{cs}V_{cd}^* + V_{ts}V_{td}^* = 0$$
$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0$$
$$V_{ub}V_{ud}^* + V_{cb}V_{cd}^* + V_{tb}V_{td}^* = 0$$
$$V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^* = 0$$

Sum of 3 complex numbers :
triangle in complex plane

Complex only if CP violating phase is large

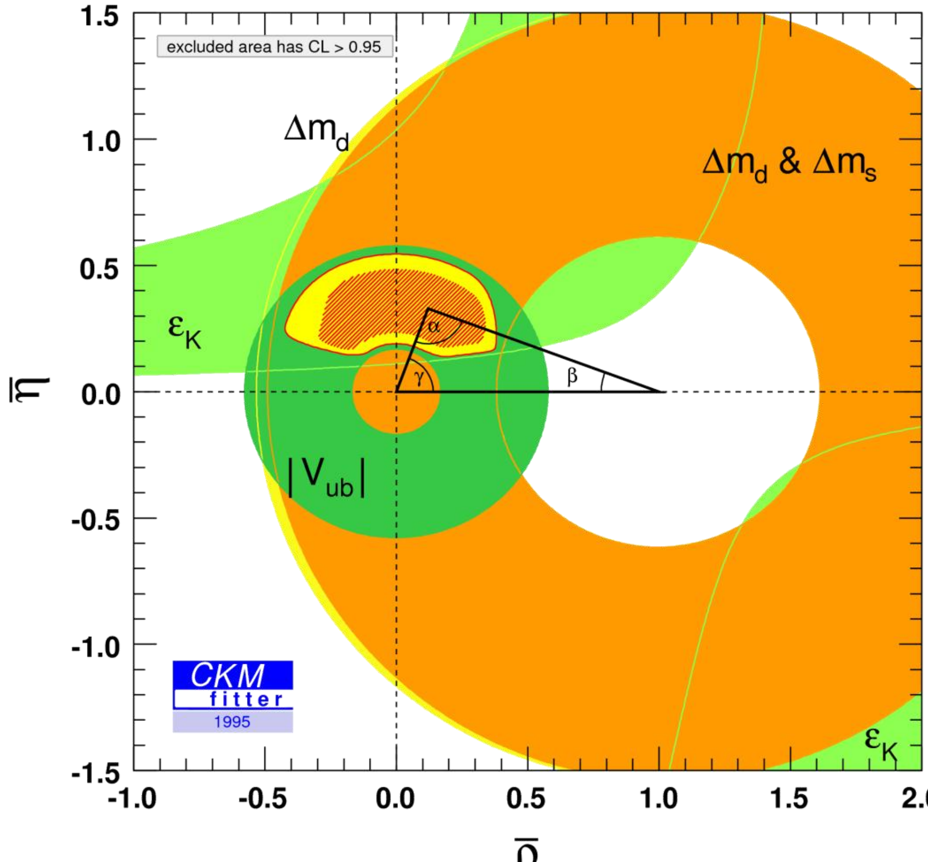
Most triangle are almost flat, except one.

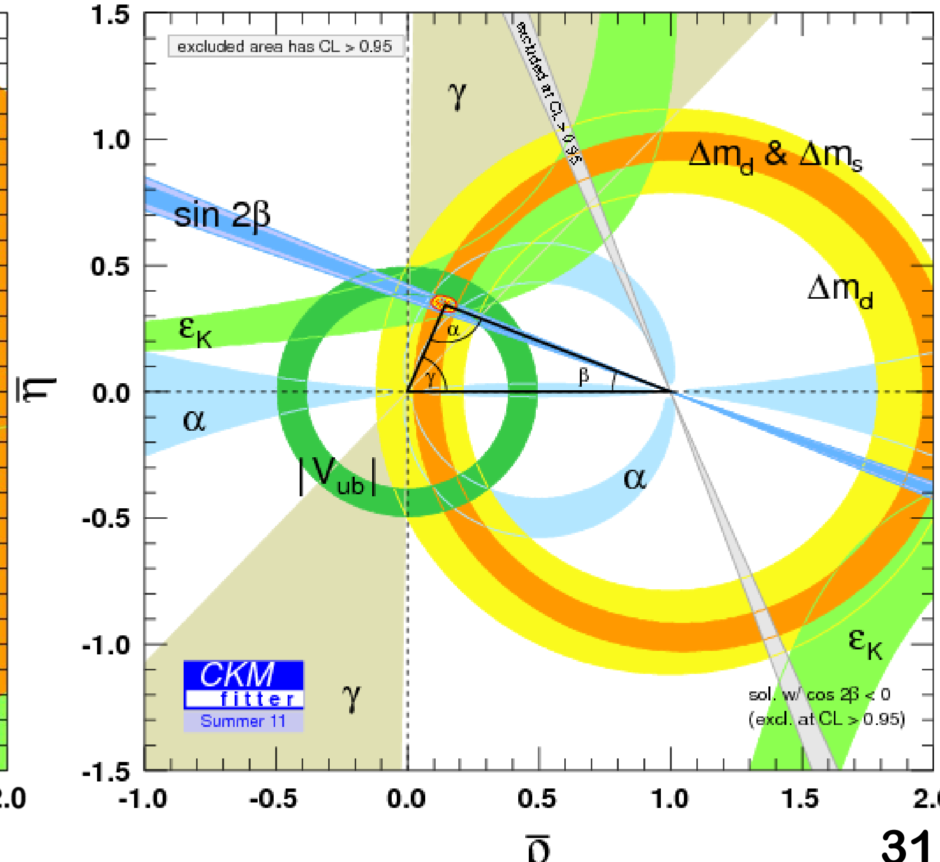


Unitarity triangles

Many decay processes, including rare decays of strange/charmed/beauty hadrons

- **Some sensitive to matrix elements V_{xy}**
- **Some sensitive mass differences (oscillations)**
- **Some sensitive to angles (CP-violation)**





