New Approaches to ElectroWeak Symmetry Breaking

Part Two

Higgs as a (Pseudo) Golstone Boson: Little Higgs Theories.
Higgs as a component of a Gauge Field: Gauge Higgs Unification.

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How to Stabilize the Higgs Potential

**Goldstone’s Theorem**
- Spontaneously broken global symmetry $\rightarrow$ Massless scalar
- ...But the Higgs has sizable non-derivative couplings

**The spin trick**
- A particle of spin $s$: $2s+1$ polarization states
  - ...With the only exception of a particle moving at the speed of light
  - ...Fewer polarization states
- Spin 1: Gauge invariance $\rightarrow$ No longitudinal polarization
- Spin 1/2: Chiral symmetry $\rightarrow$ Only one helicity
- ...But the Higgs is a spin 0 particle
- $m=0$
Symmetries to Stabilize a Scalar Potential

Supersymmetry

Higher Dimensional Lorentz invariance

\[ A_\mu \sim A_5 \]

These symmetries cannot be exact symmetry of the Nature. They have to be broken. We want to look for a soft breaking in order to preserve the stabilization of the weak scale.
Little Higgs Theories
Goldstone Boson

\[ \phi \rightarrow e^{i\alpha} \phi \]

\[ \mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - \lambda \left( |\phi|^2 - \frac{f^2}{2} \right)^2 \]

\[ \phi = \frac{1}{\sqrt{2}} \left( f + h(x) \right) e^{i\theta(x)/f} \]

\[ h \rightarrow h \]

\[ \theta \rightarrow \theta + \alpha f \]

\[ \mathcal{L} = \frac{1}{2} \partial_\mu h \partial^\mu h + \frac{1}{2} \left( \frac{f + h}{f} \right)^2 \partial_\mu \theta \partial^\mu \theta - \lambda \left( f^2 h^2 + f h^3 + \frac{1}{4} h^4 \right) \]

If the U(1) symmetry is gauged, the Goldstone boson is eaten and it becomes the longitudinal component of the massive gauge boson.
Example of Uneaten Goldstone Bosons

\[ SU(N) \rightarrow SU(N - 1) \]

\[ (N^2 - 1) - ((N - 1)^2 - 1) = 2N - 1 \quad \text{Goldstone bosons} \]

Let us assume that only SU(N-1) is gauged: then the Goldstone are uneaten.

\[ \phi = e^{i\pi} \phi_0 \]

\( (N-1) \) complex, \( \vec{\pi} \), and 1 real, \( \pi_0 \), scalars

SU(N-1)

\[ \phi \rightarrow U_{N-1} \phi = U_{N-1} e^{i\pi} U_{N-1}^\dagger U_{N-1} \phi_0 = e^{iU_{N-1} \pi U_{N-1}^\dagger} \phi_0 \]

linear transformations

\[ \pi \rightarrow \left( \begin{array}{c} U_{N-1} \pi_0 \\ \pi \end{array} \right) \left( \begin{array}{c} \pi_0^+ \\ \pi_0 \end{array} \right) \left( \begin{array}{c} U_{N-1}^\dagger \pi_0 \\ \pi^\dagger \pi_0 \end{array} \right) = \left( \begin{array}{c} \pi_0 \\ \pi^\dagger U_{N-1}^\dagger \pi_0 \end{array} \right) \]

non-linear transformations

SU(N)

\[ \frac{SU(N)}{SU(N - 1)} \]

\[ \phi \rightarrow \exp \left( i \left( \begin{array}{c} \vec{\alpha} \\ \vec{\alpha}^\dagger \end{array} \right) \right) \exp \left( i \left( \begin{array}{c} \vec{\pi} \\ \vec{\pi}^\dagger \end{array} \right) \right) \phi_0 \approx \exp \left( i \left( \begin{array}{c} \vec{\pi} + \vec{\alpha} \\ \vec{\pi}^\dagger + \vec{\alpha} \end{array} \right) \right) \phi_0 \]
The Desired Properties of a Little Higgs

- Unbroken SU(2) x U(1) gauge symmetry
- At least $2\frac{1}{2}$ doublet as a Goldstone boson
- Break the G/H shift symmetry that protects the Higgs potential in a way to allow a quartic coupling and a mass term, as well as Yukawa interactions
- to guarantee the absence of quadratic divergences at one loop

Implement the idea of Higgs = (Pseudo)-Goldstone boson

Kaplan, Georgi ‘84
Kaplan, Georgi, Dimopoulos ‘84
Collective Breaking

The global symmetry is explicitly broken but only “collectively”! The symmetry is broken when two or more couplings in the Lagrangian are non-vanishing.

