1.1

$D_3 = \{ e, c, c^2, b, bc, bc^2 \}$ $\quad b^2 = c^3 = (bc)^2 = e \quad bcbc = e$

D_3	e	c	c^2	b	bc	bc^2
e	e	c	c^2	b	bc	bc^2
c	c	c^2	e	bc^2	b	bc
c^2	c^2	e	c	bc	bc^2	b
b	b	bc	bc^2	e	c	c^2
bc	bc	bc^2	b	c^2 CL	e	c CL
bc^2	bc^2	b	bc	c CL	c^2 CL	e only possibility: Carré ~~Latin~~ Latin

* $bc^2 bc^2 \overset{!}{=} e$

* $bcbc = e \overset{\cdot c^2}{\Rightarrow} bcb = c^2 \overset{b^2=e}{\Rightarrow} cb = bc^2$

1.2

$(1\ 6\ 7\ 2)(3\ 4\ 8)$

$(1\ 3\ 4)(2\ 5\ 8\ 7\ 6\ 9)$

1.3

Ordre $(1\,6\,7\,2)(3\,4\,8)$: $n = \text{kgV}(3,4) = 12$ $\qquad$ kgV $\hat{=}$ ppcm (plus petit commun multiple)

Ordre $(1\,3\,4)(2\,5\,8\,7\,6\,9)$: $n = \text{kgV}(3,6) = 6$

Test: $(3\,4\,8)^2 = (3\,4\,8)(3\,4\,8) = (3\,8\,4)$

$(3\,4\,8)^3 = (3\,8\,4)(3\,4\,8) = (3)(4)(8) = (\,)$

Ordre : $(a_1 a_2 \dots a_{2r})(b_1 b_2 \dots b_r)$: $n = 2r$

1.4

élément neutre

$\{(2,4,6,8), \cdot \bmod 10\}$ ~~n'est pas un groupe~~ : $6 = e$

$\{1, i, -1, -i\}$: $i^2 = -1$, $(-i)^2 = -1$; -1 ; ordre $= 2$; $i, -i$: ordre $= 4$

a) $\simeq$ b)

par contre $(12)^2 = (34)^2 = ((12)(34))^2 = ()$

a) $\ncong$ c) : c) $\Rightarrow$ $(12), (34), (12)(34)$ tous ordre $= 2$

$\boxed{\text{b)} \neq \text{a)} \overset{!}{\cong} \text{d)}}$ ~~$\ncong$ a)~~

$(1234), (1432)$: ordre $= 4$

$(13)(24)$: ordre $= 2$

e)

$$\begin{pmatrix} 1 & \\ & -1 \end{pmatrix}\begin{pmatrix} 1 & \\ & -1 \end{pmatrix} = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

tous ordre $= 2$

$$\begin{pmatrix} -1 & \\ & 1 \end{pmatrix}\begin{pmatrix} -1 & \\ & 1 \end{pmatrix} = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & \\ & -1 \end{pmatrix}\begin{pmatrix} -1 & \\ & -1 \end{pmatrix} = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

$\leadsto$ $\boxed{\text{c)} \overset{!}{\cong} \text{e)}}$

Chaque fois tableau de Cayley $\overset{!}{\nabla}$

a) $\cong$ d) :

$1 \longmapsto ()$

$-1 \longmapsto (13)(24)$

$(1234)^2 = (13)(24)$

$(1432)^2 = (13)(24)$

abélien

$i \longmapsto (1234)$

$-i \longmapsto (1432)$

ou

$i \longmapsto (1432)$

$-i \longmapsto (1234)$

c) $\cong$ e) :

abélien

$() \longmapsto \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} = e$

$(12) \longmapsto \begin{pmatrix} -1 & \\ & 1 \end{pmatrix} = g_1$

$(34) \longmapsto \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} = g_2$

$(12)(34) \longmapsto \begin{pmatrix} -1 & \\ & -1 \end{pmatrix} = g_3$

Test:

	e	g_1	g_2	g_3
e	e	g_1	g_2	g_3
g_1	g_1	e	g_3	g_2
g_2	g_2	g_3	e	g_1
g_3	g_3	g_2	g_1	e

Symétrique : abélien

1.4

Cayley

a)

	i	-1	1	$-i$
i	-1	$-i$	i	1
-1	$-i$	1	-1	i
1	i	-1	1	$-i$
i	-1	$-i$	i	1

$\phi : G_a \to G_b$

b)

	2	4	6	8
2	4	8	2	6
4	8	6	4	2
6	2	4	6	8
8	6	2	8	4

$\phi(i) = 2$

$\phi(-1) = 4$

$\phi(1) = 6$

$\phi(-i) = 8$

d)

	(1 2 3 4)	(13)(24)	( )	(1 4 3 2)
(1234)	(13)(24)	(1432)	(1234)	( )
(13)(24)	(1432)	( )	(13)(24)	(1234)
( )	(1234)	(13)(24)	( )	(1432)
(1432)	( )	(1234)	(1432)	(13)(24)

$\phi : G_b \to G_d$

$\phi(6) = (\ )$, $\phi(2) = (1234)$

$\phi(4) = (13)(24)$, $\phi(8) = (1432)$

(c)

	( )	(1 2)	(3 4)	(12)(34)
( )	( )	(1 2)	(3 4)	(12)(3 4)
(1 2)	(1 2)	( )	(12)(34)	(3 4)
(3 4)	(3 4)	(12)(34)	( )	(1 2)
(12)(34)	(12 34)	(3 4)	(12)	( )

e)

$g \in G_e \ : \ g^2 = e$

$\phi : G_c \to G_e$

$\phi(( )) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$\phi((1 2)) = \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}$

$\phi((3 4)) = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$\phi((12)(34)) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

1.5

Centre $Z = \{ z \in G \mid zg = gz \quad \forall g \in G \}$

$Z < G$, abélien!

1.) $\underline{Z < G}$:

$e \in Z$ ✓

$z_1, z_2 \in Z$: $\quad \cancel{z_1 z_2 g = g z_1 z_2 \quad \forall g \in G}$

$z_1 g = g z_1 \quad \forall g$

$z_2 g = g z_2 \quad \forall g$

$z_1 z_2 g = z_1 g z_2 = g z_1 z_2 \quad \Rightarrow z_1 z_2 \in Z$ Fermeture ✓

$z \in Z$: $\quad zg = gz \quad \forall g$ $\quad , \ (zg)^{-1} = (gz)^{-1} \quad \forall g$

$\overset{!}{\Rightarrow} \quad z^{-1} g = g z^{-1}$ $\qquad g^{-1} z^{-1} = z^{-1} g^{-1} \quad \forall g$

$\Rightarrow g z^{-1} = z^{-1} g \quad \forall g$ ✓ $\Rightarrow z^{-1} \in Z$ ✓

Inverse ✓

2.) Z est abélien : trivial