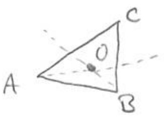
5.1 Le groupe ponctuel C_{3v}



a) $C_{3v} = \langle c, b \equiv \sigma_v \rangle \quad , \quad c^3 = b^2 = e$

$c = (A\,B\,C) \quad , \quad b = (B\,C) \qquad \Rightarrow \quad bc = (BC)(ABC) = (AC) \quad \Rightarrow (bc)^2 = e$

$\Rightarrow \quad C_{3v} \cong D_3$

b) $R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$D(c) = R(2\pi/3) = \begin{pmatrix} -1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & -1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad$ avec $\vec{e}_z := \frac{\vec{OD}}{|\vec{OD}|}$

$\vec{e}_x := \frac{\vec{OA}}{|\vec{OA}|}$

$D(b) = ?$ $\qquad \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$

$\cap$ plan OAD $\qquad \cap$ plan OAD $\qquad$ réflexion au plan OAD

$D(b) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

c) $\mathcal{D}^3 \equiv D : C_{3v} \to GL(3, \mathbb{R}) \qquad ; \quad C_{3v} < O(3)$

Il faut vérifier si cette rep. contient la rep. triviale.

Classes de conj. : $E = \{e\}$, $2C_3 = \{c, c^2\}$, $3\sigma_v = \{b, bc, bc^2\}$

Caractères :

C_{3v}	E	$2C_3$	$3\sigma_v$
$\mathcal{D}^{(0)} = A_1$	1	1	1
$\mathcal{D}^3 = V$	3	0	1

Irreps : A_1, A_2, E

$Tr[\mathrm{diag}(1,1,1)] = 3$, $Tr[D(c)] = 0$, $Tr[D(b)] = 1$

$\mathcal{D}^3 = a_0 \mathcal{D}^{(0)} + \ldots$

$a_0 = \langle \chi^V, \chi^{(0)} \rangle = \frac{1}{6}\left[1 \times 3 \times 1 + 2 \times 0 \times 1 + 3 \times 1 \times 1\right] = 1$

$a_0 \neq 0 \quad \Rightarrow \quad \vec{P}$ permanent possible

d) Le caractère pour un vecteur axiale est $\chi = (3, 0, -1)$

$\tilde{D}(b) = \begin{pmatrix} -1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$ inversion spaciale appliquée au vecteurs axiales

$\Rightarrow \quad a_0 = \frac{1}{6}(3 - 3) = 0$

$\Rightarrow$ signé opposé par rapport à un vrai vecteur

$\vec{r} \times \vec{p}$

$$\vec{r} \to \begin{pmatrix} r_1 \\ -r_2 \\ r_3 \end{pmatrix}$$

$$\vec{p} \to \begin{pmatrix} p_1 \\ -p_2 \\ p_3 \end{pmatrix}$$

$$\vec{r} \times \vec{p} \to \begin{vmatrix} e_1 & r_1 & p_1 \\ e_2 & -r_2 & -p_2 \\ e_3 & r_3 & p_3 \end{vmatrix} = \begin{pmatrix} -r_2 p_3 + p_2 r_3 \\ p_1 r_3 - r_1 p_3 \\ -r_1 p_2 + p_1 r_2 \end{pmatrix} = \begin{pmatrix} -L_1 \\ L_2 \\ -L_3 \end{pmatrix}$$

$$\vec{r} \times \vec{p} = \begin{vmatrix} e_1 & r_1 & p_1 \\ e_2 & r_2 & p_2 \\ e_3 & r_3 & p_3 \end{vmatrix} = \begin{pmatrix} r_2 p_3 - p_2 r_3 \\ p_1 r_1 - r_1 p_3 \\ r_1 p_2 - r_2 p_1 \end{pmatrix} = \begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix}$$

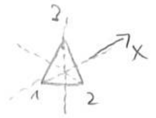
$$\Rightarrow \tilde{D}(b) = \begin{pmatrix} -1 & & \\ & 1 & \\ & & -1 \end{pmatrix}$$

(c'est une autre representation sur l'espace des vect. axiales)