Setting any one of these couplings to zero restores the symmetry and therefore the masslessness of the Little Higgs.

\[
\left[\frac{SU(3)}{SU(2)}\right]^2
\]

\(2\times(8-3)=10\) Goldstone

Let us now gauge SU(3)

\[
\mathcal{L} = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2
\]

\(D_\mu \phi_i = \partial_\mu \phi_i - igA_\mu \phi_i\)

\(SU(3)_V \rightarrow SU(2)_V\) vev

the 5 Goldstone are eaten

\[
\phi_i \rightarrow U_i \phi_i
\]

\(\phi_1\) with \(\langle \phi_1 \rangle = f \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\) and \(\phi_2\) with \(\langle \phi_2 \rangle = f \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\)

\[
L = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2
\]

\(\phi \rightarrow U \phi\) and \(A_\mu \rightarrow UA_\mu U^\dagger\)

requires \(U_1 = U_2\)

Break \(SU(3)_1 - SU(3)_2\). Gauge \(SU(3)_1 + SU(3)_2\).

in presence of the gauge coupling, only 5 Goldstone

The 5 extra would-be Goldstone boson acquire a mass through the gauge coupling.
Collective Breaking

$[SU(3)/SU(2)]^2$

$2 \times (8-3) = 10$ Goldstone

Let us now gauge $SU(3)$

$L = |D_\mu \phi_1|^2 + |D_\mu \phi_2|^2$

$D_\mu \phi_i = \partial_\mu \phi_i - igA_\mu \phi_i$

$SU(3)_V \rightarrow SU(2)_V$

the 5 Goldstone are eaten

$\phi_1$ with $\langle \phi_1 \rangle = f \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ and $\phi_2$ with $\langle \phi_2 \rangle = f \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$\phi_i \rightarrow U_i \phi_i$

requires $U_1 = U_2$

Break $SU(3)_1 - SU(3)_2$. Gauge $SU(3)_1 + SU(3)_2$. in presence of the gauge coupling, only 5 Goldstone

The 5 extra would-be Goldstone boson acquire a mass through the gauge coupling.

As soon as the coupling of either $\phi_1$ to $A_\mu$ or $\phi_2$ to $A_\mu$ is set to zero, enlarged global symmetry and 5 Goldstone in the physical spectrum.

If a diagram involves only one type of couplings, it cannot generate a mass term for the pseudo-Goldstone bosons

There is no quadratically divergent diagrams at one loop.
Absence of $\Lambda^2$ divergent Diagrams

$\Lambda^2$

SU(3)

$\phi_i^\dagger \phi_i$

$\delta \mathcal{L} = \frac{g^2 \Lambda^2}{16\pi^2} (\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2) = \frac{g^2 f^2 \Lambda^2}{8\pi^2}$

independent of the Higgs field!

$\log (\Lambda^2)$

$\phi_2^\dagger \phi_2$

$\phi_1^\dagger \phi_1$

$\delta \mathcal{L} = \frac{g^4 f^2}{16\pi^2} |\phi_1^\dagger \phi_2| \log \frac{\Lambda^2}{Q^2} = \frac{g^4 f^2}{4\pi^2} \log \frac{\Lambda^2}{Q^2} h^\dagger h$

log-divergent one-loop suppressed Higgs mass
Cancelation Mechanism of $\Lambda^2$

$$
\phi_1 = \exp \left( i \left( h^\dagger h \right) \right) \left( f \right) \approx f \left( 1 - \frac{ih}{f} \right) \left( 1 - \frac{h^\dagger h}{2f^2} \right)
$$

$$
\phi_2 = \exp \left( -i \left( h^\dagger h \right) \right) \left( f \right) \approx f \left( 1 - \frac{-ih}{f} \right) \left( 1 - \frac{h^\dagger h}{2f^2} \right)
$$

$$
\mathcal{M}^2 = g^2 f^2 \begin{pmatrix}
1 & 1 & \frac{v^2}{f^2} \\
1 - \frac{v^2}{f^2} & 1 - \frac{v^2}{f^2} & \frac{v^2}{f^2} \\
\frac{v^2}{f^2} & \frac{v^2}{f^2} & \frac{4}{3} - \frac{v^2}{f^2} - \frac{v^2}{\sqrt{3}f^2}
\end{pmatrix}
$$

