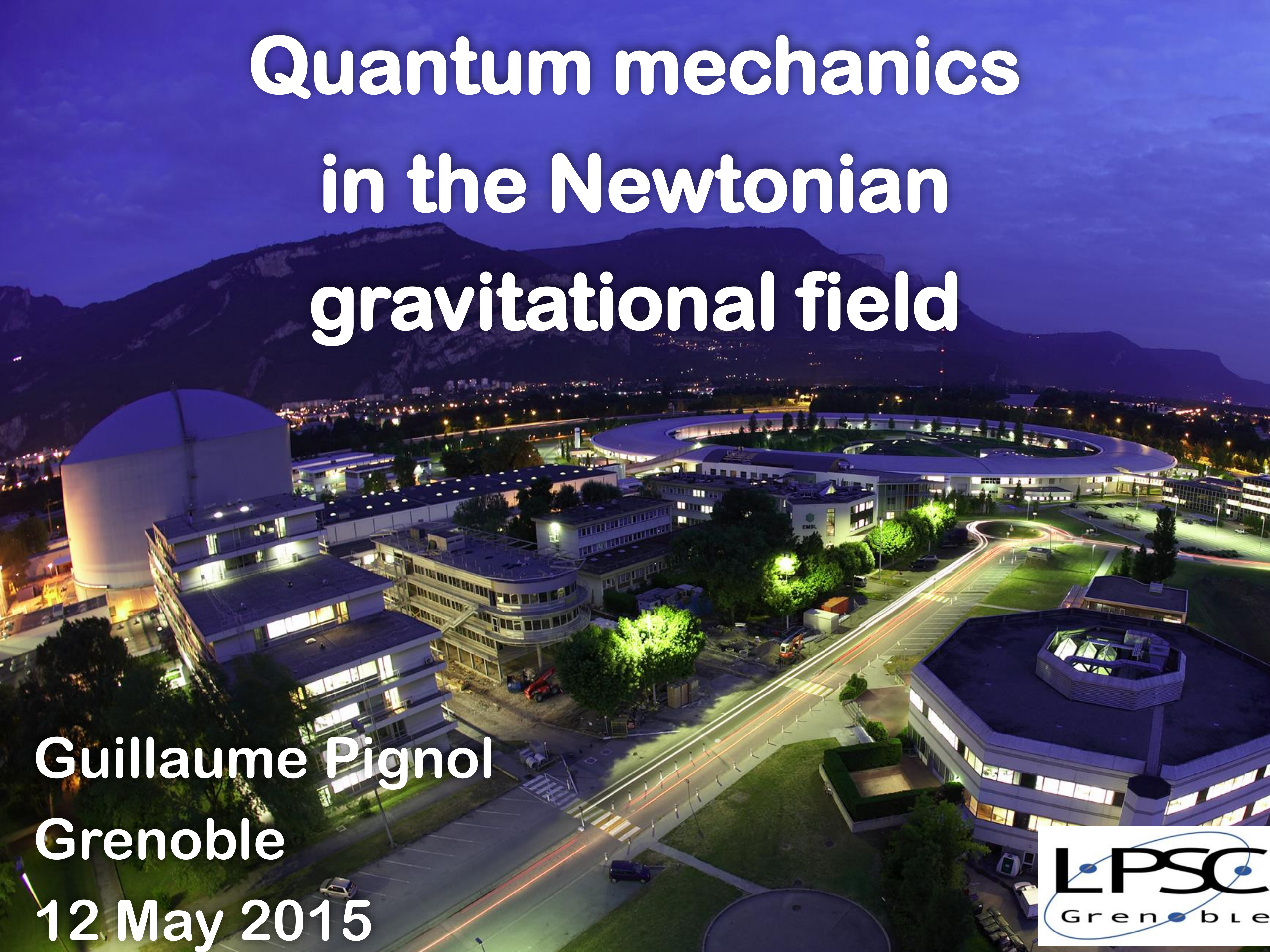
Quantum mechanics in the Newtonian gravitational field



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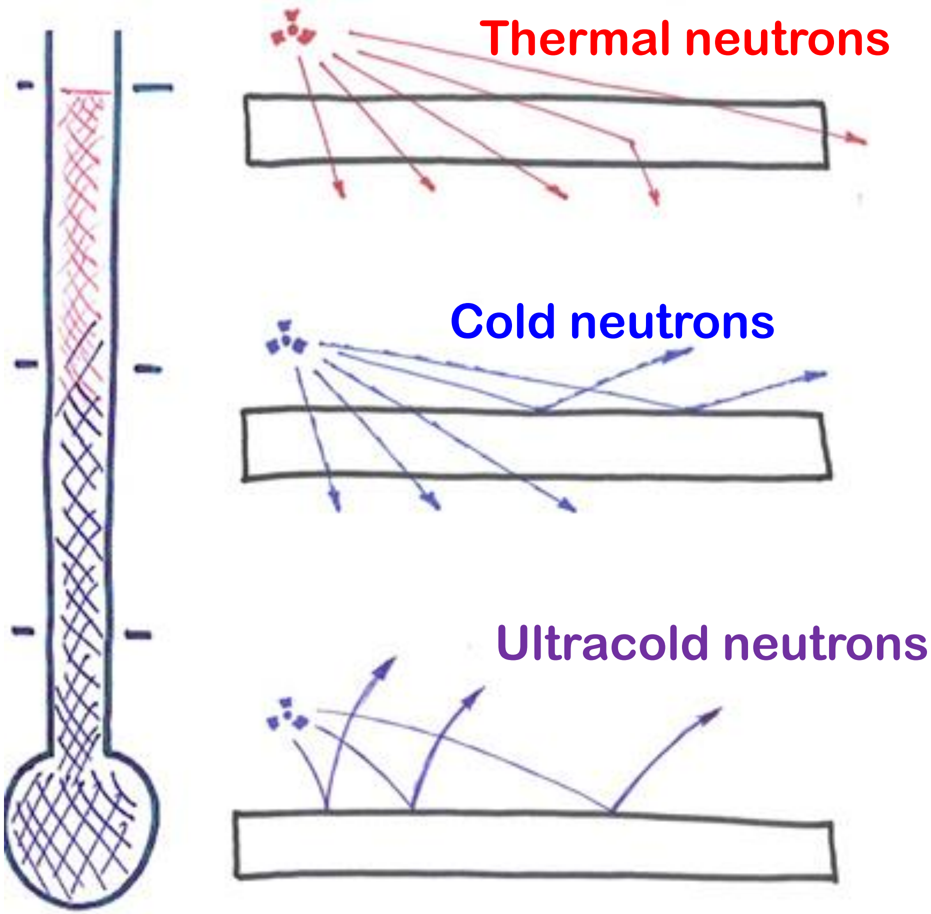
Grenoble

