Stanford conference

on dark matter

ENTROPY EVAPORATED BY A BRANEWORLD BLACK HOLE

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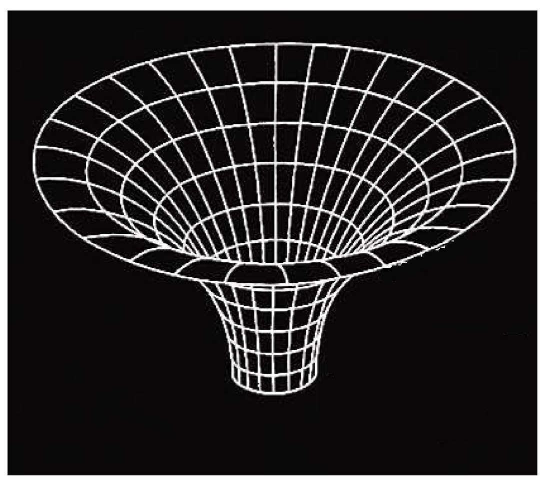
BLACK HOLES THERMODYNAMICS : INTRODUCTION

- Spacetime symmetries and Einstein field equations $\rightarrow$ Schwarzschild metric (spherical, static)
  $\rightarrow$ can be generalized to n extra-dimensions : $ds^2 = \left(1 - \left(\frac{r_H}{r}\right)^{n+1}\right) dt^2 - dr^2 / \left(1 - \left(\frac{r_H}{r}\right)^{n+1}\right) + r^2 d\Omega_{n+2}$ where r_H is the horizon radius



- Couplings between quantum fields and black holes $\rightarrow$ quasi-thermal Hawking radiation + greybody factors :

$$\frac{dN^{(s)}(\omega)}{dt} = \sum_{l,m} \sigma^{(s)}(\omega) \frac{1}{\exp(\frac{\omega}{T_H}) - (-1)^{2s}} \frac{d^{n+3}k}{(2\pi)^{n+3}}$$

- Basic thermodynamics :
  - Generalized second law (GSL) : $\Delta S = \Delta S_{BH} + \Delta S_{rad} \geq 0$
  - Bekenstein-Hawking entropy and temperature : $S_{BH} = \frac{A}{4}$, $T_{BH} = \frac{n+1}{4\pi r_H} \rightarrow T \propto M^{-\frac{1}{n+1}} \rightarrow$ calorific capacity $C = \frac{\delta M}{\delta T} < 0$

THE GENERALIZED SECOND LAW IN A WORLD WITH EXTRA-DIMENSIONS

The GSL requires : $R = \frac{\Delta S_{rad}}{|\Delta S_H|} \geq 1$

- The black hole - blackbody approximation :
  - particles in the brane : $dS_{rad} \simeq \frac{4}{3} \frac{1}{T} dE$ and $dS_{BH} = \frac{dA}{4}$
  - particles in the bulk : $dS_{rad} \simeq \frac{n+4}{n+3} \frac{1}{T} dE$ and $dS_{BH} = \frac{dA}{4}$