5.2

En cours : G_{ij} transforme selon rep. $D^{V\otimes V_+}$ du groupe $D_3 = \langle c, b \rangle$

$$D^{V\otimes V_+} = 2A_1 + \dots \quad \Rightarrow \quad G = \begin{pmatrix} a & & 0 \\ & a & \\ 0 & & b \end{pmatrix}$$



Ici : calcul explicit !

$$G' = \underset{\substack{\uparrow \\ \text{repr. vectorielle}}}{D^V(g)}\, G\, D^V(g)^{-1} \overset{!}{=} G \quad \rightsquigarrow \text{ restrictions sur } G_{ij} \qquad ; \; D^V \equiv D$$

$$\Leftrightarrow \quad \boxed{D(g)\, G \overset{!}{=} G\, D(g)}$$

b : rotation autour de l'axe x par π $\Rightarrow$ $D(b) = \mathrm{diag}(1, -1, -1)$

$$\Rightarrow \quad D(b)\, G = \begin{pmatrix} G_{11} & G_{12} & G_{13} \\ -G_{21} & -G_{22} & -G_{23} \\ -G_{31} & -G_{32} & -G_{33} \end{pmatrix} \overset{!}{=} G\, D(b) = \begin{pmatrix} G_{11} & -G_{12} & -G_{13} \\ G_{21} & -G_{22} & -G_{23} \\ G_{31} & -G_{32} & -G_{33} \end{pmatrix}$$

$$\Rightarrow \quad G_{12} = G_{21} = G_{13} = G_{31} = 0 \quad \Rightarrow \quad G = \begin{pmatrix} G_{11} & 0 & 0 \\ 0 & G_{22} & G_{23} \\ 0 & G_{32} & G_{33} \end{pmatrix}$$

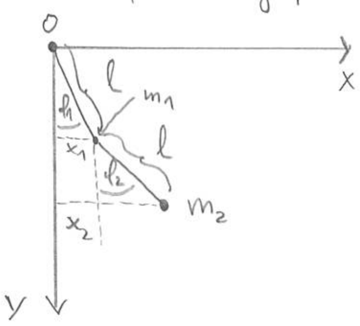
C : rotation par $\frac{2\pi}{3}$ autour de l'axe z : $D(c) = \begin{pmatrix} -1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & -1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$\Rightarrow \quad D(c)\, G = \begin{pmatrix} -1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & -1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} G_{11} & 0 & 0 \\ 0 & G_{22} & G_{23} \\ 0 & G_{32} & G_{33} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -G_{11} & -\sqrt{3}\, G_{22} & -\sqrt{3}\, G_{23} \\ \sqrt{3}\, G_{11} & -G_{22} & -G_{23} \\ 0 & 2G_{32} & 2G_{33} \end{pmatrix}$$

$$\overset{!}{=} \begin{pmatrix} G_{11} & 0 & 0 \\ 0 & G_{22} & G_{23} \\ 0 & G_{32} & G_{33} \end{pmatrix} \begin{pmatrix} -1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & -1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -G_{11} & -\sqrt{3}\, G_{11} & 0 \\ \sqrt{3}\, G_{22} & -G_{22} & 2G_{23} \\ \sqrt{3}\, G_{32} & -G_{32} & 2G_{33} \end{pmatrix}$$

$$\Rightarrow \quad G_{23} = G_{32} = 0 \, , \quad G_{22} = G_{11} \quad \Rightarrow \quad G = \mathrm{diag}(a, a, b)$$

5.3 Mécanique analytique



$x_1 = \ell \sin\varphi_1 \,,\; y_1 = \ell\cos\varphi_1$

$x_2 = x_1 + \ell\sin\varphi_2 \,,\; y_2 = y_1 + \ell\cos\varphi_2$

petites osc.:

$x_1 = \ell\varphi_1 \,,\; y_1 = \ell$

$x_2 = \ell(\varphi_1+\varphi_2) \,,\; y_2 = 2\ell$

$\dot x_1 = \ell\dot\varphi_1\cos\varphi_1 \,,\; \dot y_1 = -\ell\dot\varphi_1\sin\varphi_1$

$\dot x_2 = \dot x_1 + \ell\dot\varphi_2\cos\varphi_2 \,,\; \dot y_2 = \dot y_1 - \ell\dot\varphi_2\sin\varphi_2$