12 May 2015

LPSC Grenoble

Neutron optics, cold and ultracold neutrons



Cold neutrons
($E < kT = 25$ meV)

have large wavelength
($\lambda > 0.2$ nm)

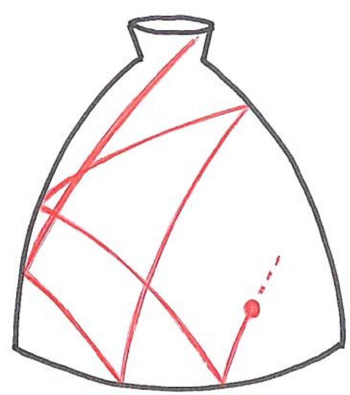
They behave like waves,

affected by the Fermi potential of matter

(order of 100 neV)

Neutrons with energy < 100 neV,

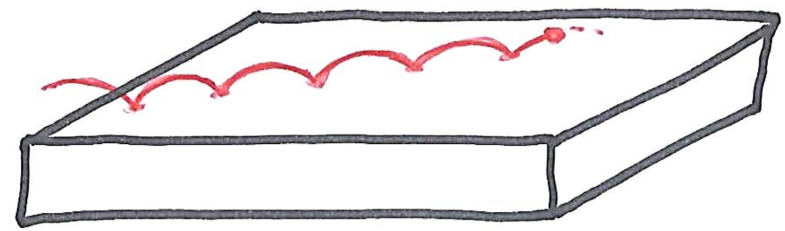
are reflected by material walls they can be stored in material bottles.



Bouncing neutrons: quantum states

Neutrons with energy < 100 neV

can bounce above a glass mirror.



The vertical motion is a simple quantum well problem

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dz^2} + mg\,z\,\psi = E\,\psi$$