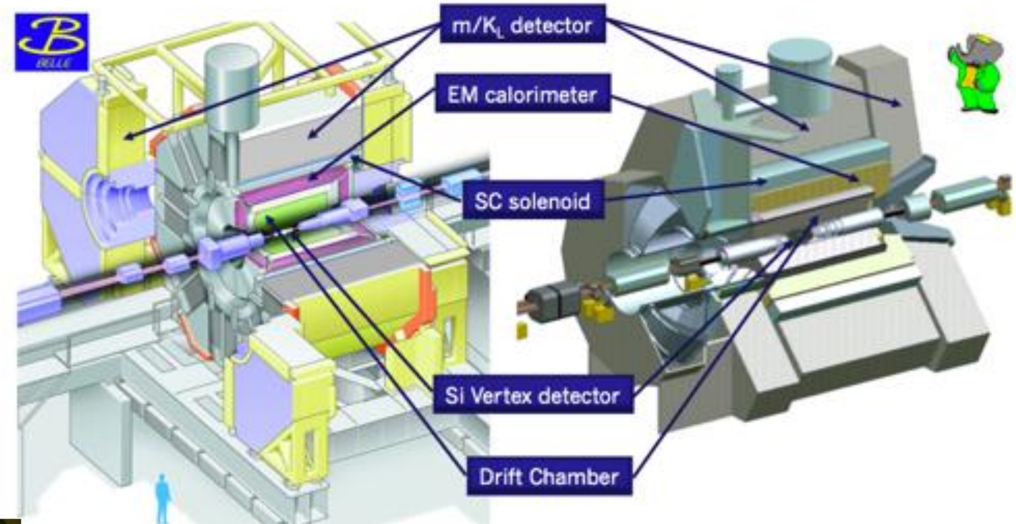
Experiments

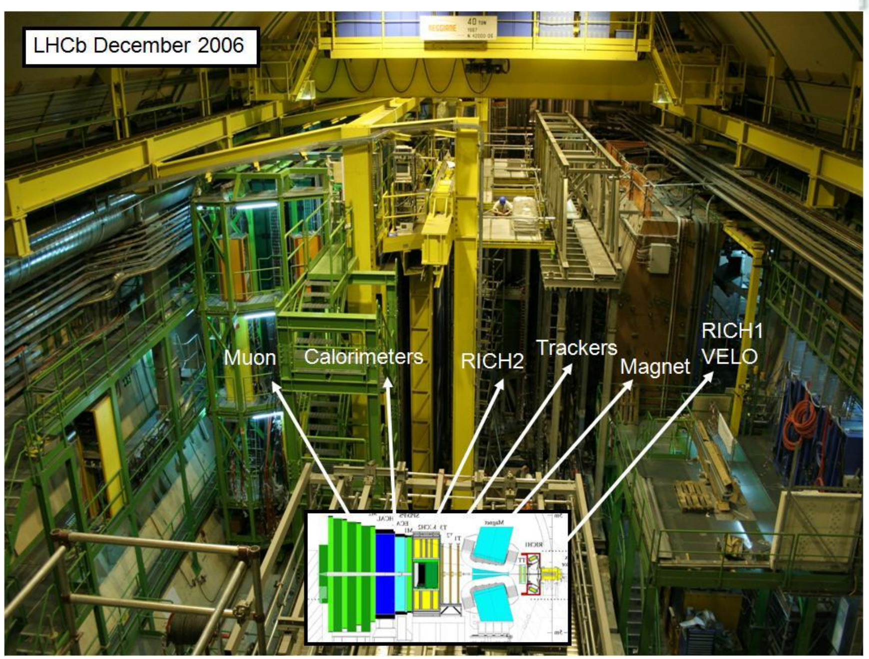
PAST :

e^+e^-, at $\psi(4S)$ resonnance (bb state)

- BELLE (KEK, Japan)
- BaBAR (SLAC, US)

pp : CDF and D0





PRESENT pp : LHCb (+ ATLAS/CMS)

FUTURE : e^+e^- SuperBELLE (Japan) SuperB (Italy)

PART 3

Top quark and electroweak physics

Top quark decays

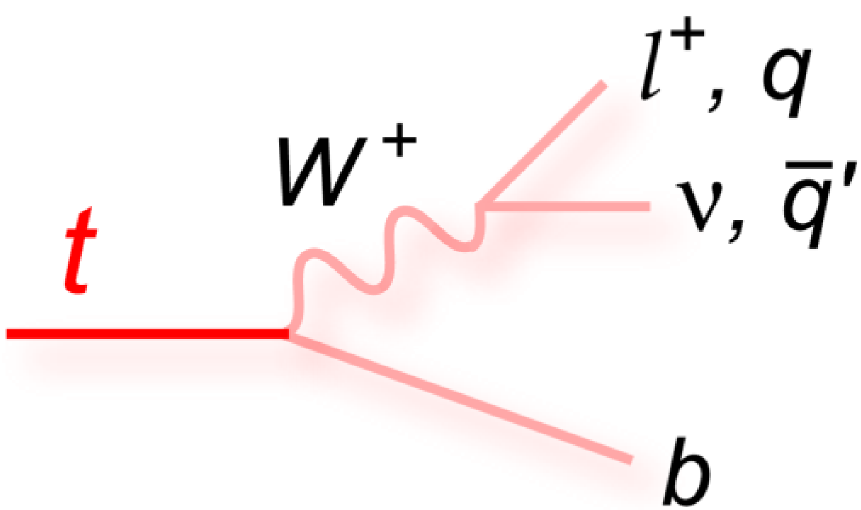
Top quark only decays through weak interaction

V_{tb} ~1 : $t \to Wb$ at 100%

$m_t > m_W + m_b$: only quark that decays with a real W

Coupling not « weak » : τ_{top} ~10^{-25}s >> $\tau_{hadronization}$

No loss of polarization.