$\text{tr}\mathcal{M}^2$ is independent of $v^2 \iff$ no quadratic divergence

$W'$ cancels the quadratic divergences generated by $W$
How to Generate a Yukawa Coupling

\[ q = \begin{pmatrix} t_L \\ b_L \end{pmatrix} \equiv 2 \text{ of } SU(2) \quad \Rightarrow \quad Q = \begin{pmatrix} t_L \\ b_L \\ T_L \end{pmatrix} \equiv 3 \text{ of } SU(3) + T_R \]

\[ \mathcal{L} = \frac{y}{\sqrt{2}} \phi_1^\dagger Q t_R + \frac{y}{\sqrt{2}} \phi_2^\dagger Q T_R \]

\[ \phi_1 \approx f \left( \frac{i h/f}{1 - (h^\dagger h)/(2f^2)} \right) \]

\[ \phi_2 \approx f \left( \frac{-i h/f}{1 - (h^\dagger h)/(2f^2)} \right) \]

\[ T_+ = \frac{1}{\sqrt{2}}(T_R + t_R) \quad T_- = \frac{i}{\sqrt{2}}(T_R - t_R) \]

\[ m_t^2 = y^2 v^2 \quad m_T^2 = y^2 f^2 \left( 1 - \frac{v^2}{f^2} \right) \]

tr\( M^2 \) is independent of \( v^2 \) \( \Leftrightarrow \) no quadratic divergence

heavy top cancels the quadratic divergences generated by top
Collective Breaking in the Top Sector

\[ \mathcal{L} = \frac{y}{\sqrt{2}} \phi_1^\dagger Q t_R + \frac{y}{\sqrt{2}} \phi_2^\dagger Q T_R \]

Without Yukawa \( \phi_i \rightarrow U_i \phi_i \) \( SU(3)^2 \) Global Symmetry

When both Yukawa are turned on \( \phi_i \rightarrow U \phi_i \) and \( Q \rightarrow U Q \)

we need both Yukawa to align the two SU(3)

more than one coupling is required to break enough global symmetry to let the Higgs acquire a potential

no quadratically divergent diagram at one-loop!

\[ \int d^4 k \frac{1}{(k_\mu \gamma^\mu - m)^4} \propto \log \Lambda^2 \]

log-divergent contribution only
Exercises

1. Bottom Yukawa:

Check that the bottom mass is generated by

\[ y \phi_1 \phi_2 Q b_R \]

2. Hypercharge:

Show that you can introduce the hypercharge by simply extending the gauge symmetry to \( SU(3) \times U(1)_X \)

hint: first determine the \( X \) charge of the two scalar triplets.
EWSB in Little Higgs

At tree-level, no Higgs potential
(well, a quartic coupling can be generated by integrating out another heavy Goldstone)

A quartic and a mass term are generated at one-loop.

As in susy, the top loop will generate a log-divergent tachyonic mass term
that will trigger EWSB.

Radiatively induced EWSB

(that’s the dynamics of the model that drives the symmetry breaking, it is not arbitrary like in the SM)

What we want

\[ v^2 = \frac{-m^2}{2\lambda} \]

- large quartic (larger than \(v/f\)) in order to get
- a physical Higgs mass of order \(v\) and not \(v^2/f\).

- small mass term (of order \(v\) and not \(f\))

Might be difficult

\[ \phi_1\phi_2 = f^2 \left( 1 - 2 \frac{h^\dagger h}{f^2} + 2 \frac{(h^\dagger h)^2}{3f^4} \right) \]

Would be better to generate a tree-level quartic

\[ v \sim f \]
Little Higgs with Tree Level Quartic

Global symmetry \( \left( SU(3)_L \times SU(3)_R / SU(3)_V \right)^4 \)

Gauge symmetry \( SU(3) \times SU(2) \times U(1) \rightarrow SU(2)_L \times U(1)_y \)

\( v \sim f / 4\pi \)

32 GB

8 eaten

24 PGB

Will generate a quartic coupling for the light PGB

Minimal moose model
Arkani-Hamed, Cohen, Katz, Nelson, Gregoire, Wacker '02

two complex doublets
1 complex triplet
1 complex singlet

f

not protected by collective breaking

\( v \sim f / 4\pi \)

two complex doublets
1 complex triplet
1 complex singlet

Minimal moose model
Arkani-Hamed, Cohen, Katz, Nelson, Gregoire, Wacker '02
The new particles @ a TeV will affect the EW precision observables

Bounds from precision data on the scale $f$ (in TeV)
for various Little Higgs models.