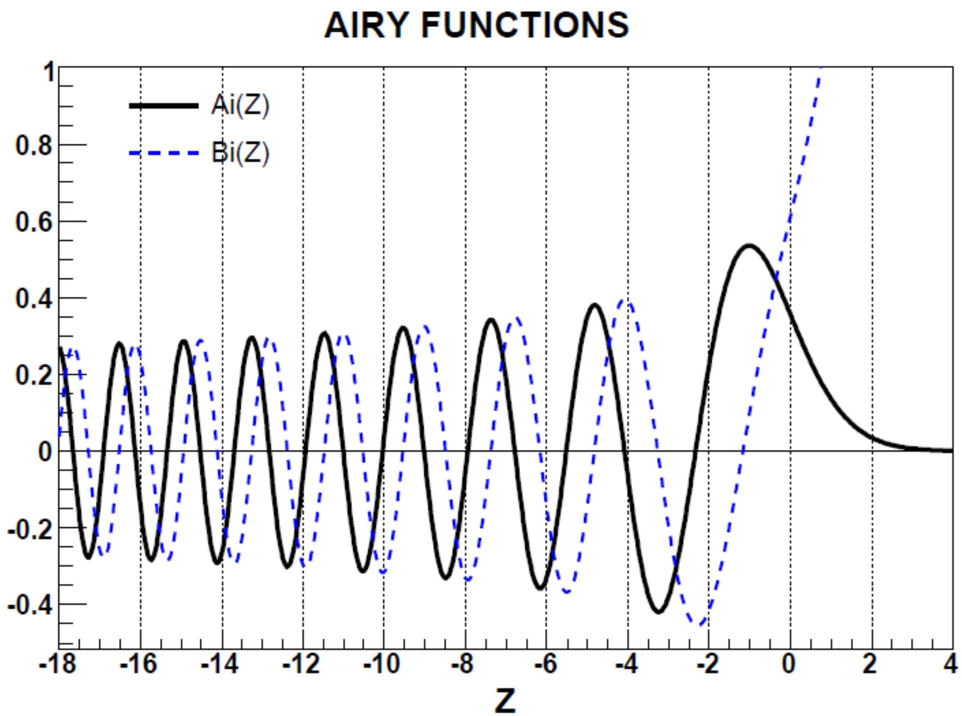
Characteristic height

$$z_0 = \left(\frac{\hbar^2}{2m^2 g}\right)^{1/3} = 5.9\ \mu\mathrm{m}$$

a-dimensional Schrodinger equation with $Z = z/z_0$ and $\epsilon = E/mgz_0$

$$-\frac{d^2\psi}{dZ^2} + (Z - \epsilon)\,\psi = 0$$

Solutions of the stationary Schrodinger equation



2d order equation

$$-\frac{d^2\psi}{dZ^2} + (Z - \epsilon)\,\psi = 0$$

General solution

$$\psi(Z) = C\ \mathrm{Ai}(Z - \epsilon) + D\ \mathrm{Bi}(Z - \epsilon)$$

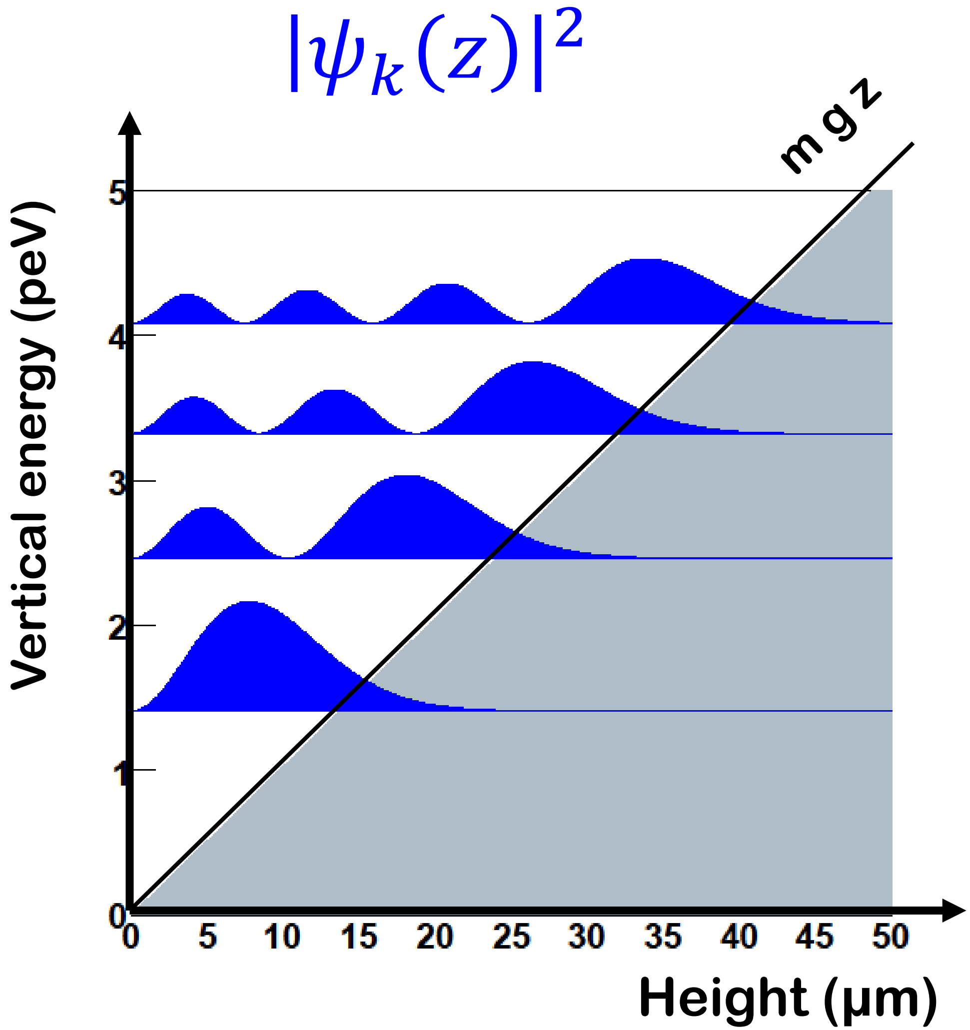
Physical solutions ($\psi_k(0) = \psi_k(\infty) = 0$) with quantized energy

$$\psi_k(Z) = C_k \times \mathrm{Ai}(Z - \epsilon_k)$$

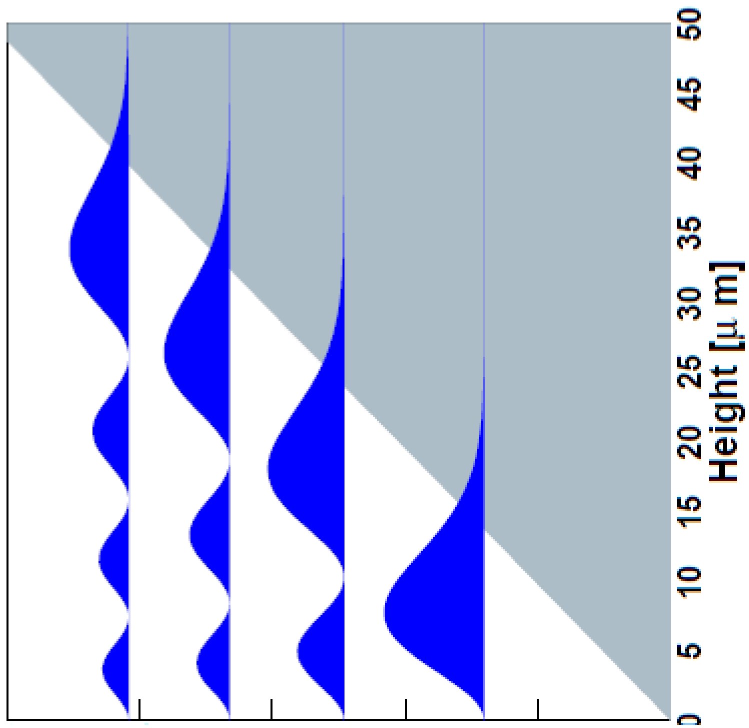
Energy levels given by the negative zeros of Ai(Z)

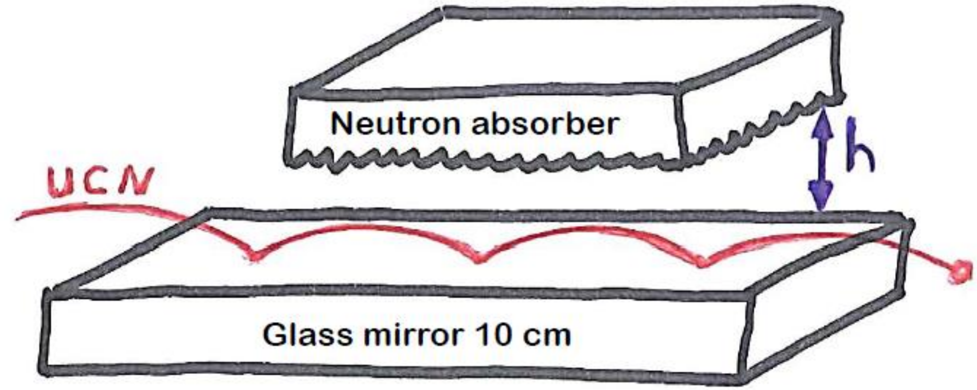
$$\epsilon_k = \{2.34, 4.09, 5.52, \cdots\}$$