Top quark signature :

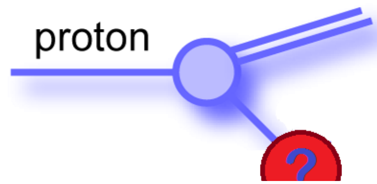
1 central b-quark jet, with high p_T (~70 GeV)

1 on-shell W boson :

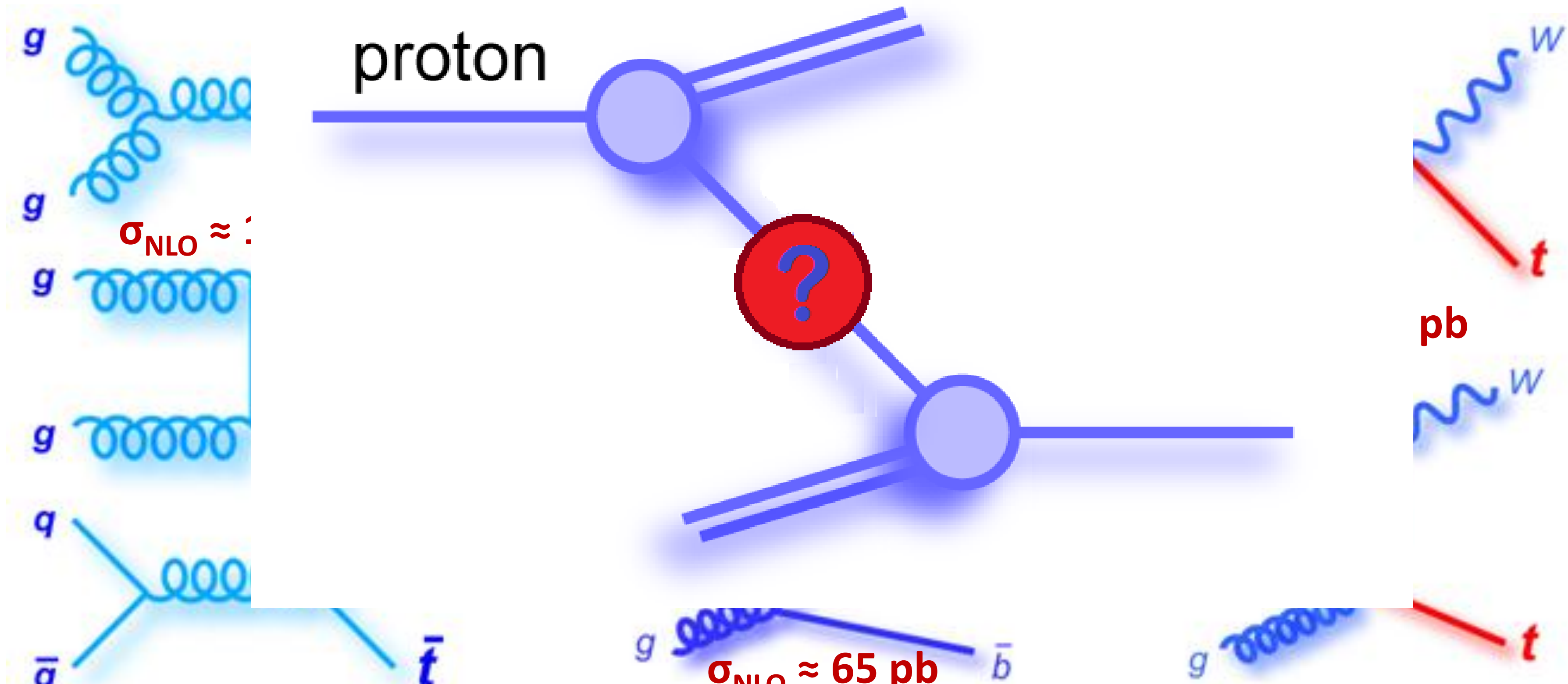
1 isolated lepton and $\not{E}_T$ (~35 GeV)

or 2 jets (~35 GeV)

Top quark production



$\sqrt{s}$=7TeV



Strong interaction
top/antitop pairs

Weak interaction
tb, tq(b)

Weak interaction
tW

Top quark and electroweak

Top quark decays through Wtb vertex

- Use pair production (largest cross-section)
- Probe V-A theory in top coupling
- Measure V_{tb} assuming 3 generations