$f \geq 3-4$ TeV instead of $f \sim 1$ TeV

Marandella, Schappacher, Strumia ‘05
Custodial Symmetry and T-parity

To improve EW fits, two useful tools:

- **little Higgs with custodial symmetry**
  - Chang, Wacker ‘03
  - based on minimal moose with SO(5)...

- **T-parity**
  - Cheng, Low ‘03

  heavy particles = odd
  light particles = even

  at each vertex, an even number of heavy fields are required

  no effective operator for light fields are generated
  by tree-level exchange of heavy fields

  tree-level exchange forbidden

  loop exchange allowed
Fine-Tuning in Little Higgs

![Graph showing fine-tuning parameter as a function of Higgs boson mass for different scenarios: MSSM, Littlest, Littlest 2, T-parity, Simplest, and Standard Model (SM)].

\[ \Delta \]

\[ m_h (\text{GeV}) \]

Casas, Espinosa, Hidalgo '05
little Higgs models require a heavy top and heavy gauge bosons
to guarantee the cancellation of the $\Lambda^2$ divergences, the couplings
of the new gauge bosons to the SM fermions is fixed

Partial width of $Z_H$ to fermions
is proportional to $\cot\theta^2$

Partial width of $Z_H$ into boson pairs
($Z_H$ and $W^+W^-$)
is proportional to $\cot\theta^2$ 2$\theta$
(this follows from the particular coupling of the Higgs to the two SU(2) gauge groups)

cot$\theta$=$g_1/g_2$ (two SU(2) gauge couplings)
Gauge-Higgs Unification
Higher Dimensional Lorentz invariance

\[ A_\mu \sim A_5 \]

4D spin 1 \hspace{1cm} 4D spin 0

These symmetries cannot be exact symmetry of the Nature. They have to be broken. We want to look for a soft breaking in order to preserve the stabilization of the weak scale.
How to Get a Doublet from an Adjoint

Consider a 5D gauge symmetry $G$

$H \sim A^a_5$ will belong to the adjoint rep. of $G$

The SM Higgs is not an adjoint of $SU(2) \times U(1)$, it is a doublet!

Consider a bigger gauge group

$G \rightarrow SU(2)_l \times U(1)_y$

$\text{Adj} \rightarrow \text{doublet} + \text{other rep.}$

\[
\frac{1}{2} \begin{pmatrix}
W_3 + W_8/\sqrt{3} & W_1 - iW_2 \\
W_1 + iW_2 & -W_3 + W_8/\sqrt{3} \\
W_4 + iW_5 & W_6 + iW_7 \\
W_4 + iW_5 & W_6 + iW_7
\end{pmatrix}
\]

$SU(2) \times U(1)$:

$\begin{pmatrix} \text{Adj} \\ 2/\sqrt{3}/2 \end{pmatrix}$
U(1) Charge of the Doublet

$$\delta_T W = g [T, W]$$

$$T = \frac{1}{2\sqrt{3}} \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}$$

$$h_+ = W_4 - iW_5$$

$$h_0 = W_6 - iW_7$$

$$\delta_T W = g \frac{1}{2\sqrt{3}} \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} h_+^* & h_0^* \\ h_+ & h_0 \end{pmatrix} - g \frac{1}{2\sqrt{3}} \begin{pmatrix} h_+^* & h_0^* \\ h_+ & h_0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}$$

$$= g \frac{1}{2\sqrt{3}} \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} h_+^* & h_0^* \\ -2h_+ & -2h_0 \end{pmatrix} - g \frac{1}{2\sqrt{3}} \begin{pmatrix} h_+^* & h_0^* \\ -2h_+ & -2h_0 \end{pmatrix}$$

$$= g \frac{3}{2\sqrt{3}} \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} h_+^* & h_0^* \\ -h_+ & -h_0 \end{pmatrix}$$

$$\text{U(1) charge of the doublet} = \frac{3}{2\sqrt{3}}$$
Weak Mixing Angle

\[
\delta_{U(1)} \left( \begin{array}{c} h_+ \\ h_0 \end{array} \right) = \frac{3g}{2\sqrt{3}} \left( \begin{array}{c} h_+ \\ h_0 \end{array} \right) = \frac{\sqrt{3}g}{2} \left( \begin{array}{c} h_+ \\ h_0 \end{array} \right)
\]