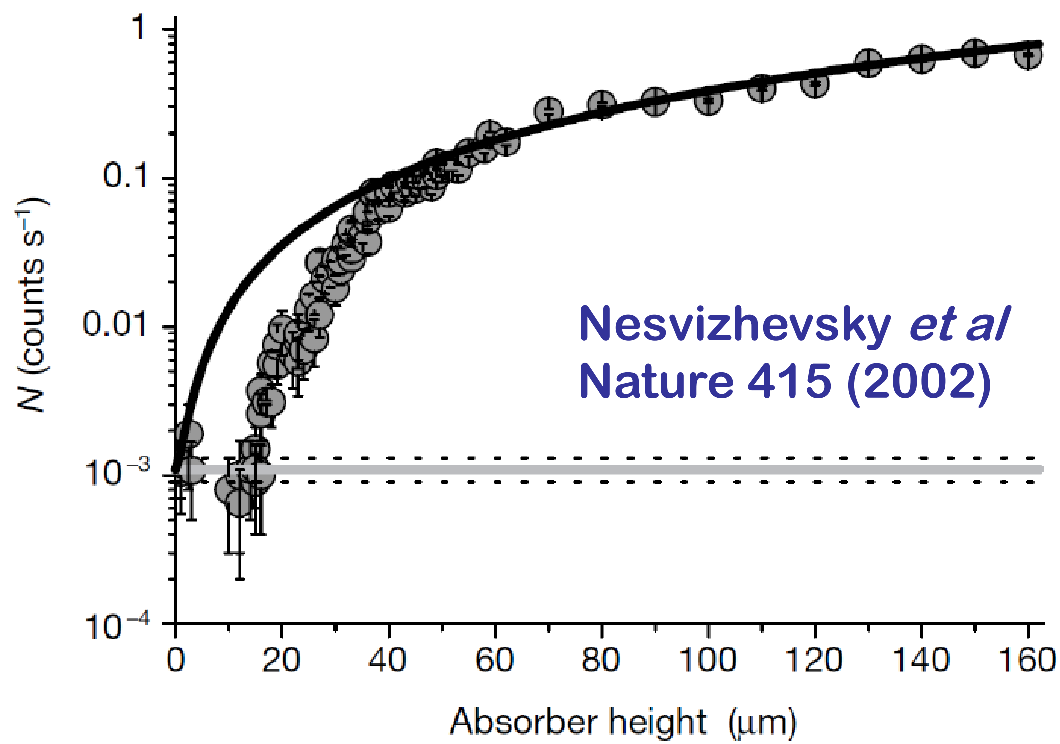
Position probability density of the states



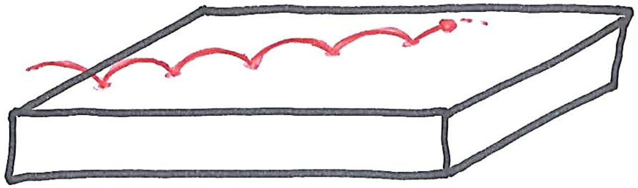
Discovery of the quantum states at ILL Grenoble