Electroweak production of the top quark

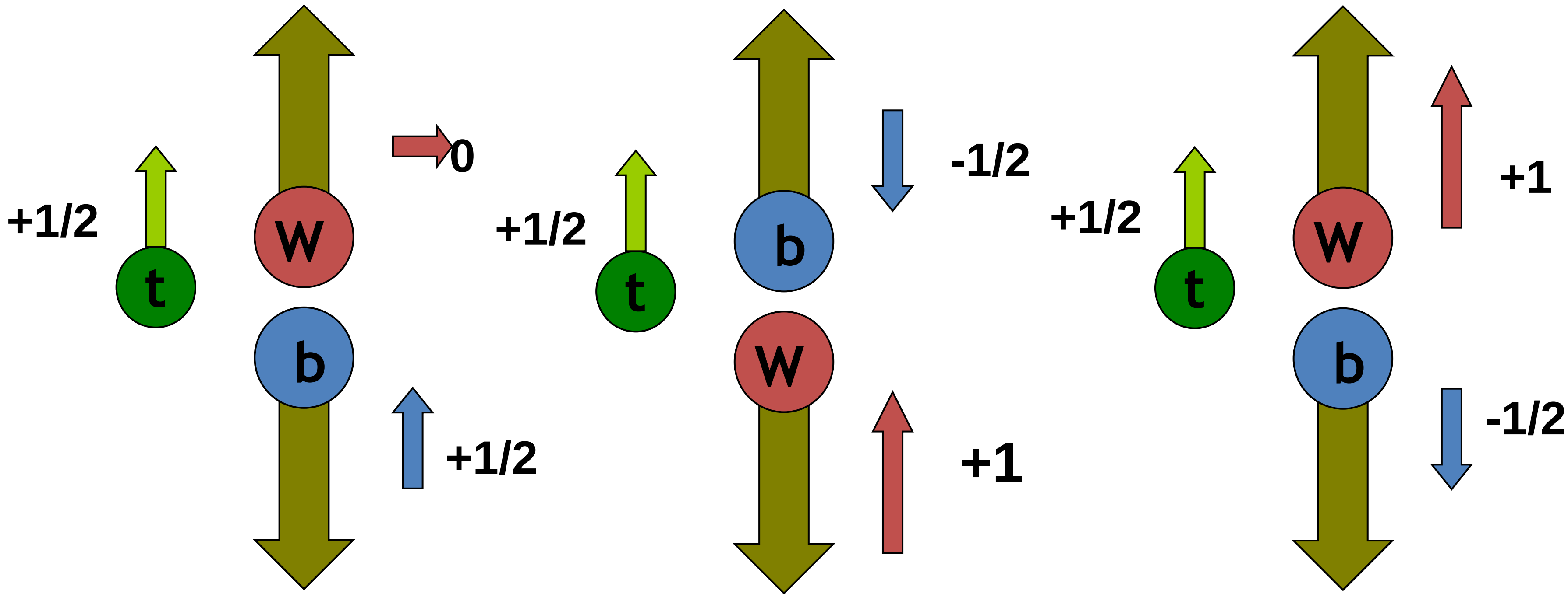
- lower cross-section, more background
- cross-section gives access to V_{tb} without assumptions on unitarity
- sensitivity to new physics (W', 4th generation…)

Precision measurement of mass and coupling

- all electroweak quantities are linked at higher order
- Sensitivity to the Higgs sector (high mass)

W helicity in top decays (1)

Longitudinal W	Left-handed W	Right-handed W
$F_0 \approx 0.7$	$F_L \approx 0.3$	$F_R \approx 0$



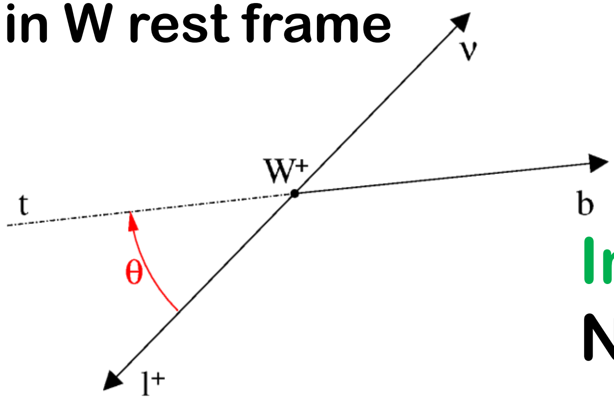
Like for muon decays : $m_b << m_t, m_W$

$\Rightarrow$ chirality = helicity

$\Rightarrow$ right-handed W is strongly suppressed

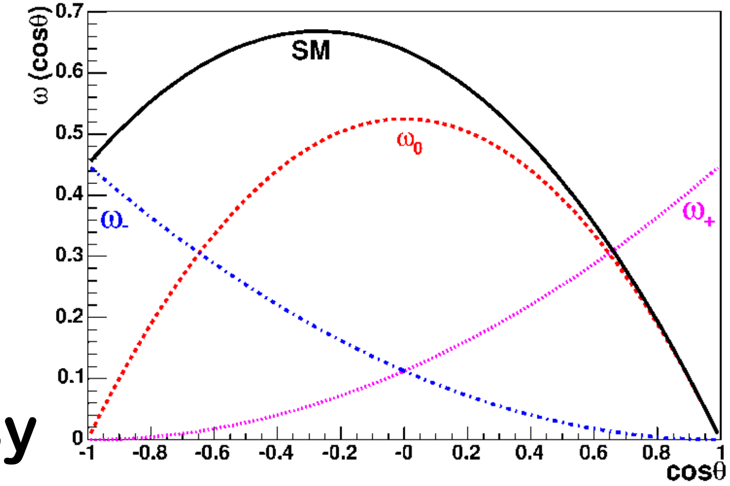
W helicity in top decays (2)

Angle between top and lepton in W rest frame



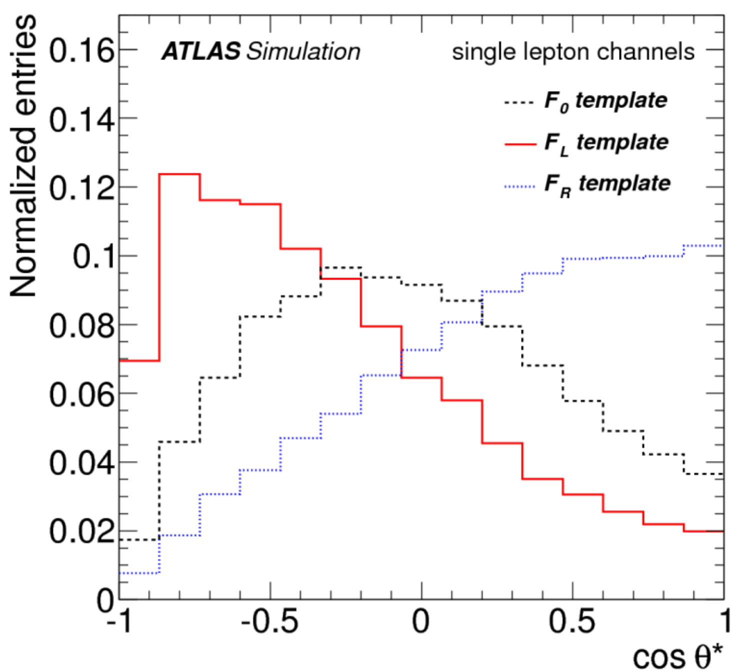
In theory :