Proper U(1) normalization
(such that the doublet has a hypercharge 1/2)

\[
g' = \sqrt{3}g
\]

\[
U(1)_y = T_8/\sqrt{3}
\]

\[
\sin^2 \theta_W = \frac{g'^2}{g^2 + g'^2} = \frac{3g^2}{g^2 + 3g^2} = \frac{3}{4}
\]

By embedding SU(2)xU(1) into a simple group, we got a prediction for the weak mixing angle

experimentally: \( \sin^2 \theta_W \approx 0.23 \)
How to Get a Good Weak Mixing Angle

Repeat the previous construction with another gauge group

\( G_2 \) gauge group

adjoint = 14

\[ 14 = 8 + 3 + \bar{3} \]

fundamental = 7

\[ 7 = 3 + \bar{3} + 1 \]

**SU(3) decomposition**

\[ 14 = 8 + 3 + \bar{3} \]

\[ 7 = 3 + \bar{3} + 1 \]

**SU(2) x U(1) decomposition**

\[ T_8 \text{ normalization} \]

\[ 14 = \left( 3_0 + \left( 2 + \bar{2} \right) \sqrt{3}/2 + 1_0 \right) + \left( 2 + \bar{2} \right)_{1/2 \sqrt{3}} + \left( 1 + \bar{1} \right)_{-1/\sqrt{3}} \]

\[ 7 = \left( 2 + \bar{2} \right)_{1/2 \sqrt{3}} + \left( 1 + \bar{1} \right)_{-1/\sqrt{3}} + 1_0 \]

\[ U(1)_y \text{ normalization} \]

\[ 14 = \left( 3_0 + \left( 2 + \bar{2} \right) 3/2 + 1_0 \right) + \left( 2 + \bar{2} \right)_{1/2} + \left( 1 + \bar{1} \right)_{-1} \]

\[ 7 = \left( 2 + \bar{2} \right)_{1/2} + \left( 1 + \bar{1} \right)_{-1} + 1_0 \]

\[ U(1)_y = \sqrt{3} T_8 \]

\[ \sin^2 \theta_W = 1/4 \]
G2 Gymnastic

fundamental rep.
14 7x7 matrices

\[ T^a = \frac{1}{2\sqrt{2}} \begin{pmatrix} \lambda^a & -\lambda^{a\dagger} \\ \lambda^{a\dagger} & \lambda^a \end{pmatrix} \quad \text{for} \quad a = 1 \ldots 8 \]

\[ T^9 = \frac{i}{2\sqrt{3}} \begin{pmatrix} \lambda^7 \\ v^1 \end{pmatrix} \quad T^{10} = -\frac{i}{2\sqrt{3}} \begin{pmatrix} \lambda^5 \\ v^{2\dagger} \end{pmatrix} \quad T^{11} = \frac{i}{2\sqrt{3}} \begin{pmatrix} \lambda^2 \\ v^{3\dagger} \end{pmatrix} \]

\[ T^{12} = T^9\dagger \quad T^{13} = T^{10\dagger} \quad T^{14} = T^{11\dagger}, \]

with

\[ v^1 = \begin{pmatrix} -i\sqrt{2} \\ 0 \\ 0 \end{pmatrix} \quad v^2 = \begin{pmatrix} 0 \\ i\sqrt{2} \\ 0 \end{pmatrix} \quad v^3 = \begin{pmatrix} 0 \\ 0 \\ -i\sqrt{2} \end{pmatrix}. \]
6D for a Tree-level Quartic Coupling