$\dot x_1 = \ell\dot\varphi_1 \,,\; \dot y_1 = 0$

$\dot x_2 = \ell(\dot\varphi_1+\dot\varphi_2) \,,\; \dot y_2 = 0$

$$T_1 = \frac{1}{2} m_1 \ell^2 \dot\varphi_1^2$$

$$T_2 = \frac{1}{2} m_2 \Big[\ell^2\dot\varphi_1^2 + \ell^2\dot\varphi_2^2 + 2\ell^2\big(\dot\varphi_1\dot\varphi_2 \underbrace{\cos\varphi_1\cos\varphi_2}_{\simeq 1} + \dot\varphi_1\dot\varphi_2 \underbrace{\sin\varphi_1\sin\varphi_2}_{\to 0}\big)\Big]$$

$$\underset{\text{petites osc.}}{=} \frac{1}{2} m_2 \ell^2 (\dot\varphi_1 + \dot\varphi_2)^2 + \mathcal{O}(\varphi_i^4)$$

$$T = \frac{1}{2}\ell^2\left[M\dot\varphi_1^2 + 2m\dot\varphi_1\dot\varphi_2 + m\dot\varphi_2^2 \right] \,,\quad M = m_1+m_2 \,,\quad m = m_2$$

$$V = -m_1 g\ell \underbrace{\cos\varphi_1}_{\simeq 1-\frac{1}{2}\varphi_1^2} - m_2 g\ell(\underbrace{\cos\varphi_1}_{1-\frac{1}{2}\varphi_1^2} + \underbrace{\cos\varphi_2}_{1-\frac{1}{2}\varphi_2^2}) \overset{\text{peti.osc.}}{=} \frac{1}{2}\underbrace{g\ell}_{\omega_0^2\ell^2}\left[M\varphi_1^2 + m\varphi_2^2\right] + \text{cte} + \mathcal{O}(\varphi_i^4)$$

$$\omega_0^2 := \frac{g}{\ell}$$

$$L = T - V = L[\varphi_1, \varphi_2, \dot\varphi_1, \dot\varphi_2]$$

E.-L.: $\dfrac{d}{dt}\dfrac{\partial L}{\partial\dot\varphi_i} - \dfrac{\partial L}{\partial\varphi_i} = 0$

$$\frac{\partial L}{\partial\dot\varphi_1} = \frac{1}{2}\ell^2(2M\dot\varphi_1 + 2m\dot\varphi_2) \,,$$

$$\frac{\partial L}{\partial\varphi_1} = -\frac{1}{2}\ell^2\omega_0^2(2M\varphi_1)$$

$$\frac{\partial L}{\partial\dot\varphi_2} = \frac{1}{2}\ell^2(2m\dot\varphi_1 + 2m\dot\varphi_2)$$

$$\frac{\partial L}{\partial\varphi_2} = -\frac{1}{2}\ell^2\omega_0^2(2m\varphi_2)$$

$$\Rightarrow \quad 2M\ddot\varphi_1 + 2m\ddot\varphi_2 + \omega_0^2\, 2M\varphi_1 = 0$$

$$2m\ddot\varphi_1 + 2m\ddot\varphi_2 + \omega_0^2\, 2m\varphi_2 = 0$$

$$\begin{pmatrix} M & m \\ m & m \end{pmatrix}\begin{pmatrix}\ddot\varphi_1 \\ \ddot\varphi_2\end{pmatrix} + \omega_0^2\begin{pmatrix} M & 0 \\ 0 & m\end{pmatrix}\begin{pmatrix}\varphi_1 \\ \varphi_2\end{pmatrix} = 0$$

Ansatz:

$$\varphi_1 = \varphi_{1,0}\, e^{-i\Omega t}, \quad \varphi_2 = \varphi_{2,0}\, e^{-i\Omega t} \qquad \varphi_{i,0} \in \mathbb{C}$$

$$\Rightarrow \ddot{\varphi}_i = -\Omega^2 \varphi_i$$

$$\Rightarrow \left[-\Omega^2 \begin{pmatrix} M & m \\ m & m \end{pmatrix} + \omega_0^2 \begin{pmatrix} M & 0 \\ 0 & m \end{pmatrix} \right] \begin{pmatrix} \varphi_{1,0} \\ \varphi_{2,0} \end{pmatrix} = 0$$