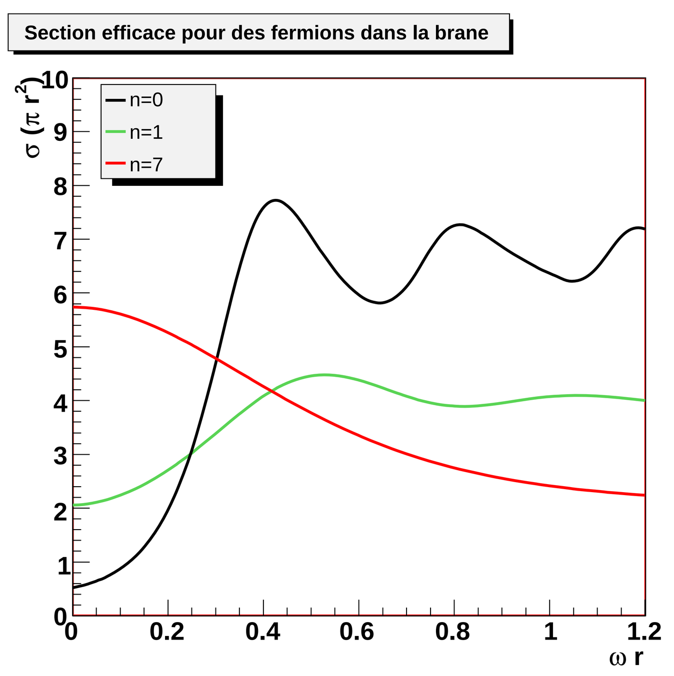
- Exact results : $\frac{dS_{BH}}{dt} = -\frac{1}{T_H}\frac{dM}{dt} = -\frac{1}{T_H} \sum_{\ell,m,s} \int \frac{d\omega}{2\pi} \omega \underbrace{\frac{|A_{\ell,s}|^2}{e^{\omega/T_H} \mp 1}}_{\langle n_{\omega,\ell,m} \rangle}$ and $\frac{dS_{rad}}{dt} = \sum_{\ell,m,s} \int \frac{d\omega}{2\pi} \left[\underbrace{\frac{|A_{\ell,s}|^2}{e^{\omega/T_H} \mp 1} \ln\left(\frac{e^{\omega/T_H} \mp 1}{|A_{\ell,s}|^2} \pm 1\right) \pm \ln\left(1 \pm \frac{|A_{\ell,s}|^2}{e^{\omega/T_H} \mp 1}\right)}_{\langle S_{\omega,\ell,m} \rangle} \right]$

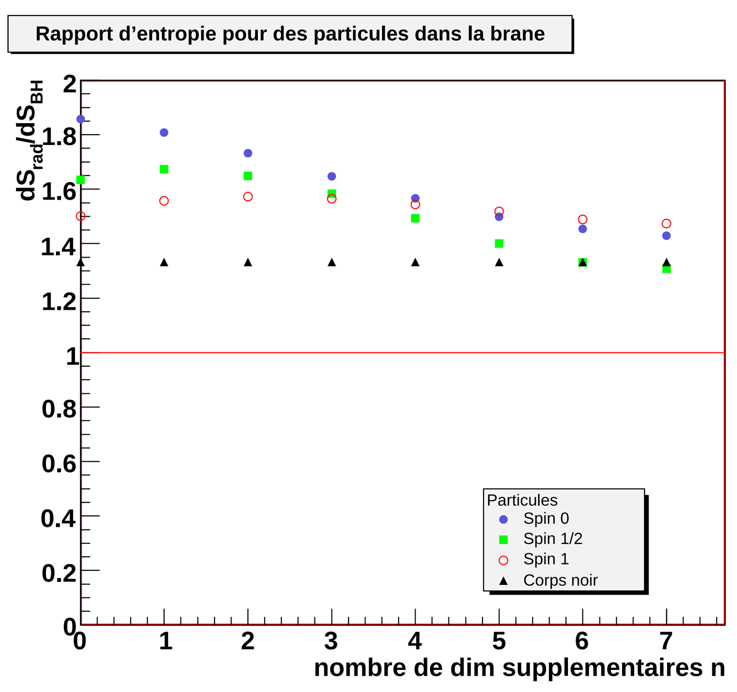
FIELDS ON THE BRANE

- "black hole = blackbody" approximation : $R = \frac{4}{3}$.

The entropy ratio is **independent** of the number n of extra dimensions.

- Exact results :