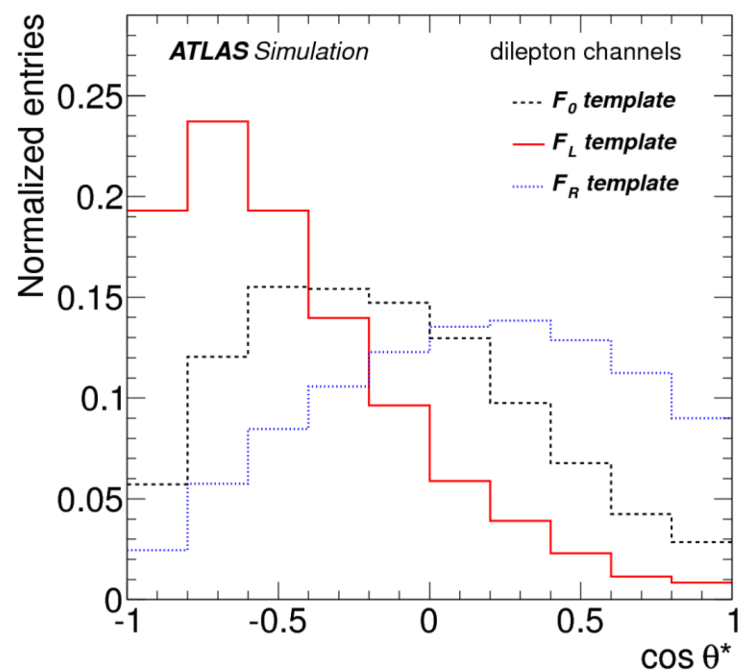
Nice and easy



In practice :