$$\det \begin{pmatrix} M(\omega_0^2 - \Omega^2) & -m\Omega^2 \\ -m\Omega^2 & m(\omega_0^2 - \Omega^2) \end{pmatrix} = 0 \qquad \rightsquigarrow \Omega_\pm$$

$$\Leftrightarrow \quad Mm(\omega_0^2 - \Omega^2)^2 - m^2 \Omega^4 = 0$$

$$\Leftrightarrow \quad (\Omega^2 - \omega_0^2)^2 - \frac{m}{M}\Omega^4 = 0$$

$$\Leftrightarrow \quad \left(\Omega^2 - \omega_0^2 - \sqrt{\tfrac{m}{M}}\,\Omega^2\right)\left(\Omega^2 - \omega_0^2 + \sqrt{\tfrac{m}{M}}\,\Omega^2\right) = 0$$

$$\Leftrightarrow \quad \boxed{\Omega_\pm^2 = \omega_0^2 \frac{1}{1 \mp \sqrt{\frac{m}{M}}}} = \omega_0^2 \frac{1 \pm \sqrt{\frac{m}{M}}}{1 - \frac{m}{M}} = \frac{M}{m_1}\omega_0^2 \left(1 \pm \sqrt{\tfrac{m}{M}}\right)$$

$$\Omega_\pm^2 - \omega_0^2 = \omega_0^2 \left(\frac{1}{1 \mp \sqrt{\frac{m}{M}}} - 1 \right) = \omega_0^2 \left(\frac{\pm \sqrt{\frac{m}{M}}}{1 \mp \sqrt{m/M}} \right) = \pm \sqrt{\tfrac{m}{M}}\, \Omega_\pm^2 = \pm \epsilon\, \Omega_\pm^2$$

$$\epsilon := \sqrt{\tfrac{m}{M}}$$

Modes: Ω_+

$$\begin{pmatrix} -M\epsilon\Omega_+^2 & -m\Omega_+^2 \\ -m\Omega_+^2 & -m\epsilon\Omega_+^2 \end{pmatrix} \begin{pmatrix} \varphi_{1,0} \\ \varphi_{2,0} \end{pmatrix} = 0 \quad \Leftrightarrow \quad \overbrace{\begin{pmatrix} M\epsilon & m \\ m & m\epsilon \end{pmatrix}}^{\det = 0\ \checkmark} \begin{pmatrix} \varphi_{1,0} \\ \varphi_{2,0} \end{pmatrix} = 0$$

$$\left.\begin{matrix} M\epsilon\,\varphi_{1,0} + m\,\varphi_{2,0} = 0 \\ m\,\varphi_{1,0} + m\epsilon\,\varphi_{2,0} = 0 \end{matrix}\right\} \Rightarrow \boxed{\varphi_{2,0} = -\frac{M\epsilon}{m}\varphi_{1,0}} = -\sqrt{\frac{M}{m}}\,\varphi_{1,0} = -\frac{1}{\epsilon}\varphi_{1,0}$$

Mode normal: $a \begin{pmatrix} 1 \\ -\sqrt{\frac{M}{m}} \end{pmatrix} = \tilde{a} \begin{pmatrix} \sqrt{m} \\ -\sqrt{M} \end{pmatrix}$ $\qquad a, b \in \mathbb{C}$

Ω_-

$$\begin{pmatrix} -M\epsilon & m \\ m & -m\epsilon \end{pmatrix} \begin{pmatrix} \varphi_{1,0} \\ \varphi_{2,0} \end{pmatrix} = 0 \Rightarrow \varphi_{2,0} = \sqrt{\frac{M}{m}}\,\varphi_{1,0}, \quad \tilde{b} \begin{pmatrix} \sqrt{m} \\ \sqrt{M} \end{pmatrix} = b \begin{pmatrix} 1 \\ \sqrt{\frac{M}{m}} \end{pmatrix}$$

$$T = \frac{1}{2} \ell^2 \left[M \dot{\varphi}_1^2 + 2m \dot{\varphi}_1 \dot{\varphi}_2 + m \dot{\varphi}_2^2 \right] \quad , M = m_1 + m_2 , \; m = m_2$$

$$V = \frac{1}{2} \ell^2 \omega_0^2 \left[M \varphi_1^2 + m \varphi_2^2 \right] \quad , \omega_0^2 = \frac{g}{\ell}$$

$$T = \frac{1}{2} \dot{\varphi}_i M_{ij} \dot{\varphi}_j \quad , \; V = \frac{1}{2} \varphi_i K_{ij} \varphi_j$$

avec $\quad M = \ell^2 \begin{pmatrix} M & m \\ m & m \end{pmatrix} \quad , \; K = \ell^2 \omega_0^2 \begin{pmatrix} M & 0 \\ 0 & m \end{pmatrix}$