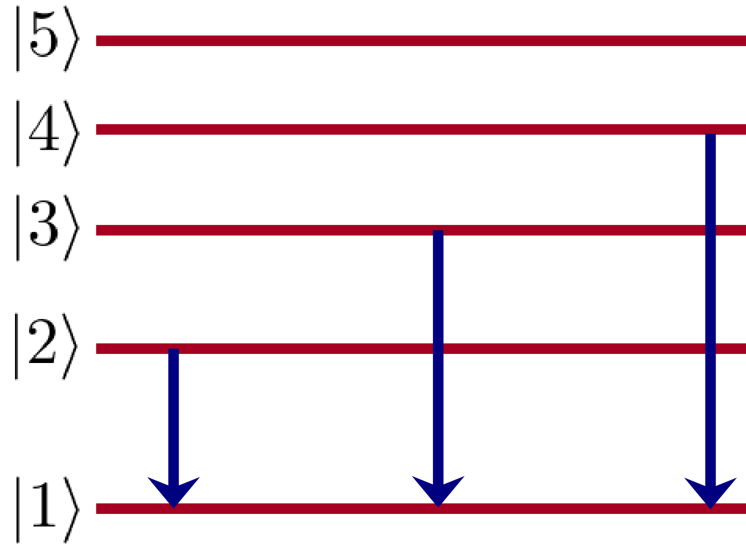






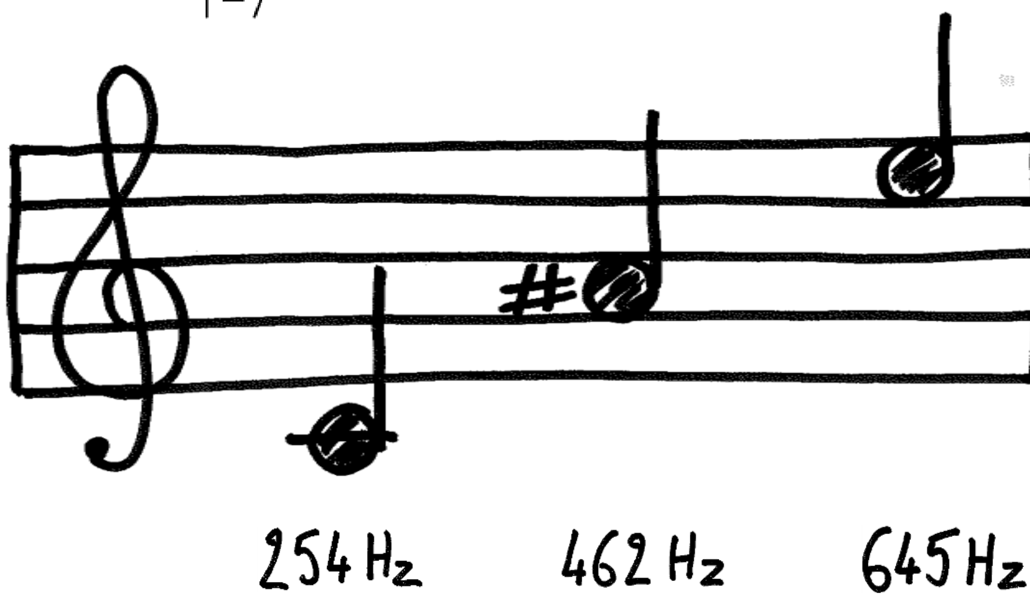
Bouncing neutron: quantum music





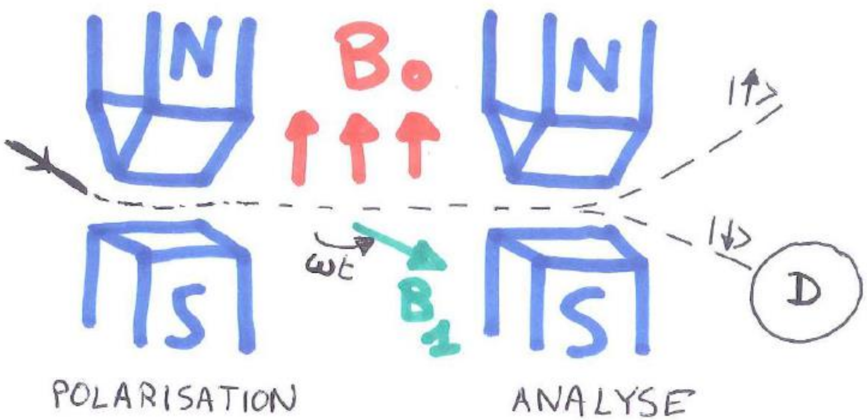
$$\Delta E = h \times f$$

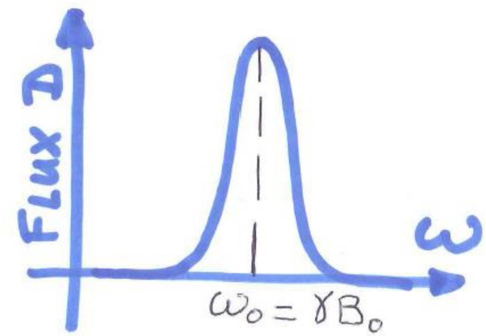
$$\frac{1}{2\pi}\sqrt[3]{\frac{mg^2}{2\hbar}} \times \left| \begin{array}{l} 1{,}75 \\ 3{,}18 \\ 4{,}45 \\ \vdots \end{array} \right.$$



Gravity resonance spectroscopy

Rabi experiment (1939)





Rabi formula

$$P_{2\to 1} = \frac{\sin^2\left(\sqrt{\delta\omega^2 + \Omega^2}\; t/2\right)}{1 + \delta\omega^2/\Omega^2}$$

$$\Omega = \left\langle 2 \middle| \widehat{V_0} \middle| 1 \right\rangle / \hbar$$

Gravity resonance spectroscopy

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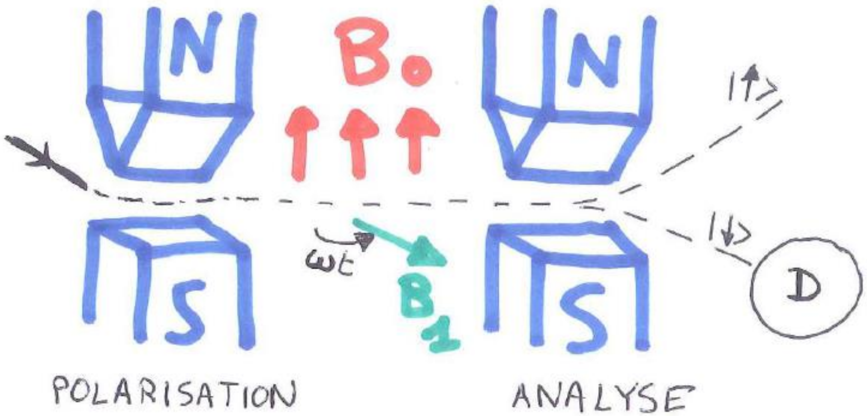
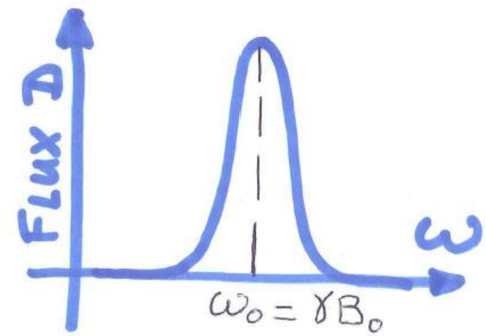
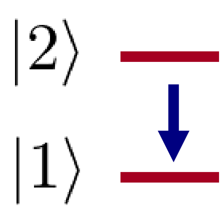




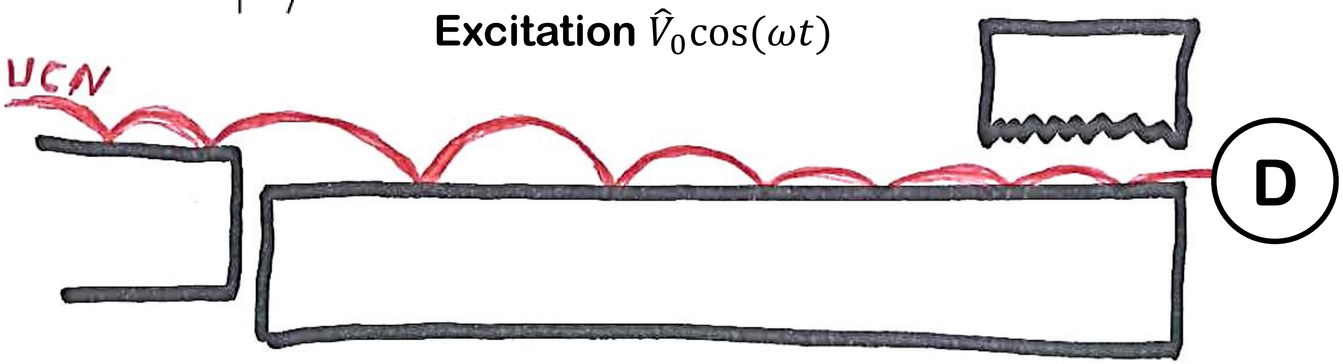
State selection

$|2\rangle$



Excitation $\hat{V}_0 \cos(\omega t)$

$|1\rangle$ **?**



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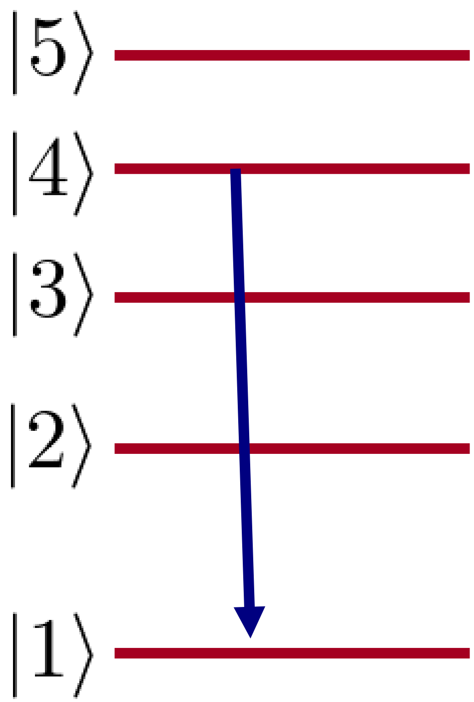
$$\Omega = \langle 2|\widehat{V_0}|1\rangle/\hbar$$

How to excite resonant transitions?