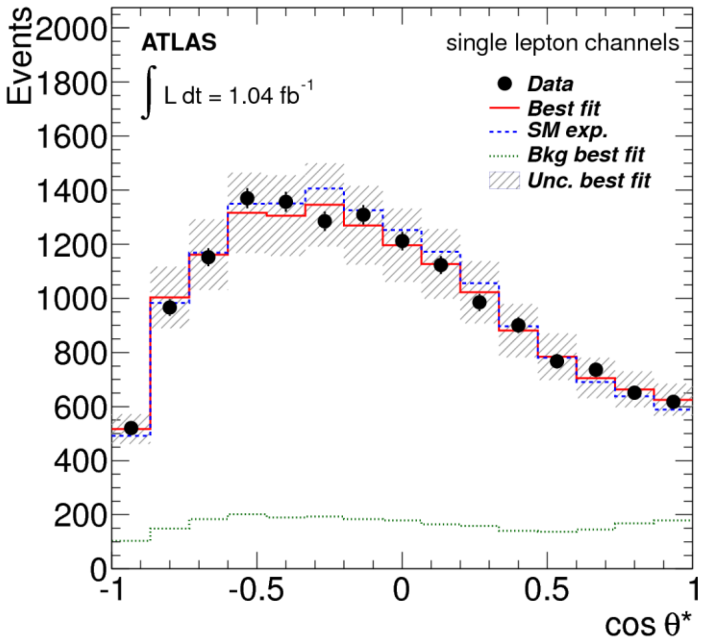
Expected shapes for ATLAS experiment

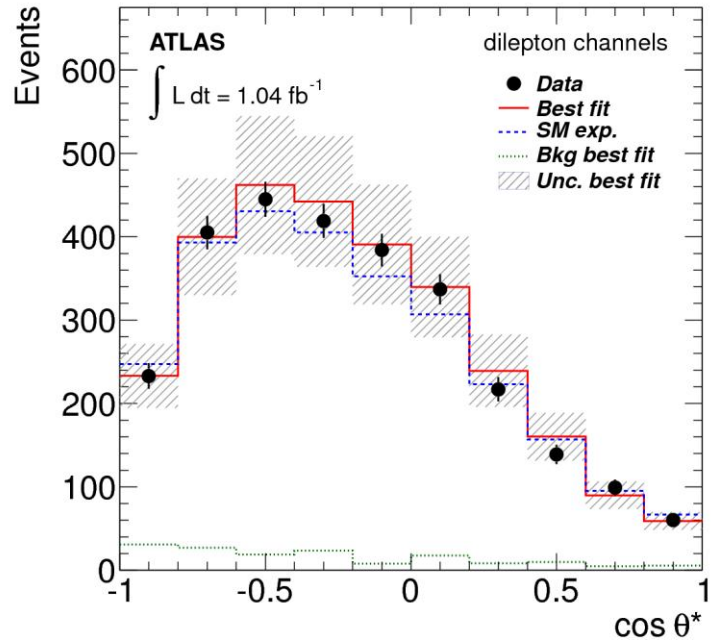


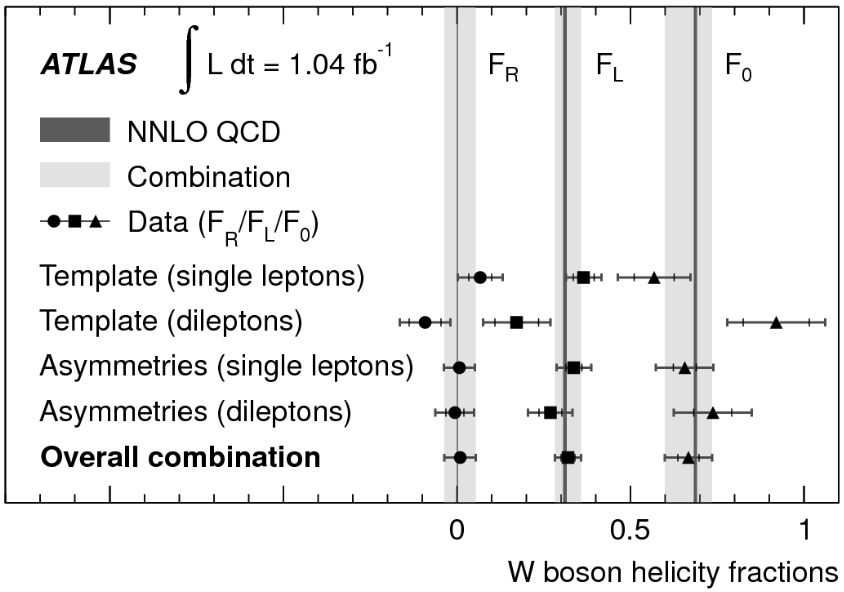


Also possible : unfolding…

ATLAS results 1fb^{-1}







The **Model** stays boringly **Standard**

Top decays and V_{tb}

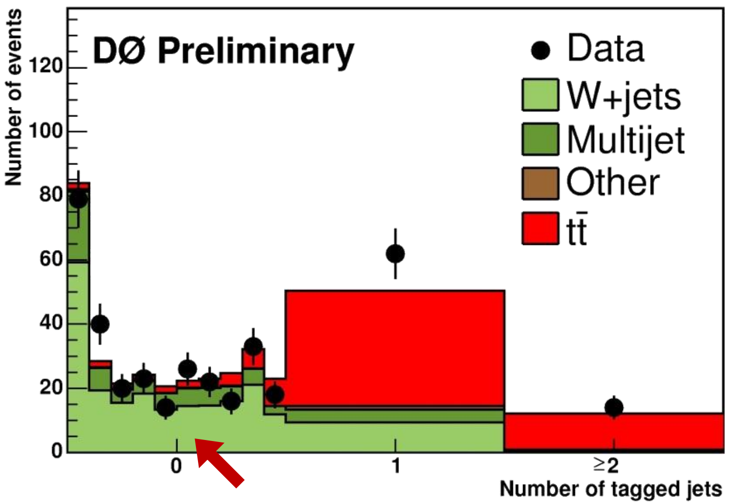
B-jets can be identified : long lifetime of b-hadrons, larger mass, fragmentation…

ε_{btag}~60%, mistag~0.1%

Use events with 0 b-tag, 1 b-tag and 2 b-tag

Simutaneous measurment of σ_{tt} and R

$$R = \frac{B(t \to Wb)}{B(t \to Wq)} = \frac{|V_{tb}|^2}{|V_{ts}|^2 + |V_{td}|^2 + |V_{tb}|^2} = |V_{tb}|^2$$

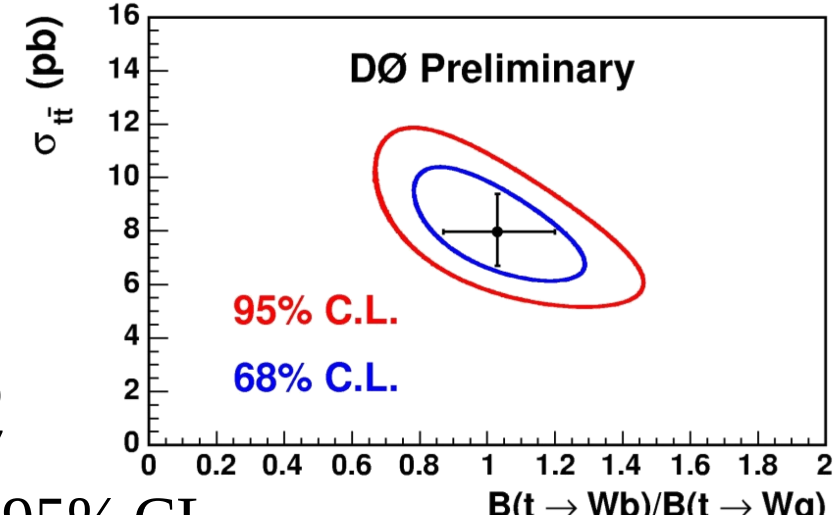


Likelihood discriminant

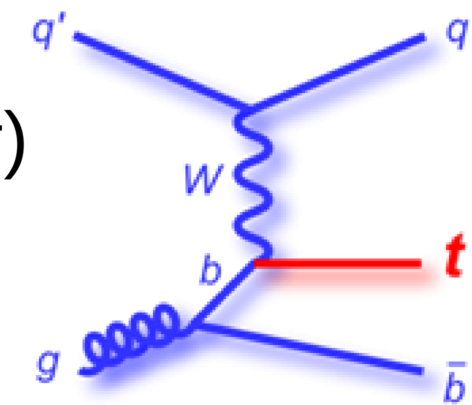
Very old D0 result Lepton+jets (~230 pb^{-1})