Ansatz: $\varphi_i = \varphi_{i,0} \, e^{-i\Omega t} \quad , \; \dot{\varphi}_i = -i\Omega \varphi_i$

$$\Rightarrow \quad E = T + V = \frac{1}{2} \varphi_i \left(-\Omega^2 M_{ij} + K_{ij} \right) \varphi_j$$

$$E = \text{cte} \iff \varphi_{i,0} \left(-\Omega^2 M_{ij} + K_{ij} \right) \varphi_{j,0} = 0$$

$$\iff \vec{\varphi}_0^{\,T} \left(K - \Omega^2 M \right) \vec{\varphi}_0 = 0$$

$$\Rightarrow \quad \det \left(K - \Omega^2 M \right) = 0$$

$$\begin{vmatrix} M(\omega_0^2 - \Omega^2) & -m\Omega^2 \\ -m\Omega^2 & m(\omega_0^2 - \Omega^2) \end{vmatrix} = 0 \quad \rightsquigarrow \Omega^2_\pm$$

$$\Rightarrow \quad \Omega^2_\pm = \omega_0^2 \frac{1}{1 \mp \sqrt{\frac{m}{M}}} \quad , \; \omega_0^2 - \Omega_\pm^2 = \mp \sqrt{\frac{m}{M}} \, \Omega_\pm^2 = \mp \epsilon \Omega_\pm^2$$

$$\epsilon := \sqrt{\frac{m}{M}}$$

$\Rightarrow$ Modes: $(K - \Omega_\pm^2 M) \vec{\varphi}_0 = 0$

$$\iff \begin{pmatrix} M\epsilon & \pm m \\ \pm m & m\epsilon \end{pmatrix} \vec{\varphi}_0 = 0 \quad \Rightarrow \quad \Omega_+ : \vec{\varphi}_0^{\,+} = a \begin{pmatrix} \sqrt{m} \\ -\sqrt{M} \end{pmatrix} \quad ; \; \Omega_- : \vec{\varphi}_0^{\,-} = b \begin{pmatrix} \sqrt{m} \\ \sqrt{M} \end{pmatrix}$$

Solution générale : Superposition

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Superposition :

$$\mathrm{Re}\left\{ a \begin{pmatrix} \varphi_{1,0} \\ \varphi_{2,0} \end{pmatrix}_+ e^{-i\Omega_+ t} + b \begin{pmatrix} \varphi_{1,0} \\ \varphi_{2,0} \end{pmatrix}_- e^{-i\Omega_- t} \right\}, \quad a, b \in \mathbb{C}$$

ou $$A \begin{pmatrix} \sqrt{m} \\ -\sqrt{M} \end{pmatrix} \cos(\Omega_+ t + \varphi_+) + B \begin{pmatrix} \sqrt{m} \\ \sqrt{M} \end{pmatrix} \cos(\Omega_- t + \varphi_-) \quad , A, B \in \mathbb{R}$$

$$\varphi_+, \varphi_- \in \mathbb{R}$$

(5.4) $O_3 > D_6 > D_3 > C_3$

Table de caractères : (→ Ex. 4.5)

D_6	E = $\{e\}$	$2C_6$ = $\{c, c^5\}$	$2C_6^2$ = $\{c^2, c^4\}$	C_6^3 = $\{c^3\}$	$3C_2$ = $\{b, bc^2, bc^4\}$	$3C_2'$ = $\{bc, bc^3, bc^5\}$
A_1	1	1	1	1	1	1
A_2	1	1	1	1	−1	−1
B_1	1	−1	1	−1	1	−1
B_2	1	−1	1	−1	−1	1
E_1	2	1	−1	−2	0	0
E_2	2	−1	−1	2	0	0
	$\theta=0$	$\theta=\pi/3$	$\frac{2\pi}{3}$	π	π	π

$r = k$!

irreps = # classes de conj.