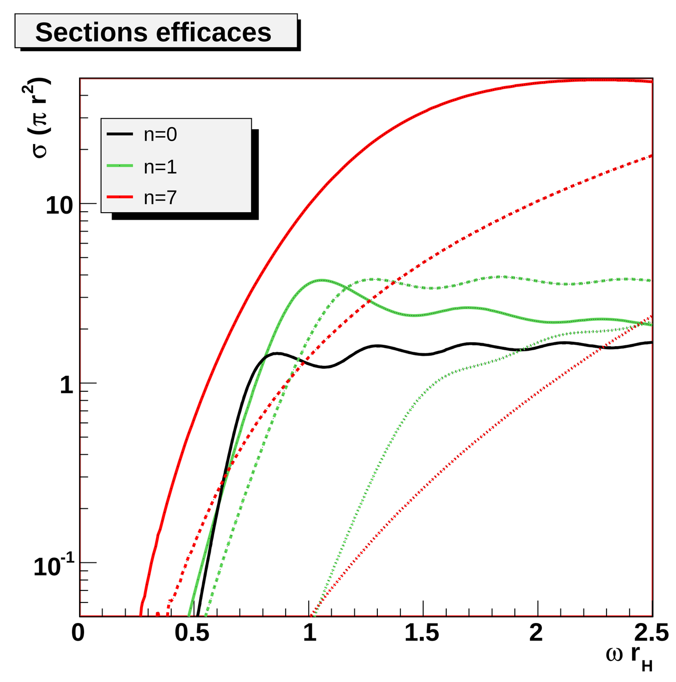


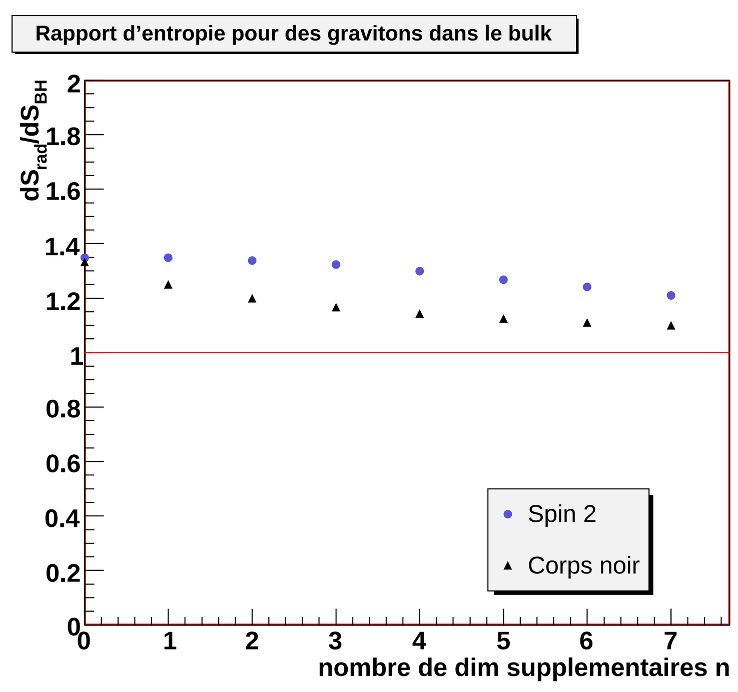
FIELDS IN THE BULK

- ="black hole = blackbody" approximation : $R = \frac{n+4}{n+3}$.

The entropy ratio **depends** on the number n of extra dimensions.

- Exact results:





ENTROPY EVAPORATED AND RELATED *gedenken* EXPERIMENTS

- The generalized second law is respected in the braneworld (ADD scenario).

- By solving the master equation (generalized Dirac equation in a curved background) :

$$\left[\Delta^s \frac{d}{dr}\left(\Delta^{1-s}\frac{d}{dr}\right)\left(\frac{\omega^2 r^2}{h} + 2is\omega r - \frac{is\omega r^2}{h}\frac{dh}{dr} - \lambda\right)\right] P_s = 0$$

where $\Delta = r^2 h(r)$ and $P_s(r) = \Delta^s R_s(r)$ and $\lambda = j(j+1) - s(s-1)$, the greybody factors can be exactly computed. This allows for complete determination if the evaporated entropy as a function of the number of degrees of freedom (scalar fields, fermions, gauge bosons and gravitons).

In, respectively, a 4-dimensional and 11-dimensional spacetimes, this reads as :

$dS_{rad}/dt(10^{-3} r_H^{-1}) = 6.948 n_0 + 3.368 n_{1/2} + 1.269 n_1 + 0.260 n_2$ and $dS_{BH}/dt(10^{-3} r_H^{-1}) = 3.740 n_0 + 2.060 n_{1/2} + 0.846 n_1 + 0.193 n_2$

$dS_{rad}/dt(10^{-3} r_H^{-1}) = 932.3 n_0 + 846.0 n_{1/2} + 950.7 n_1 + 28386 n_2$ and $dS_{BH}/dt(10^{-3} r_H^{-1}) = 661.3 n_0 + 646.5 n_{1/2} + 645.0 n_1 + 23466 n_2$

- By considering a black hole in thermal equilibrium with a surrounding heat bath, one can prove the inverse Bekenstein conjecture.