$$R = 1.03^{+0.19}_{-0.17}$$

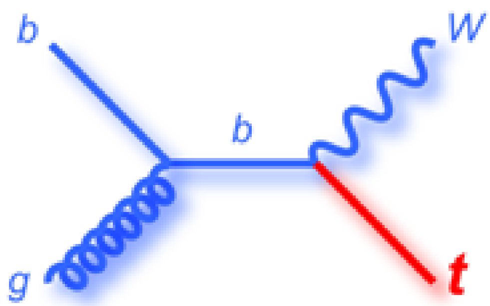
$$R > 0.64 \,@\, 95\%\ \mathrm{CL}$$



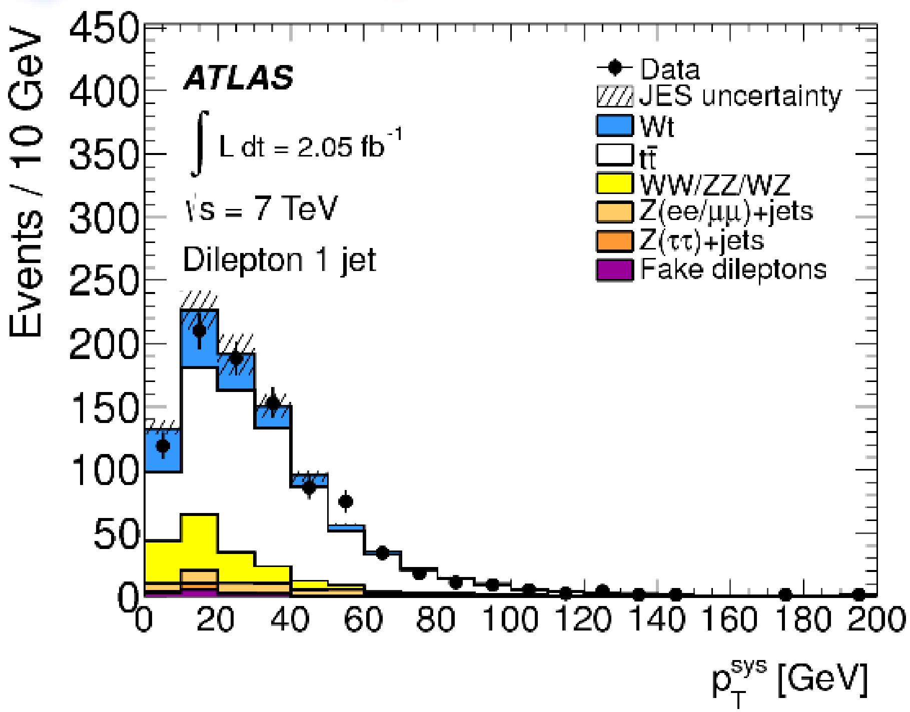
Single top

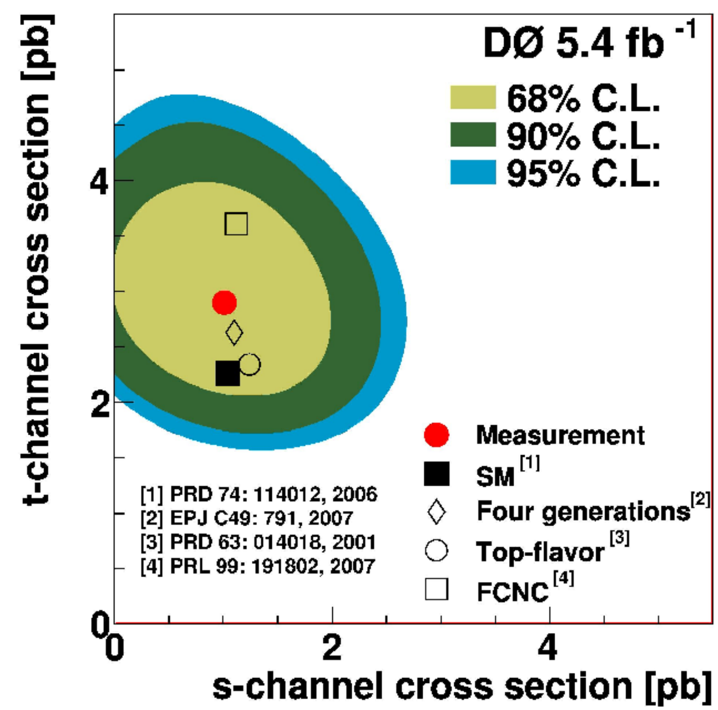


Only t-channel has been clearly (>5σ) seen at Tevatron and LHC.



First 3σ evidence for tW (ATLAS)