D_3: $B_1 = A_1$
$B_2 = A_2$

$E_1 = E_2 = E$

a) $$\chi^{(\ell)}(\theta) = \frac{\sin(\ell+1/2)\theta}{\sin\frac{1}{2}\theta} \xrightarrow[\theta\to 0]{} \frac{(\ell+1/2)}{1/2}$$

$$\chi^{(\ell=1)}(\theta) = \frac{\sin 3/2\,\theta}{\sin 1/2\,\theta}, \quad \chi^{(\ell=2)}(\theta) = \frac{\sin 5/2\,\theta}{\sin 1/2\,\theta}$$

$(\ell=1): D^{(1)} = D^V$, $\chi^V = 1 + 2\cos\theta$

$$\chi^{(\ell=1)} = \left(3, \frac{\sin \pi/2}{\sin \pi/6} = 2, \frac{\sin\pi}{\sin \pi/3} = 0, \frac{\sin\frac{3}{2}\pi}{\sin \pi/2} = -1, -1, -1\right)$$

$$\chi^{(\ell=2)} = (5, 1, -1, 1, 1, 1)$$

$$D^{(1)}: \chi^{(1)} = (3, 2, 0, -1, -1, -1) = \sum_{\substack{\text{irreps}\\ i}} \langle \chi^{(1)}, \chi_i\rangle \chi^i = \chi^{A_2} + \chi^{E_1} \quad \text{irreps } i = A_1, A_2, B_1, B_2, E_1, E_2$$

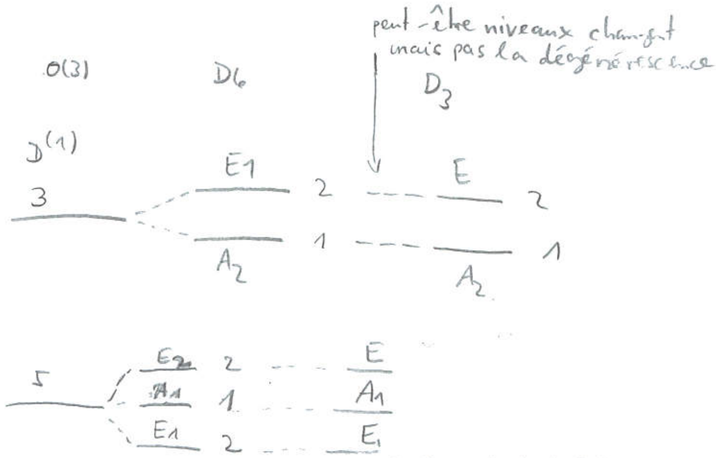
$$D^{(2)}: \chi^{(2)} = (5, 1, -1, 1, 1, 1) = \chi^{A_1} + \chi^{E_1} + \chi^{E_2}$$

$$\underset{\substack{\uparrow\\ \text{irrep}\\ \text{par rapport}\\ \text{à } SO(3)}}{D^{(1)}} = \underbrace{D^{(A_2)} \oplus D^{(E_1)}}_{\text{irreps par rapport à } D_6}$$

$$3 \to 1 + 2$$

$$D^{(2)} = D^{(A_1)} + D^{(E_1)} + D^{(E_2)}$$

$$5 \to 1 + 2 + 2$$



b) D_3: $\theta = 0, \frac{2\pi}{3}, \pi$; seulement caractères avec c^2

$$\chi^{(\ell=1)} = (3, 0, -1) = \chi^{A_2} + \chi^E \qquad 3 \to 1+2$$

$$\chi^{(\ell=2)} = (5, -1, 1) = \chi^{A_1} + 2\chi^E \qquad 5 \to 1+2+2$$

Pas de séparation supplémentaire

D_3	E = $\{e\}$	$2C_3$ = $\{c, c^2\}$	$3C_2$ = $\{b, bc, bc^2\}$
A_1	1	1	1
A_2	1	1	−1
E	2	−1	0
θ	0	$2\pi/3$	π

c) C_3 abélien → irreps 1-dimensionelles ⇒ $3 \to 1+1+1$

$5 \to 1+1+1+1+1$