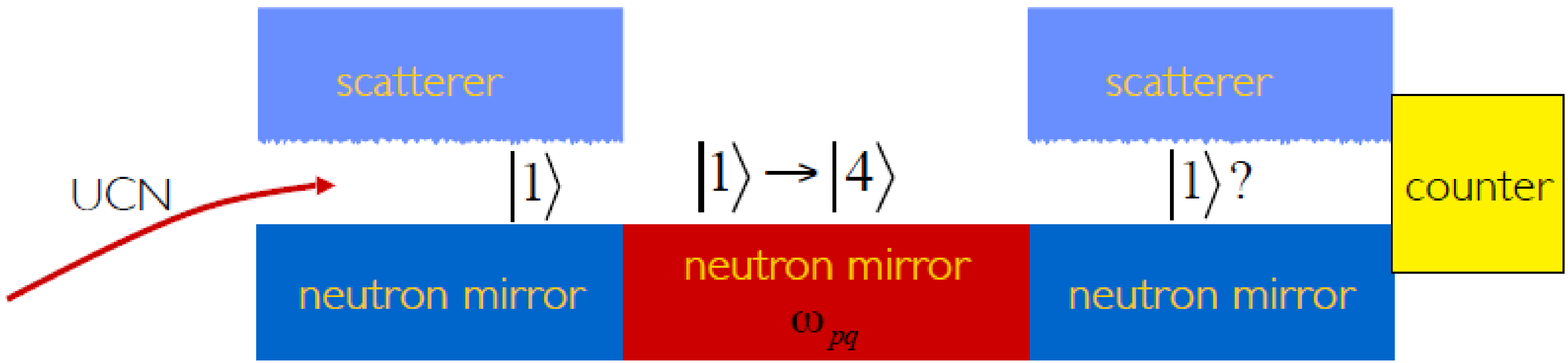
1) Mirror vibrations $h(t)$

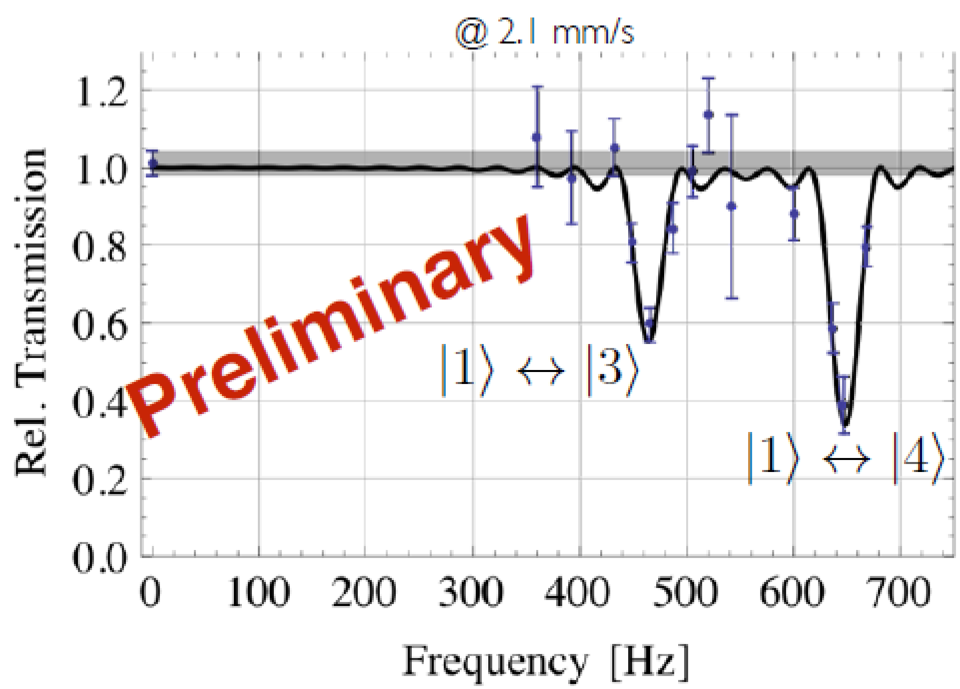
$$\hat{V}(t) = -\frac{dh}{dt}\,\widehat{p_z}$$