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} F_{\mu 5}^2 + \frac{1}{2} F_{\mu 6}^2 - \frac{1}{2} F_{56}^2 \]

\[ F_{56} = \partial_5 A_6 - \partial_6 A_5 - g[A_5, A_6] \]

\[ A_5 = H + \ldots \quad A_6 = H + \ldots \]

\[ \mathcal{L} = -g^2 H^4 \]

the 6D gauge kinetic term contains a Higgs quartic coupling

and

\[ \lambda \equiv g^2 \]
Explicit Tree-level Quartic

\[
A_5 = \begin{pmatrix}
\frac{1}{2} h_+ \\
\frac{1}{2} h_0 \\
\frac{1}{2} h_+^* \\
\frac{1}{2} h_0^*
\end{pmatrix},
A_6 = \begin{pmatrix}
\frac{-i}{2} h_+ \\
\frac{-i}{2} h_0 \\
\frac{i}{2} h_+^* \\
\frac{i}{2} h_0^*
\end{pmatrix}
\]

SU(3) model

\[
A_\mu = \frac{1}{2} \begin{pmatrix}
A_3^\mu + \frac{1}{3} A_8^\mu & A_1^\mu - i A_2^\mu \\
A_1^\mu + i A_2^\mu & -A_3^\mu + \frac{1}{3} A_8^\mu \\
& -\frac{2}{\sqrt{3}} A_8^\mu
\end{pmatrix}
\]

\[
\mathcal{L} = -\frac{1}{2} \text{Tr} F_{\mu \nu}^2 + \text{Tr} F_{\mu 5}^2 + \text{Tr} F_{\mu 6}^2 - \text{Tr} F_{56}^2
\]

\[
\mathcal{L} = D_\mu H^\dagger D_\mu H - \frac{g^2}{2} (H^\dagger H)^2
\]
Towards a Complete Construction

so far we haven’t broken any symmetry... we even enlarged the gauge group

We need to break $G$ down to $SU(2) \times U(1)$

we can achieve this breaking while compactifying the extra-dimension

**Orbifold Compactification Breaking**

circle $y \sim y + 2\pi R$

orbifold $y \sim -y$

- signs are compensating

zero mode: $A_\mu$ is independent of $y$

$$A_\mu(-y) = U A_\mu(y) U^\dagger$$
$$A_5(-y) = -U A_5(y) U^\dagger$$

$$A_\mu = U A_\mu U^\dagger$$
$$A_5 = -U A_5 U^\dagger$$

gauge symmetry breaking + chiral fermions
Orbifold Projection as Boundary Conditions

\[ G \rightarrow H \] by orbifold projection

**H subgroup**

\[ A^H_\mu (-y) = A^H_\mu (y) \]

\[ A^H_5 (-y) = -A^H_5 (y) \]

which is equivalent to the BCs at the fixed points

\[ \partial_5 A^H_\mu = 0 \]

\[ A^H_5 = 0 \]

**G/H coset**

\[ A^{G/H}_\mu (-y) = -A^{G/H}_\mu (y) \]

\[ A^{G/H}_5 (-y) = A^{G/H}_5 (y) \]

which is equivalent to the BCs at the fixed points

\[ A^{G/H}_\mu = 0 \]

\[ \partial_5 A^{G/H}_5 = 0 \]
SU(3) → SU(2) x U(1) 5D Orbifold Breaking

\[ U = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad U \in SU(3) \quad U^2 = 1 \]

massless \( A_\mu \)

\[ [A_\mu, U] = 0 \quad A_\mu = \frac{1}{2} \begin{pmatrix} A_\mu^3 + A_\mu^8/\sqrt{3} & A_\mu^1 - iA_\mu^2 \\ A_\mu^1 + iA_\mu^2 & -A_\mu^3 + A_\mu^8/\sqrt{3} \end{pmatrix} \]

massless \( A_5 \)

\[ \{A_5, U\} = 0 \quad A_5 = \frac{1}{2} \begin{pmatrix} A_5^4 - iA_5^6 \\ A_5^6 - iA_5^7 \end{pmatrix} \]
6D Orbifold Breaking

$$T^2/Z^4 \text{ orbifold}$$

$$(y, z) \sim (-z, y)$$

$$A_\mu(-z, y) = U A_\mu(y, z) U^\dagger$$

$$A_y(-z, y) = -U A_z(y, z) U^\dagger$$

$$A_z(-z, y) = U A_y(y, z) U^\dagger$$

$$U = \begin{pmatrix} i & i \\ i & -1 \end{pmatrix}$$

$$U^4 = 1$$

$$A_y = \begin{pmatrix} \frac{1}{2} h_+ \\ \frac{1}{2} h_0 \end{pmatrix}$$

$$A_z = \begin{pmatrix} \frac{-i}{2} h_+ \\ \frac{-i}{2} h_0 \end{pmatrix}$$
Residual Symmetry

\[ \delta A^A_M = \partial_M \epsilon^A + g f^{ABC} A^B_\mu \epsilon^C \]