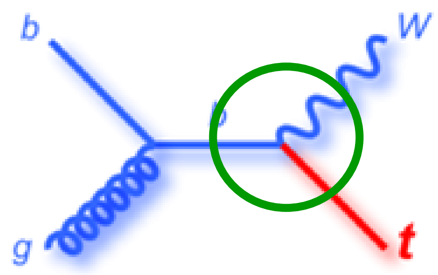


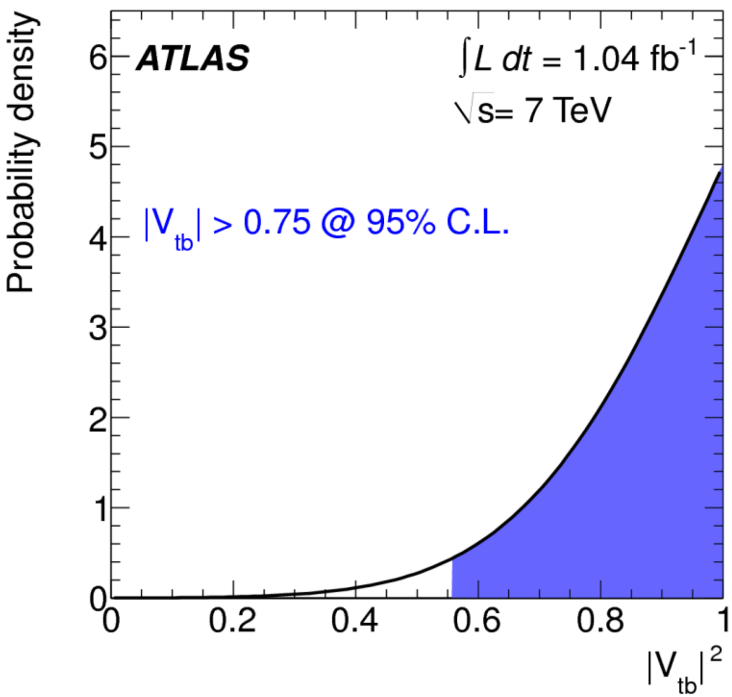


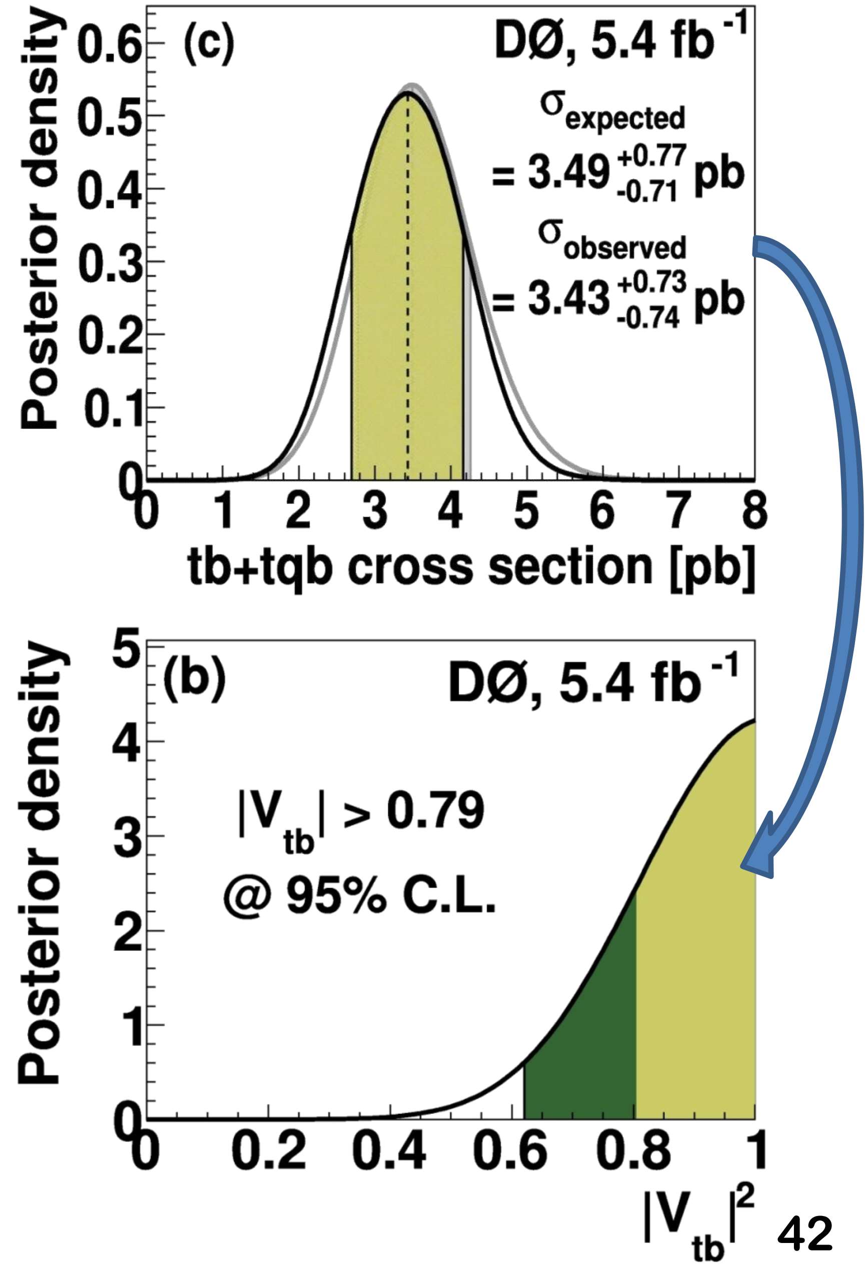
Direct V_{tb} measurement

All 3 diagrams feature 1Wtb vertex

$\sigma \propto |V_{tb}|^2$



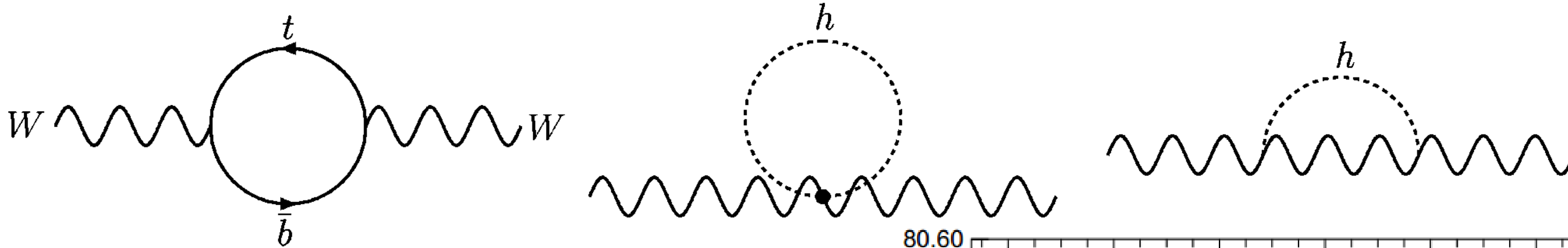




Standard Model consistancy

Top mass is one of the ingredients of the electroweak fit

indirect contraints on the Higgs mass



Radiative corrections (1 loop) to the W mass

$$\mathbf{m_W} = \frac{\pi\alpha}{\sqrt{2}\mathbf{G_F}}\frac{\mathbf{1}}{\sin^2\theta_\mathbf{W}(\mathbf{1}+\mathbf{\Delta r})}$$

$$\mathbf{\Delta r_{top}} \approx -\frac{\mathbf{3G_F m_t^2}}{\mathbf{8}\sqrt{\mathbf{2}}\pi^2\tan^2\theta_\mathbf{W}} \propto \mathbf{m_t^2}$$

$$\mathbf{\Delta r_{Higgs}} \approx \frac{\mathbf{11G_F M_Z^2}\cos^2\theta_\mathbf{W}}{\mathbf{24}\sqrt{\mathbf{2}}\pi^2}\ln\frac{\mathbf{m_h^2}}{\mathbf{M_Z^2}} \propto \ln(\mathbf{m_h})$$