2) Magnetic excitation

$$\hat{V}(t) = \pm\mu\,|B|(t) \approx \pm\mu\frac{d|B|}{dz}(t)\,\hat{z}$$

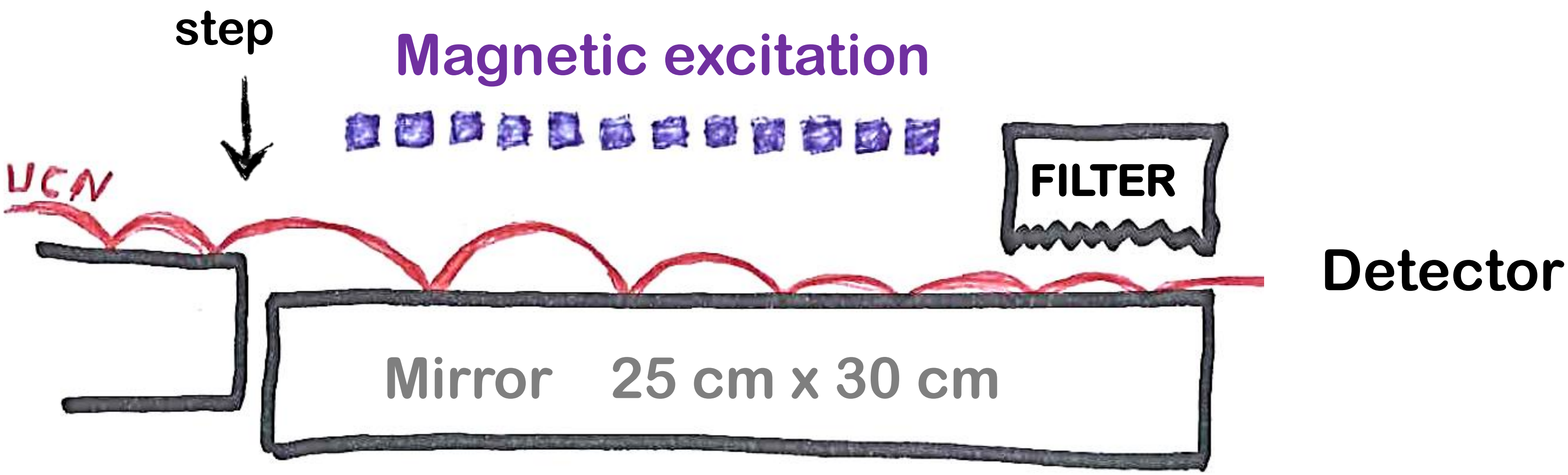
Vibrating mirror: Qbounce result

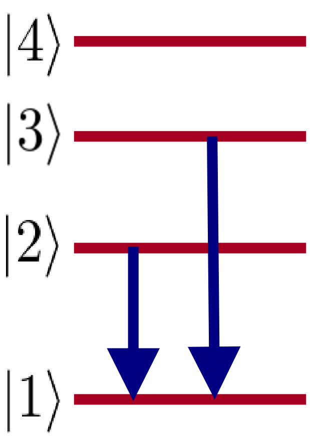


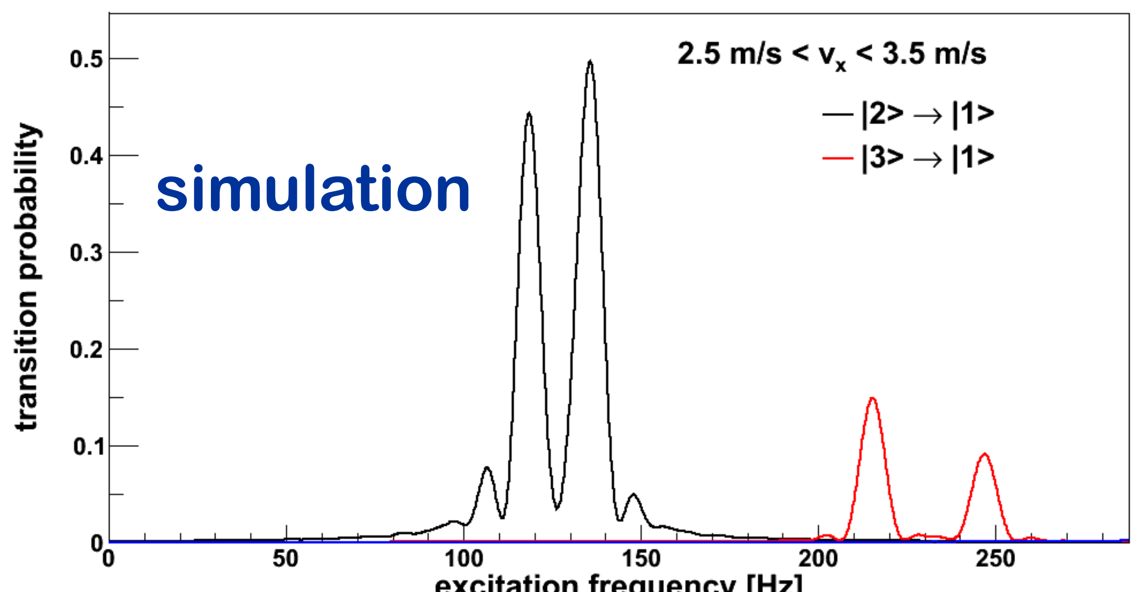


n.b. this slide is stolen from the Vienna group

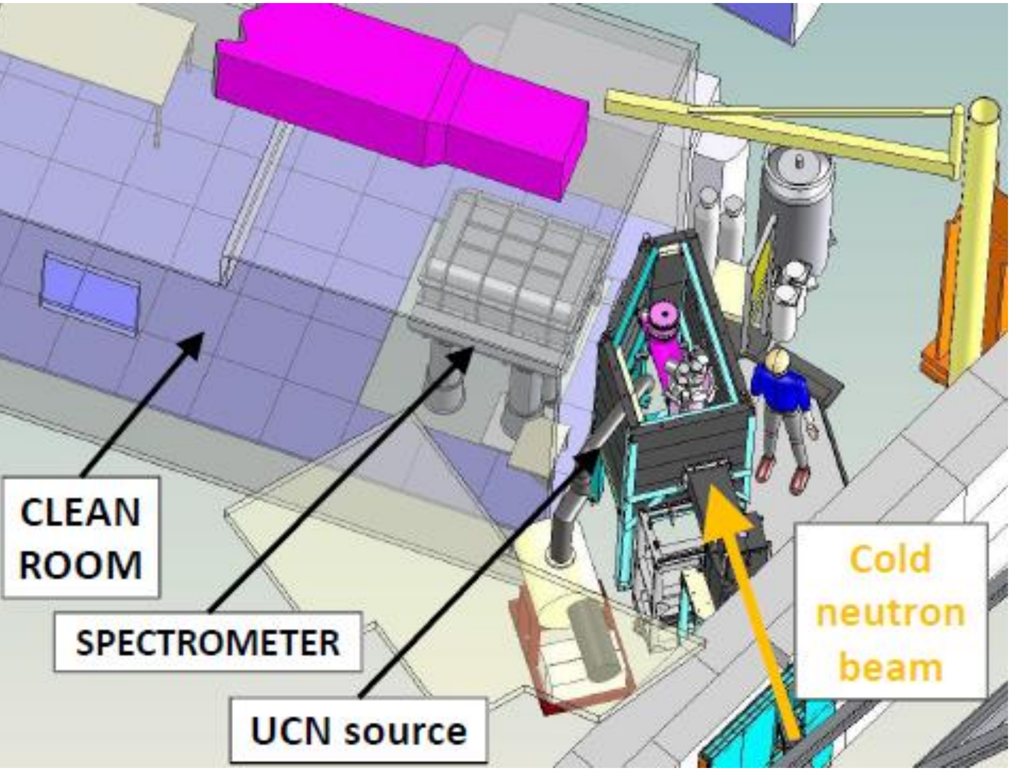
Resonant transitions in GRANIT

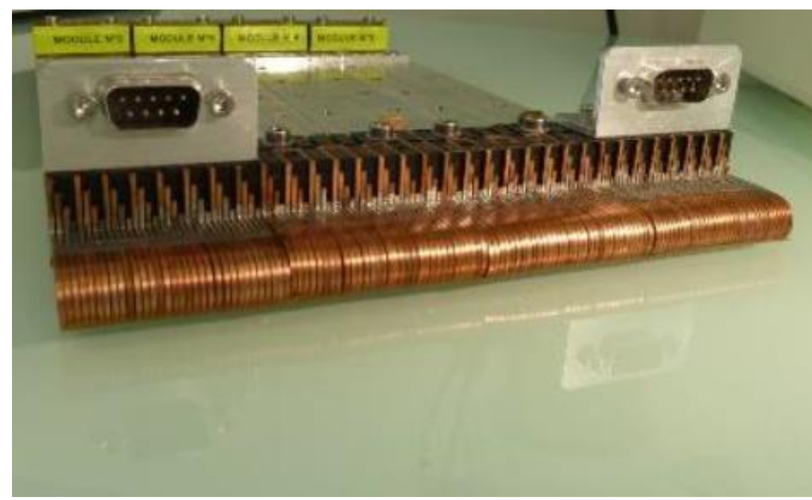






The GRANIT instrument
at ILL level C





First UCNs in GRANIT in 2013.

Producing UCNs is a delicate art.

We are getting ready for measuring something interesting…

The equivalence principle with a classical test mass

Newton's law of motion $$m_i \vec{a} = m_g \vec{g}$$

Weak equivalence principle

$$m_i = m_g$$

Universality of free fall

$$\vec{a} = \vec{g}$$

Einstein E.P.

Gravity is locally like in an accelerating elevator

Quantum test of the equivalence principle