Gersdorff, Irges, Quiros ‘02

In the bulk

higher dimensional gauge invariance forbids a mass term for \( A_5 \)

At the fixed points

\[ G \rightarrow H \]

\[ \begin{align*}
\partial_5 A^H_\mu &= 0 \\
A^H_5 &= 0 \\
\partial_5 \epsilon^H &= 0 \\
A_\mu^{G/H} &= 0 \\
\partial_5 A^{G/H}_5 &= 0 \\
\epsilon^{G/H} &= 0
\end{align*} \]

(since \([H, H] \subset H\) \([H, G/H] \subset G/H\), the only non-vanishing structure constants are \( f^{HHH}, f^{G/H G/H H} \))

\[ \delta A^H_\mu = \partial_\mu \epsilon^H + g f^{HHH} A^H_\mu \epsilon^H \]

\[ \delta A^H_5 = 0 \]

\[ \delta A^{G/H}_\mu = 0 \]

\[ \delta A^{G/H}_5 = \partial_5 \epsilon^{G/H} + g f^{G/H G/H H} A^{G/H}_5 \epsilon^H \]

\[ \text{G/H acts non-linearly on the Higgs at the fixed points.} \]

\[ \text{No local mass term allowed} \]
Finite Mass from Wilson Line

there is no local counter term that will give a mass to the Higgs

the mass term can only be generated non locally

the mass term will be finite then

$$W_n = e^{i \int_0^{2n\pi R} A_5 dy}$$

Wilson’s line. Gauge invariant object.

The radiative corrections will generate a (finite) effective potential

$$V_{eff}(A_5) = f(W_n)$$

controlled by finite size effects

UV insensitive
**Tadpole Operator**

the situation complicates in 6D

$F_{56}$ contains a mass term for the Higgs

$\text{Tr} (U F_{56})$ is a local invariant operator at a fixed point

$\text{Tr} (U F_{56}(0)) \rightarrow \text{Tr} (U g(0) F_{56}(0) g^{-1}(0)) = \text{Tr} (U F_{56}(0) g^{-1}(0) g(0)) = \text{Tr} (U F_{56}(0))$

at the fixed points, $U$ and $g$ commute

Unless forbidden by a discrete symmetry, we expect this operator to be generated at one loop

For $Z_2$ or $Z_2 \times Z_2$ orbifolds can define a parity that forbids the tadpole

For $Z_4$ orbifold, no such symmetry
a $\Lambda^2$ tadpole is generated at one loop
Fermion Masses

4D chiral matter

Non local couplings: 

\[ W = \mathcal{P} e^{i \int A_i dx^i} \]

localize fermions at the fixed points

\[ \Lambda^2 \text{ divergences} \]

forbidden by G/H shift symmetry

Direct coupling:

Higgs Coupling

Fermion Masses

U(1)_y fractional charges

Extra Massive Fermion in the Bulk

\[ \int d^5 x \left( \bar{\psi} (i \sigma^\mu \partial_\mu - m) \psi + \delta(y_1) \psi \chi + \delta(y_2) \psi \xi \right) \]

\[ \int d^4 x e^{-m|y_1 - y_2|} \xi(y_2) e^{i \int A_i dx^i} \chi(y_1) \]

integrating out massive fermion

Froggatt-Nielsen mass suppression
Experimental Signatures

the collider signals haven't been studied in details
(lack of fully realistic models?)

the main predictions of these models are

- KK excitations of $W,Z$ around 500 GeV ~ 1 TeV
- KK excitations of $G/H$: gauge bosons with the quantum numbers of the Higgs doublets
- extra scalar fields
- extra fermions (to cancel the top loop quadratic divergence)
Open Issues

In 6D models

- generate a large Higgs quartic
- good prediction for the weak mixing angle
- enlarge the gauge group?
- start with SU(3) and add kinetic terms at the fixed points to change the prediction of the weak mixing angle?

In 5D models

- generate a large Higgs quartic coupling at one-loop
- add large boundary kinetic terms
- add appropriate matter fields in the bulk
- warped space

Seems to be a promising way to proceed.
Weakly coupled dual of composite Higgs models

See Agashe, Contino, Pomarol ‘04-’05