$$-\frac{\hbar^2}{2m_i}\frac{d^2\psi}{dz^2} + m_g g\, z\, \psi = E\, \psi$$

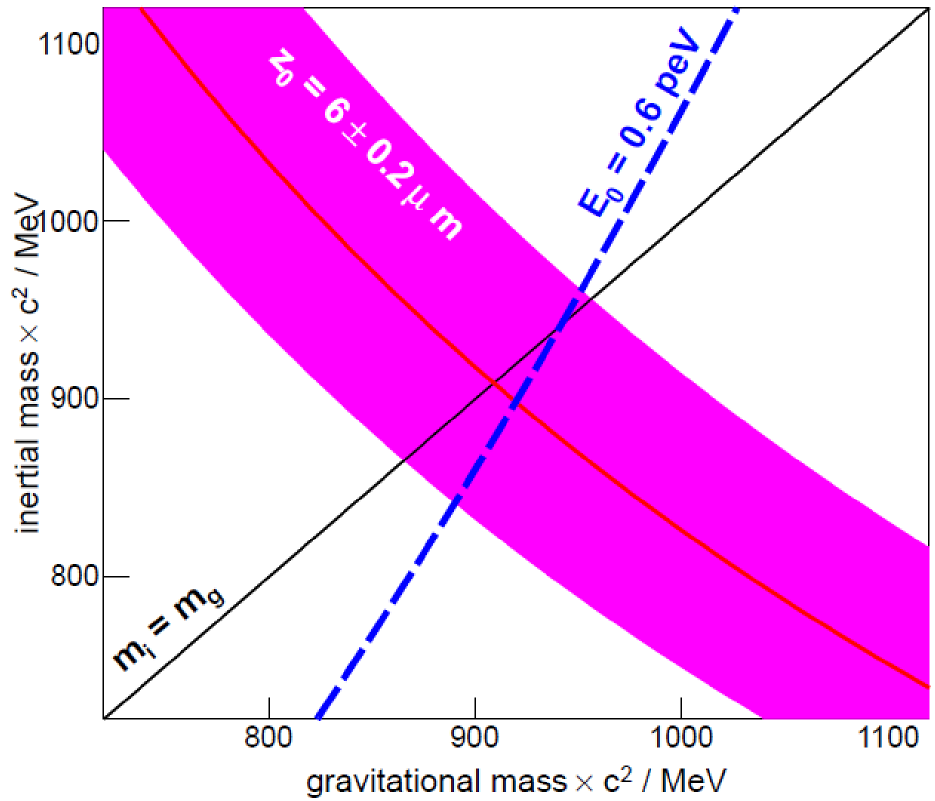
Inertial Mass **Gravitational Mass**

Measuring the wavefunctions one access

$$z_0 = \left(\frac{\hbar^2}{2m_i m_g g}\right)^{1/3}$$

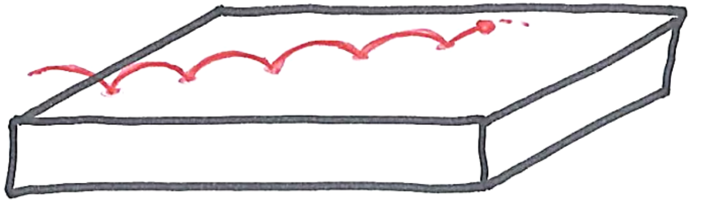
Measuring transition frequencies one access

$$E_0 = m_g g z_0 = \left(\frac{m_g^2}{m_i}\frac{g^2\hbar^2}{2}\right)^{1/3}$$



One can tell separately the inertial and gravitational mass !

Concluding remarks

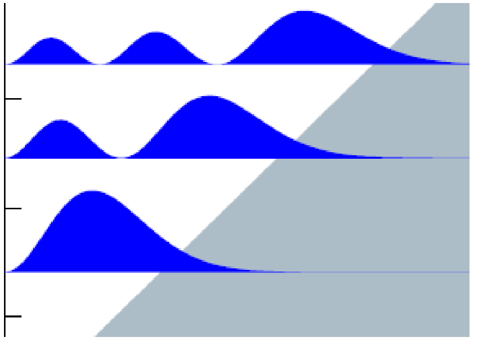


An intriguing system where energy quantization is due to weight.

Caution: this is not quantum gravity!

Two features due to the extreme weakness of gravity:

1. Super-large wave functions visible to the naked eye



2. Super-low (audio frequency) transition energies

Measuring both allows a new test of the equivalence principle